CLNS-131
November 1970
September 1973

(Preliminary Version)

Some Experiments on Multiple Production*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University,
Ithaca, New York 14850

A program of experiments is described mainly on secondary particle spectra to test scaling hypotheses derived from the multiperipheral model. It is assumed that diffraction dissociation and multiperipheral processes are distinct effects, and the consequences of this for the scaling laws are explained. Feynman's analogy linking multiple production to the statistical mechanical distribution functions of a gas is outlined, and based on this analogy it is suggested that one look for a correlation length in the two particle spectrum of secondaries.

*Supported in part by the National Science Foundation.

High energy scattering cross sections of pions and protons show a preponderance of multiple production processes. For example, the mean multiplicity of charged secondaries is about 4 for 30 GeV p-p interactions, and cosmic ray data suggest a mean multiplicity of order 10 at much higher energies.[1] Multiple production will be a dominant effect at the NAL accelerator and CERN storage rings, and there is currently much interest in developing an experimental program to study it.

Actually, multiple production is already dominant at the energies of the present Brookhaven and CERN accelerators, but experimental studies of it have been sporadic and always subordinate to the study of elastic and quasi-elastic cross sections.[2] A major problem preventing more systematic study of inelastic events has been the question of what variables to observe in highly multiple events. A program to measure the complete differential cross sections for a process with, for example, 5 secondary particles is prohibitive in both time and expense; one must select particular aspects of the cross section to measure.

In the case of elastic or quasi-elastic scattering, one must also be selective in any given experiment. One looks at a limited range of incident energy and scattering angle, and one selects particular final states. Usually the choice is dictated by a desire to test a particular theoretical model: a Regge pole fit, an optical model calculation, etc. However as the total number of experiments becomes large one collects a reservoir of data on elastic and quasi-elastic scattering covering most of the range of energies, angles and final states accessible to experiment; that is the total range covered by all data from all experiments is more limited by experimental than by theoretical considerations. The importance of this is that a future theory of strong interactions is likely to find its best test in different ranges of energy, angle, etc., than any particular model now under consideration so one would prefer not to have the range of data collected in the sum total of high energy experiments be limited by currently popular model theories.

It is out of the question to use the same approach in experiments on multiple production; because of the number of variables involved even the sum total of all multiple production experiments will provide only a small fraction of the data that is experimentally accessible. Therefore it is important to think about the choice of experiments to be performed, in particular to try to maximize the possibility that the experimental data collected will remain useful despite continuing changes in theoretical fashions.

The purpose of this paper is to propose an experimental program for studying multiple production, which takes into account the problems mentioned above. The program is a set of specific and feasible experiments, whose immediate aim is to test some currently popular theoretical models.[3] However it will be argued that the results of these experiments will help to characterize the general features of multiple production independently of any model. As a basis for setting up the experiments and predicting their outcome, it will be supposed that there are three types of multiple production processes, namely: a) multiperipheral events, b) diffraction and diffraction dissociation, and c) multi-Regge exchange events.[4] We shall give a simple but qualitative definition for each of the three types of processes; these definitions will not involve specific models requiring detailed calculation and parameter fits in order to compare with experimental data. The experiments to be proposed will test whether multiple production processes can be separated into these three categories.

Before discussing these ideas in detail, some general comments will be made about the purposes for doing high energy experiments.

There does not exist a real theory of strong interactions at present, and the models of high energy processes one studies at present are no substitute for such a theory. It is the ultimate aim of experiment and theory to try to obtain a real theory, so it is worth considering how particular experiments will affect the finding and testing of such a theory. First, though, one must say what one means by a "real theory of strong interactions". My view is that there are four essential requirements for a real theory:

A. It must be derived from a few fundamental principles comprehensible to both experimentalists and theorists.

B. Any free parameters in the theory must appear explicitly and obviously as a consequence of the fundamental principles (just as e and $\hbar$ are explicit and obvious in ordinary quantum mechanics, and c is explicit and obvious in relativity) and there should be no arbitrary functions in the theory.

C. The fundamental principles should imply a set of equations containing the fundamental parameters whose solution will describe all aspects of strong interactions including the complete S matrix (even the S matrix for n particles going to m particles for any n and m) and all matrix elements of the weak and electromagnetic currents. For a given set of values of the parameters the equations should have one and only one solution; if this cannot be proven there should at least be plausible physical arguments suggesting it.

D. One should be able to determine qualitative features of the solution of the equations from qualitative features of the equations, or better from qualitative statements of the fundamental principles without using the equations at all.

One has to be an idealist to believe that a theory will be found satisfying these four requirements. So the author is prepared to be flexible but would be very skeptical about any proposed theory that seriously violates any of these requirements.

At present there are two principles (in addition to the principles of quantum mechanics) which one hopes will be a part of a real theory when it is found, namely, locality and Gell-Mann's current commutators. Model-independent tests of these principles are of vital importance. Locality can be tested, at least partially, by checking forward dispersion relations and rigorous bounds on high energy cross sections. However these tests do not involve detailed experiments on multiple production processes, so they will not be discussed further here.[5] One has some ideas on what the fundamental parameters of strong interactions are, namely, the strengths of the $SU(3) \times SU(3)$ breaking terms in the Lagrangian are probably fundamental parameters. It is difficult to learn much about these parameters in a model-independent way from high energy cross sections. We have not even a glimmering of an idea what the equations of strong interactions would be, let alone how to obtain qualitatively or quantitatively the solution of these equations.

It is likely that the equations of strong interactions will be as complicated as the many-body equations of non-relativistic quantum mechanics or classical physics. In consequence it will probably be very difficult to get detailed quantitative solutions of them; the best one can hope for is to determine the basic qualitative features of their solution.

Because of this I think the experimental program on multiple production should be aimed at finding clear-cut qualitative features of these processes rather than trying to have precise numerical data on particular processes just to test a particular model. Also, I think it is more important to understand processes with large cross sections than to study processes with small cross sections. For example, the elastic cross sections at large angles where the cross sections are $\sim 10^{-33}$ or less may well be useless for either finding or testing a real theory because there can be very many competing small effects which would become important in calculating a cross section of this size. Where a cross section is large there is more hope that a few qualitative features of the theory will be sufficient to determine the behavior of the cross section.

There are already known a number of simple properties of high energy cross sections which are good examples of the "clear-cut qualitative features" that one should look for. We know that (at presently accessible large energies) total cross sections are constant (at least roughly). Elastic or quasi-elastic cross sections requiring exchange of internal quantum numbers (isospin, strangeness, etc.) fall with s roughly like $s^{(\alpha-1)}$ where α depends only on what is exchanged and not the particular process. In multiple production processes, we know that the transverse momentum of secondaries is bounded, having a mean value around 300 MeV independent of the incident energies. The probability of finding a secondary with transverse momentum $p_\perp$ much larger than 500 GeV falls rapidly as $p_\perp$ increases, perhaps exponentially. As new energy ranges open up it is important to check that these results continue to hold.

Now the three types of high energy events discussed in this paper will be defined. It will be assumed that total cross sections are strictly constant at high energy (i.e. they do not increase with energy, even logarithmically). First consider diffraction and diffraction dissociation.[6] These are the processes that would be described by single Pomeron exchange, if one can describe these processes by Regge theory. They include elastic and quasi-elastic scattering where no quantum numbers are exchanged. They also include processes in which the incident or target particle (or both) fragment into several particles where the several particles are not just the decay products of an N^* or ρ or etc.[7] It is assumed that the cross sections for diffraction or diffraction dissociation to a fixed final state are constant at high energy. By a "fixed final state", I mean that the fragments of the target particle have fixed momenta in the lab system independent of the incident energy, while the fragments of the incident particle have longitudinal momenta which are fixed fractions of the incident energy (fixed x in the Feynman language) and fixed transverse momenta.

The assumption that total cross sections are constant puts a strong restriction on the nature of diffraction dissociation. Consider the partial cross section σ_n(diff) for producing n secondaries by diffraction dissociation at high energies. Because of the constant total cross section, $\Sigma_n \sigma_n$(diff) must be finite. Since σ_2, σ_3, etc. are fixed with energy, this means σ_n(diff) $\to 0$ as $n \to \infty$. This means that diffraction dissociation will go predominantly to low multiplicity intermediate states. In other words the mean multiplicity in diffraction dissociation must be constant independent of energy. If the average multiplicity of all inelastic events increases indefinitely with energy there must be other processes besides diffraction dissociation.

By the same argument the probability that a pion of longitudinal momentum k_z is emitted as a fragment of the target in diffraction dissociation to a final state of fixed multiplicity must decrease rapidly as k_z increases; to be precise the cross section must fall faster than $1/k_z$, since the integral over k_z must converge.[8] To produce pions of large k_z is possible only if large multiplicities are allowed so that large k_z pions appear simultaneously with low k_z pions rather than in separate events. But by the previous argument large multiplicities are unimportant for diffraction dissociation.

So our picture of diffraction dissociation is that diffraction dissociation cross sections are constant with energy for fixed final states, that the final states resulting from diffraction dissociation are predominantly of low multiplicity, and that final state particles will mostly be of low energy either in the lab system or the projectile system. There will be a tail of events not satisfying these criteria, but it will be assumed that this tail is negligible compared to multiperipheral cross sections.

Secondly, consider double Regge pole exchange. A typical double Regge pole exchange process is shown in Fig. 1: $p + p \rightarrow p + p + \rho^o$ where the ρ^o is at rest in the center-of-mass system, with pions exchanged between the ρ^o and each proton. Double Regge pole exchange is expected to apply when the particles emitted at the intermediate vertex are highly relativistic with respect to both incident particles; in other words when the particle s across each Regge pole is large (s_1 and s_2 of Fig. 2 must be large). For any exchange other than double Pomeron exchange these double exchanges should fall rapidly with energy and hence be a negligible part of the total cross section. If there is double Pomeron exchange these processes are important, and wreck the picture of this paper because with double Pomeron exchange one has a process with a fixed number of particles in the final state where secondaries can come out with any k_z. However, because diffraction may well not be an exchange process at all, and because multiple Pomeron exchange causes theoretical difficulties[9], it will be assumed here that double Pomeron exchange is either non-existent or very small. One of the experiments described later is to test this assumption.

Finally, consider multiperipheral processes. The multiperipheral model of Amati, Fubini, and Stanghellini[10] is at first sight a special model based on multiple pion exchanges which one would not want to take very seriously. But as Amati *et al*. found, the multiperipheral model exhibits simple scaling laws for inelastic processes at high energies[10] (including the scaling law for the single particle spectrum rediscovered 7 years later by Feynman[3]). These scaling laws were shown by Amati *et al*. to be independent of the details of the model, such as the values of coupling constants; one can also have other particle exchanges besides pions. By multiperipheral processes I mean any processes satisfying the scaling laws predicted from the multiperipheral model. The author has stated elsewhere the rules for constructing such scaling laws.[10] The best way to introduce these scaling laws is, I think, to use an analogy invented by Feynman.[11] This analogy links multiparticle production cross sections to the multiparticle distribution functions of a classical gas, with the total cross section becoming the partition function of a gas. This analogy is very much on Feynman's mind when he discusses his parton model of high energy collisions, although it is not discussed in his papers.

In the Feynman gas analogy, a secondary particle with momentum $\underset{\sim}{k}$ and energy k_o corresponds to a gas particle at position $\underset{\sim}{r}$; the components of $\underset{\sim}{r}$ are

$$x = k_x \tag{1}$$

$$y = k_y \tag{2}$$

$$z = \ln[(k_z + k_o)/m_\perp] \tag{3}$$

and
$$m_\perp = (m^2 + k_x^2 + k_y^2)^{1/2} \tag{4}$$

Here m is the mass of the secondary and k_x and k_y are the transverse components of the momentum. The variable z is Feynman's "rapidity"[3] and is used because a Lorentz transformation in the z direction on k is equivalent to a translation of z.

It will be assumed that multiple production cross sections are defined using the invariant form of phase space d^3k/k_o for each secondary. When one changes variables from $\underset{\sim}{k}$ to $\underset{\sim}{r}$, invariant phase space becomes just d^3r. We also need the form of the energy-momentum conservation δ-functions written in terms of the $\underset{\sim}{r}$ variables. One notes that

$$k_o + k_z = m_\perp e^z \tag{5}$$

$$k_o - k_z = \frac{m_\perp^2}{k_o + k_z} = m_\perp e^{-z} \tag{6}$$

Let $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$ be the $\underset{\sim}{r}$ variables of the incident particles while $\underset{\sim}{r}_1, \ldots, \underset{\sim}{r}_n$ are the coordinates of the secondaries. Let the incident momenta be $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$, the secondary momenta $\underset{\sim}{k}_1 \ldots \underset{\sim}{k}_n$. Let $p = p_1 + p_2 - k_1 - \ldots - k_n$; then the energy-momentum conservation δ functions are

$$\delta(p_o)\delta^3(p) = 2\delta(p_o + p_z)\delta(p_o - p_z)\delta^2(\underset{\sim}{p}_\perp) \tag{7}$$

One now has

$$p_+ = p_o + p_z = \mu_1 e^{\zeta_1} + \mu_2 e^{\zeta_2} - m_{1\perp} e^{z_1} - \ldots - m_{n\perp} e^{z_n} \qquad (8)$$

$$p_- = p_o - p_z = \mu_1 e^{-\zeta_1} + \mu_2 e^{-\zeta_2} - m_{1\perp} e^{-z_1} - \ldots - m_{n\perp} e^{-z_n} \qquad (9)$$

$$\underset{\sim}{p}_\perp = \underset{\sim}{s}_{1\perp} + \underset{\sim}{s}_{2\perp} - \underset{\sim}{r}_{1\perp} - \ldots - \underset{\sim}{r}_{n\perp} \qquad (10)$$

where ζ_1 is the z component of $\underset{\sim}{s}_1$ and ζ_2 the z component of $\underset{\sim}{s}_2$; μ_1 and μ_2 are the masses of the incoming particles (which have no transverse momentum so $\mu_1 = \mu_{1\perp}$, $\mu_2 = \mu_{2\perp}$). The vector $p_\perp$ contains the transverse components (p_x, p_y) of p.

The total cross section, written in $\underset{\sim}{r}$ variable form, is a function

$$\sigma_T(\underset{\sim}{s}_1, \underset{\sim}{s}_2) = \Sigma_{n=2}^{\infty} \frac{1}{n!} \int d^3 r_1 \ldots \int d^3 r_n \, 2\delta(p_+)\delta(p_-)\delta^2(\underset{\sim}{p}_\perp)$$
$$\sigma_n(\underset{\sim}{r}_1 \ldots \underset{\sim}{r}_n, \underset{\sim}{s}_1, \underset{\sim}{s}_2) \qquad (11)$$

where $\sigma_n(\underset{\sim}{r}_1, \ldots, \underset{\sim}{r}_n, \underset{\sim}{s}_1, \underset{\sim}{s}_2)$ is the invariant cross section[12] for producing n secondaries with $\underset{\sim}{r}$ variables $\underset{\sim}{r}_1 \ldots \underset{\sim}{r}_n$. One must also sum over particle species (π^+, π^o, π^-, K^+, etc.) but this will not be written explicitly.

In the Feynman analogy $\sigma_T(\underset{\sim}{s}_1, \underset{\sim}{s}_2)$, with $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$ fixed, is a partition function of a gas, and the functions $\sigma_n(\underset{\sim}{r}_1, \ldots, \underset{\sim}{r}_n, \underset{\sim}{s}_1, \underset{\sim}{s}_2)$ are the n-particle distribution functions for the gas. The gas has some strange features, namely the restriction on the positions of particles given by the δ-functions and the fact that transverse coordinates x and y are measured in GeV; these features will not upset the usefulness of the analogy.

The most important effect of the δ-functions is to bound the longitudinal positions z_i of the gas particles. Assume the invariant $s = (p_1 + p_2)^2$ is large, and that one is in the lab system. Then one finds

$$\zeta_1 = \ell n[s/\mu_1\mu_2] \tag{12}$$

$$\zeta_2 = 0 \tag{13}$$

The δ function for p_+ puts an upper bound on the z_i, namely they cannot be much larger than ζ_1 (z_i can be larger than ζ_1 if $m_{i\perp}$ is less than μ_1, but the maximum value of $z_i - \zeta_1$ is $\ell n(\mu_1/m_i)$ where m_i is the mass of the secondary, and this quantity is a constant independent of s). Similarly, the δ function for p_- puts a lower bound on z_i, namely it cannot be much less than $\zeta_2 = 0$ (z_i cannot be less than $-\ell n(\mu_2/m_i)$. Hence the quantities ζ_1 and ζ_2 define boundaries in the z direction for the gas; one can imagine the gas being confined between walls at ζ_1 and ζ_2. The separation of the walls is proportional to $\ell n\, s$ and goes to infinity as $s \to \infty$. The wall at ζ_1 will be called the "incident particle wall"; the wall at ζ_2 is the "target wall". The Feynman gas is illustrated in Fig. 2.

In the transverse direction there are also kinematic bounds but these are not noticeable except near the walls at ζ_1 and ζ_2; away from these walls the dynamic property of bounded transverse momenta keeps the gas largely confined to a tube of radius ~300 MeV.

In a real gas with over 10^{23} particles one does not even think of measuring the distribution functions for these 10^{23} particles. Instead one measures thermodynamic (statistical) average properties of the gas, such as the density. A density $\rho(\underset{\sim}{r})$ is readily defined for the Feynman gas; to be precise it is a function $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$. The density-density correlation function is another statistical average which is readily defined for the Feynman gas; it is a function $g(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\underset{\sim}{s}_1,\underset{\sim}{s}_2)$. In real gases a knowledge of the density and density correlation functions (as a function of temperature and pressure, say) determines all the properties of the gas of practical interest. By analogy a knowledge of the density and density-density correlations of the Feynman gas should be invaluable for characterizing its properties.

The density $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ is simply the invariant single particle spectrum for the scattering problem. The definition of the density ρ is that $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)d^3r$ is the average number of particles to be found in the volume d^3r. From the partition function of Eq. (11) it follows that ρ is

$$\rho(r,s_1,s_2) = \frac{1}{\sigma_T(s_1,s_2)} \Sigma_{n=2}^{\infty} \frac{1}{(n-1)!} \int d^3r_1 \ldots \int d^3r_{n-1}$$
$$2\delta(p_+)\delta(p_-)\delta^2(\underset{\sim}{p}_\perp)\sigma_n(\underset{\sim}{r},\underset{\sim}{r}_1,\ldots,\underset{\sim}{r}_{n-1},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \tag{14}$$

This is just the probability density per collision to produce a secondary with position variable $\underset{\sim}{r}$ [i.e. momentum $\underset{\sim}{k}$ given by Eqs. (1) - (4)]. If one converts from position variables to momentum variables, ρ becomes a function $\rho(k,p_1,p_2)$. Since d^3r is d^3k/k_o, $\rho(k,p_1,p_2)d^3k/k_o$ is the probability per collision to produce a secondary of momentum k in a range d^3k. This ρ is an inclusive quantity in Feynman's language; all events are included which have a secondary in this range. In practice one defines separate densities for each particle species--π^+, π^o, π^-, K^+, etc. I have normalized the spectrum to the total cross section instead of the total inelastic cross section as is sometimes done; the question of normalization will be reconsidered later.

To define the two particle correlation function one first defines a joint probability density $P(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ for finding one particle at $\underset{\sim}{r}$, another at $\underset{\sim}{r}'$. P is given by

$$P(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) = \frac{1}{\sigma_T(s_1,s_2)} \Sigma_{n=2}^{\infty} \frac{1}{(n-2)!} \int d^3r_1 \ldots \int d^3r_{n-2}$$

$$2\delta(p_+)\delta(p_-)\delta^2(\underset{\sim}{p}_\perp)\sigma_n(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{r}_1,\ldots,\underset{\sim}{r}_{n-2},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \qquad (15)$$

If the particles of the gas were uncorrelated the joint probability density $P(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ would be a product of two single particle densities. The correlation function is defined by subtracting this product from P:

$$g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) = P(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) - \rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)\rho(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2). \qquad (16)$$

The most interesting property of the correlation function for a real gas is the correlation length ξ. Qualitatively ξ measures the maximum separation $|\underset{\sim}{r}-\underset{\sim}{r}'|$ for which $g(\underset{\sim}{r},\underset{\sim}{r}')$ is appreciably different from zero. Quantitatively ξ is defined from the expectation that $g(\underset{\sim}{r},\underset{\sim}{r}')$ falls off exponentially in the separation $|\underset{\sim}{r}-\underset{\sim}{r}'|$ when this separation is large. One then defines ξ by the asymptotic form

$$g(\underset{\sim}{r},\underset{\sim}{r}') \sim \exp\left\{-|\underset{\sim}{r}-\underset{\sim}{r}'|/\xi\right\} \tag{17}$$

apart from a power of $|\underset{\sim}{r}-\underset{\sim}{r}'|$.

By analogy with the real gas one should define a correlation length ξ for the Feynman gas. This makes sense only at high energies where large values of $|\underset{\sim}{r}-\underset{\sim}{r}'|$ are possible and one might be able to see experimentally the function $g(\underset{\sim}{r},\underset{\sim}{r}')$ dropping to zero as $|\underset{\sim}{r}-\underset{\sim}{r}'|$ increases. Consider, to be specific, the π^+-π^- correlation function; let $\underset{\sim}{r}$ refer to the π^+, $\underset{\sim}{r}'$ to the π^- and let k and k' be the corresponding momenta. Let s_{+-} be the invariant mass of the π^+-π^- pair; in terms of the $\underset{\sim}{r}$ variables, one has

$$s_{+-} = 2m_\pi^2 + 2m_\perp m_\perp' \cosh(z-z') - 2\underset{\sim}{k}_\perp \cdot \underset{\sim}{k}_\perp' \tag{18}$$

(m_π is the π mass). The "perpendicular masses" $m_\perp$ and $m_\perp'$ are typically approximately equal to $k_\perp$ and $k_\perp'$, all being of order 300 MeV. So for s_{+-} to be large, z-z' must be large. One expects strong correlation between the π^+ and π^- when $s_{+-} \simeq m_\rho^2$ (m_ρ is the ρ mass). The best one can hope for is that $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ becomes small when $s_{+-} \gg m_\rho^2$. To see this fall-off one has to let z-z' become large. In principle one can also make s_{+-} large by going to large transverse momenta, but this means going well out into the tail of transverse momentum distribution. This involves looking at small cross sections of the kind one would prefer to avoid.[13] So what one should look for is a correlation length in $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ as a function of the longitudinal variables z and z'; an experiment to measure this correlation function is included in the program.

The Feynman gas analogy makes clear, I think, that any program to study multiple production should include experiments on the single particle spectrum and two-particle correlation functions.

To motivate the general predictions of the multiperipheral model, imagine that the particles of the Feynman gas interact only through short range forces. In particular, assume the range of the forces stays fixed as s increases, so that for large s the range of forces is small compared to the separation of the walls of the gas. I shall not give a precise definition for the idea of "short range forces", as it is hard to formulate as a mathematical property of the distribution functions σ_n. Further discussion of this problem is given in the Appendix. The hypothesis suggests some properties of the density and correlation functions for large s. The part of the gas that is well away from the walls has no direct interactions with the walls, in fact being many interaction lengths away from them. One would then expect the properties of the gas in this region to be independent of the precise location of the walls. This means $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ should be independent of $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$ when $\zeta_1 - z$ and $z - \zeta_2$ are large: $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \rightarrow \rho(\underset{\sim}{r})$ for $\zeta_1 - z$ and $z - \zeta_2 \rightarrow \infty$. By Lorentz invariance in the collision plane, which is translational invariance along the z axis for the Feynman gas, $\rho(\underset{\sim}{r})$ cannot depend on z so $\rho(\underset{\sim}{r})$ depends only on x and y, i.e. only on $k_\perp$.[14] The part of the gas near the incident particle wall ($z = \zeta_1$) is many interaction lengths away from the target wall; for this range of z, ρ should depend on $\underset{\sim}{s}_1$ but not on $\underset{\sim}{s}_2$: $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \rightarrow \rho(\underset{\sim}{r},\underset{\sim}{s}_1)$ when $z - \zeta_2 \rightarrow \infty$ holding $\zeta_1 - z$ fixed. By translational invariance $\rho(\underset{\sim}{r},\underset{\sim}{s}_1)$ can depend only on $k_\perp$ and $z - \zeta_1$. Translated into momentum variables, this means that $\rho(k,p_1,p_2)$ depends only on $k_\perp$ and k_o/p_{1o} (Feynman's x variable[15]) when $p_{1o} \rightarrow \infty$ with x held fixed. These are just the scaling predictions of Amati, Fubini, Stanghellini[10], Feynman[3], and Yang et al.[3] for the single particle spectrum.

Similarly, the correlation function $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ should be independent of $\underset{\sim}{s}_2$ when z, z', and ζ_1 all go to ∞, and in this limit should depend on z, z', and ζ_1 only in terms of the differences $z - z'$ and $z' - \zeta_1$. So if one holds $k_\perp$, $k_\perp'$, and $z' - \zeta_1$ fixed (i.e. fixed inelasticity x' for the particle with momentum r'), the correlation function should depend only on $z - z'$ and not on ζ_1 (i.e. not on s). Hence the correlation length ξ should also be independent of s, for fixed $k_\perp$, $k_\perp'$, and $z' - \zeta_1$. More generally, the correlation length cannot become large as s increases no matter what values one uses for $k_\perp$, $k_\perp'$, or z', since a dependence on s is possible only if $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ depends simultaneously on $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$, and for sufficiently large s this is excluded by the short range force picture. (Once one has a function independent of $\underset{\sim}{s}_1$ or $\underset{\sim}{s}_2$, one can write a scaling law for that function from which one shows that ξ is bounded.)

If the density of the Feynman gas is independent of z except near the walls, the mean number $\bar{n}$ of particles in the Feynman gas is of order $\zeta_1 - \zeta_2 \sim \ln s$. There can be large fluctuations in the density over small ranges of z, but these are unlikely to be coherent over the entire volume of the gas so fluctuations in the number n of particles about $\bar{n}$ should be small relative to $\bar{n}$, presumably of order $(\bar{n})^{1/2}$.[16] In particular it is unlikely that n will be 2 or 3 or 4 when s is sufficiently large. But this is in clear contradiction with the assumption that diffractive cross sections are constant as $s \to \infty$. This provides the motivation for trying to distinguish diffractive processes from multiperipheral processes; we imagine that it is only the multiperipheral part of the inelastic cross sections which act like the distribution functions of a gas with short range forces. It would then only be the multiperipheral cross sections which produce a multiplicity proportional to $\ln s$ on the average, with fluctuations of order $[\ln s]^{1/2}$. Since diffractive cross sections are mostly of low multiplicity, while the multiplicity of multiperipheral processes increases indefinitely with s, they should be clearly separated at sufficiently large s. In particular, for very large s there should be values of the multiplicity n with $1 \ll n \ll \ln s$ for which both the diffractive and multiperipheral cross sections should be very small. One of the experiments is designed to test whether multiperipheral processes can be distinguished by their multiplicity from diffractive processes.

We must now reexamine the problem of how to normalize the single particle spectrum. This normalization is important due to the definition of the two particle correlation function $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$. In particular, whether $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ goes to zero as $|\underset{\sim}{r}-\underset{\sim}{r}'|$ becomes large depends on how $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ and $P(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ are normalized [see Eq. (16)]. If it is only the multiperipheral part of the inelastic cross sections which behave like distribution functions of a gas with short range forces then it is desirable to define a partition function which is a sum only over multiperipheral cross sections and to define the density and correlation functions using this partition function. What this means is that both P and ρ should be normalized using σ_M, the multiperipheral part of the cross section, rather than either σ_T or the total inelastic cross section. Let us denote by $\rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ and $g_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ the density and correlation function for the multiperipheral cross sections normalized to σ_M. That is, $\rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)d^3r$ is the probability per multiperipheral interaction that a secondary is found in a multiperipheral interaction with position variable $\underset{\sim}{r}$ in a range d^3r. It will now be assumed that it is $\rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ and $g_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ (rather than $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ and $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ which satisfy the Amati-Fubini-Stanghellini scaling laws.

One now has the problem of determining whether ρ and g will also satisfy scaling laws or whether one must measure ρ_M and g_M separately. The answer is that some scaling laws apply also to ρ and g, some do not. For example, the scaling law $\rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \rightarrow \rho_M(\underset{\sim}{r},\underset{\sim}{s}_1)$ when ζ_1 and $z \rightarrow \infty$ holds also for ρ.

To see this one defines $\rho_D(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ to be the single particle spectrum for diffractive events, normalized to the total diffractive cross section σ_D. At high energies we assume the total cross section σ_T is approximately equal to $\sigma_D + \sigma_M$; then

$$\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) = \frac{\sigma_D}{\sigma_D + \sigma_M}\, \rho_D(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) + \frac{\sigma_M}{\sigma_D + \sigma_M}\, \rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2) \quad (19)$$

One further assumption: assume that σ_M approaches a constant as $s \rightarrow \infty$(i.e. does not go to zero). Now it is easily seen that $\rho_D(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ becomes independent of $\underset{\sim}{s}_2$ when z and $\zeta_1 \rightarrow \infty$ due to diffractive cross sections individually being constant for large s. Since σ_D and σ_M are constants for large s, the combined spectrum ρ is also independent of s_2, in the limit z and $\zeta_1 \rightarrow \infty$.

An example of a result holding for $g_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ which does <u>not</u> hold for the full correlation function $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ is the result that $g_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) \rightarrow 0$ when $|z - z'| \gg \xi$ where ξ is a fixed correlation length. Suppose for example that ζ_1 is very large, $z' \sim \zeta_1$ and z lies in the range $0 \ll z \ll \zeta_1$ (so that $\zeta_1 - z$ and $z - \zeta_2$ are both $\gg 1$). Then there are no diffractive events which give secondaries of position $\underset{\sim}{r}$; as a result the correlation function $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ can be written

$$g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) = \frac{\sigma_M}{\sigma_M + \sigma_D}\, P_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) - \frac{\sigma_M}{\sigma_M + \sigma_D}\, \rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$$

$$\frac{\sigma_M \rho_M(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) + \sigma_D \rho_D(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)}{\sigma_M + \sigma_D}$$

$$= \frac{\sigma_M}{\sigma_M + \sigma_D}\, g_M(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) - \frac{\sigma_M \sigma_D}{(\sigma_M + \sigma_D)^2}\, \rho_M(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)\, \rho_D(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) - \rho_M(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2) \quad (20)$$

It follows from this formula that the full correlation function $g(\underset{\sim}{r},\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ will not be zero for $|z-z'|$ large unless the diffractive spectrum ρ_D is equal to the multiperipheral spectrum ρ_M. This is impossible for all $\underset{\sim}{r}'$ because $\rho_M(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ is constant when $\zeta_2 \ll z' \ll \zeta_1$ while $\rho_D(\underset{\sim}{r}',\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ is zero in this range.

The problem of separating multiperipheral and diffractive events will be discussed in connection with individual experiments.

To conclude the discussion of multiperipheral events there are two miscellaneous observations to be made. First, there has always been some reluctance to measure secondary pion spectra on the grounds that most of them are probably decay products of ρ's, A_1's, etc., and so one would prefer to know the spectra of ρ's and A_1's. However, the scaling laws of Amati, Fubini, and Stanghellini for pion spectra are equally valid whether or not the π's are mostly decay products.[10] The intuitive reason for this is that the decay process is itself a short range effect in $\underset{\sim}{r}$-space [for example a π^+ and π^- with an invariant mass of order m_ρ cannot have a large separation in z, from Eq. (18)]. Hence, if the cross sections for producing ρ's and other resonances obey the multiperipheral scaling laws, then so will the cross sections for the π's resulting from ρ decays. This means that to test the multiperipheral scaling laws it is pointless to distinguish π's from ρ decay from uncorrelated π's.

The second observation is this. In p-p collisions it is possible for pions to be emitted backwards in the lab system due to the low mass of the pion. The minimum z for the pion is $-\ln(m_p/m_\pi)$ where m_p is the proton mass (this was shown earlier). This value for z corresponds to a rather fast backward pion. It is not easy to find dynamic mechanisms that would produce such fast backward pions; they are too fast to be decay products of low-lying N^*'s. So there should not be many such pions. By symmetry there cannot be many pions going much faster than the incident proton either, for such pions would be going backwards in the rest system of the incident proton. However a pion having the same velocity as the incident proton and a transverse momentum of about 300 MeV has only 1/3 the energy of the incident proton. Hence the spectrum of pions in p-p collisions should fall rapidly as the longitudinal momentum k_z of the pion increases once $k_z/E > 1/3$ where E is the incident proton energy. This is the explanation for the exponential fall-off in k_z seen experimentally for $k_z/E > 1/4$ in pion spectra in p-p collisions.[17] This exponential tail is useful for experimental purposes, as will be discussed later. No similar tail is expected for high energy π's in π-p interactions; in π-p interactions the π spectrum should be reasonably flat up to the kinematic limit $k_z/E \simeq 1$.

Now a specific program of experiments will be described. They test the theoretical ideas already described. However, if the predictions are correct the experiments can be interpreted without making numerical fits to a particular model; rather the experiments will define non-trivial properties of multiple production which all future models would have to agree with. The experiments should all be feasible with present-day techniques. I have tried to define the simplest and most practical experiment to test each theoretical prediction; I hope that these experiments will not be replaced by more elaborate "improvements" of them. The individual experiments are not new; what is important is that they form a sensible program.

Experiment 1: Partial Cross Sections as a Function of Multiplicity

In this experiment one considers p-p interactions at fixed but large s. One measures the partial cross section $\sigma_{nc}(s)$ for producing n charged secondaries, as a function of n. No distinction is made between π^+, π^-, p, $\overline{P}$, etc.: all charged particles are counted, and all neutrals are ignored. What one is looking for is a dip in a plot of $\sigma_{nc}(s)$ versus n. Namely, at sufficiently high s the partial cross section $\sigma_{nc}(s)$ should first decrease with n as one moves out of the diffraction dissociation region, and then increase again as n approaches the mean multiplicity $\overline{n}_c(s)$ of charged secondaries from multiperipheral processes. One expects $\overline{n}_c(s)$ to be proportional to $\ell n\, s$, so the larger s is the more pronounced the dip should be.

There is no guarantee that a dip will occur at the energies of NAL or the CERN ISR, even if the theoretical picture of diffraction dissociation plus multiperipheralism is correct.

So an absence of a dip, while disappointing, does not disprove the picture. The importance of the experiment is that a dip, if found,. would be clear cut and model independent evidence for the existence of two separate processes in multiple production.

If one could detect neutrals as easily as charged particles one would measure $\sigma_n(s)$, the partial cross section for producing n secondaries including neutrals instead of $\sigma_{nc}(s)$. Because the mean number $\overline{n}(s)$ of all secondaries in multiperipheral processes is larger than $\overline{n}_c(s)$, the dip is likely to appear at a lower energy in the function $\sigma_n(s)$ than in $\sigma_{nc}(s)$. But I believe the charged multiplicity remains the simplest experiment to do even if large bubble chambers can see γ-rays, and I urge that an experiment on charged multiplicities not be held up in hopes of doing a measurement including neutrals.

The next group of experiments concern the single particle spectrum. For purposes of this discussion I shall write ρ as a function of $k_\perp$, k_z, and E, where $k_\perp$ is the transverse momentum of the secondary, k_z is the longitudinal momentum in the <u>lab</u> system of the secondary, and E is the incident energy. Phase space is d^3k/k_o: $\rho(k_\perp,k_z,E)d^3k/k_o$ is the probability per collision to find a secondary of momentum $\underset{\sim}{k}$ in a range d^3k. There are separate spectra for π^+,π^-,K^+, etc.; there are also according to theory separate spectra $\rho_D(k_\perp,k_z,E)$ and $\rho_M(k_\perp,k_z,E)$ for diffractive events and multiperipheral events. The spectrum ρ_M is normalized to the total multiperipheral cross section $\sigma_M(s)$

instead of the total cross section; ρ_D is normalized to the total diffractive cross section. When it is necessary to separate ρ_M or ρ_D from ρ we will discuss how to do it, if possible.

I beg all experimentalists not to plot single particle spectra as angular distributions. It is $k_\perp$, not an angle, which has an average value around 300 MeV independent of k_z; this fact makes plots vs. $k_\perp$ and k_z much simpler to interpret than angular distributions. I personally prefer the use of the lab variable k_z or else the rapidity z to a center-of-mass variable, since the multiperipheral scaling laws look simpler in terms of these variables.

Experiment 2: Beam Survey from a Hydrogen Target

In this experiment one measures the spectrum $\rho(k_\perp,k_z,E)$ of high energy secondary π^- mesons from p-p collisions. The measurement is made varying E holding $k_\perp$ and k_z/E fixed (fixed x in Feynman's language). The prediction to be tested is that $\rho(k_\perp,k_z,E)$ is independent of E when $k_\perp$ and k_z/E are held fixed. The range of E should be ~30 GeV and up; $k_\perp$ should be of order 300 MeV and $k_z/E \sim 1/3$. The choice of $k_\perp$ and k_z is meant to ensure that one is in a region where the spectrum is large. The choice of π^- over other particles is for simplicity, since all but a few negative secondaries are π^-. A good experiment would be to hold $k_\perp$ fixed, measure the spectrum as a function of k_z and E and try to fit the data to an exponential depending only on k_z/E.

The experiment can also be done in π^+p or π^-p collisions but in this case one does not have the exponential behavior in k_z which makes for a good experiment. On the other hand, one can get to higher values of k_z for the secondary π if the incident particle is a π, since the limitation $k_z/E \lesssim 1/3$ no longer applies.

There is no need to separate diffractive from multiperipheral contributions since both are predicted to obey the Amati-Fubini-Stanghellini-Feynman scaling law.

Experiment 3: Factorization in the Single Particle Spectrum

In this experiment one measures the spectrum $\rho(k_\perp,k_z,E)$ for backwards π^- in the lab in both p-p and π-p collisions. The energy E is high and fixed; I suggest holding $k_\perp$ fixed and fitting to an exponential in k_z. What one is testing for is whether these spectra are equal for p-p and π-p collisions, that is whether $\rho(k_\perp,k_z,E)_{\pi p} = \rho(k_\perp,k_z,E)_{pp}$. According to the picture described earlier the multiperipheral spectrum $\rho_M(k_\perp,k_z,E)$ should be the same for high energy π^-p and pp collisions as long as one is looking at low energy secondaries; this is because a low energy particle is a particle near the target wall in the Feynman gas and cannot tell what particle was incident at the other wall which is many interaction lengths away. (This prediction is another general consequence of the multiperipheral model[10].) It is not clear whether or not the diffractive component $\rho_D(k\ ,k_z,E)$ should be the same for π-p and p-p collisions. If diffraction is described by a simple

Regge pole then the factorization property of Regge poles requires that $\rho_D(k_\perp, k_z, E)$ be the same for π-p and p-p. If diffraction is something else than Regge exchange then $\rho_D(k_\perp, k_z, E)$ could very well be different (at least in magnitude, if not in shape) for π-p and p-p scattering. If this is the case then one will see a violation of factorization when one performs this experiment. I do not expect there will be large violations of factorization, so I think it is important to do this experiment carefully so that one can make accurate comparisons of πp and pp collisions at the same values of $\underset{\sim}{k}$; also the experiment will have to be done at several incident energies E to see if any violations of factorization persist as E increases. One does not have to use incident π's and p's of the same energy; one might instead use π's and p's with the same velocity, which means the π would have 1/7 of the energy of the proton. I would suggest a compromise: Let the incident π's have 1/3 the energy of the incident protons.

There is no way in this experiment to separate ρ_M from ρ_D; a separate experiment will be proposed later to test factorization in multiperipheral processes separately.

This experiment tests whether factorization holds when ρ has both diffractive and multiperipheral contributions. Even if factorization breaks down, it is still possible for ρ_D and ρ_M separately to factorize; if $\rho_D \neq \rho_M$ and if the ratio σ_M/σ_{TOT} is different for πp and pp collisions, then ρ will not factorize (see Eq. (19)).

Experiment 4: The dk_z/k_z Law

In this experiment one measures the spectrum $\rho(k_\perp,k_z,E)$ of secondary π^- in p-p collisions. The energy E is held fixed and should be the highest energy available. The spectrum is measured as a function of k_z holding $k_\perp$ fixed. As usual $k_\perp$ should be of order 300 MeV, while k_z should be in the intermediate range, say $1\ \text{GeV} < k_z < E/4$. What one is testing for is whether $\rho(k_\perp,k_z,E)$ is independent of k_z in the range $1\ \text{GeV} \ll k_z \ll E/4$. This prediction is called the "dk_z/k_z" law because longitudinal phase space has the form dk_z/k_z in this region.

The problem with this experiment is that the prediction that ρ is independent of k_z is valid only for secondaries which in the Feynman gas analogy must be many interaction lengths away from both walls. In experiments 2 and 3 the only requirement is that the walls be separated by many interaction lengths. So if "many interaction lengths" turns out to be a distance z_o, then the bound for s in experiments 2 and 3 is $\zeta_1 - \zeta_2 = \ell n(s/\mu_1\mu_2) > z_o$ but the bound for this experiment is $\ell n(s/\mu_1\mu_2) > 2z_o$. To double $\ell n\, s$ means squaring s; for example, if experiments 2 and 3 require $s > 30\ \text{GeV}^2$, this experiment requires $s > 900\ \text{GeV}^2$! (μ_1 and μ_2 are of order 1 in GeV for p-p collisions and can be neglected in these units.) One is further squeezed in this experiment because the pions must have z's considerably less than ζ_2 which means energies $\ll E/3$ instead of energies $\ll E$. This latter squeeze can be avoided by using

πp instead of pp collisions; if one can get enough incident pions with more than 1/3 the beam energy the πp experiment is probably better.

The prediction is precisely that $\rho_M(k_\perp, k_z, E)$ is independent of k_z in the intermediate range; but according to the picture of diffraction described earlier $\rho_D(k_\perp, k_z, E)$ is negligible in this range so $\rho(k_\perp, k_z, E)$ should also be independent of k_z in the intermediate range.

Experiment 5: Search for Double Pomeron Exchange[18]

In this experiment one measures the cross section for the specific reaction $p + p \rightarrow p + p + \pi^+ + \pi^-$. One looks in particular at π^+-π^- pairs whose center of mass is at rest in the center-of-mass system of the incident particles. The cross section is studied as a function of the incident energy E and the invariant mass squared s' of the pair. What one is looking for is

a) the energy dependence of the reaction, for fixed s', and

b) whether the ρ peak broadens as E increases.

The theory of this experiment is as follows. Double pomeron exchange (see Fig. 1), if it exists, can only produce $I = 0$ π pairs and cannot produce ρ's. Pomeron exchange with the incident particle plus ρ exchange with the target particle can produce ρ's. Double Pomeron exchange if it exists should become the dominant process compared to other exchanges at sufficiently high energies. There should not be a narrow resonance like the ρ in the $I = 0$ channel; if $I = 0$ pairs become important the ρ peak in the π^+-π^- mass distribution should appear to be broadened due to these pairs. The cross section for producing ρ's should fall with energy; for example, if ρ's are produced by Pomeron exchange + ρ exchange, one finds the cross section should behave as $E^{-1/2}$, E being the incident energy. (To be precise the cross section is the cross section for producing π^+-π^- pairs with fixed windows in the center of mass for the π^+ and π^- and no restrictions on the final state of the protons.) A naive double Pomeron model predicts a constant cross section for π^+-π^- pairs with $I = 0$; there are enough problems with double Pomeron exchange[9] that I am unwilling to make this or any other prediction for the $I = 0$ pair cross section.

There is no sign of $I = 0$ production at 30 GeV energies, so it is unlikely to overwhelm ρ production at NAL or CERN energies; at best the $I = 0$ cross section might become comparable to the ρ cross section.

In summary one looks at the energy dependence of the ρ cross section to see if it falls with E according to a Pomeron + ρ exchange model or another exchange model; one looks for a broadening of the ρ peak to see if there is double Pomeron exchange.

Experiment 6: Correlation Length Experiment

In this experiment one measures the two-particle spectrum $P(k_\perp, k_z, k'_\perp, k'_z, E)$ for production of a π^+ of momentum $\underset{\sim}{k}$ and a π^- of momentum $\underset{\sim}{k}'$ in $\pi^- p$ collisions. That is the probability of finding a π^+ of momentum $\underset{\sim}{k}$ and a π^- of momentum $\underset{\sim}{k}'$ in ranges d^3k and d^3k' in the final state, per collision, is

$P\, d^3k\, d^3k'/k_o k_o'$. This is an inclusive experiment in the Feynman terminology. For fixed and large E, one holds $k_\perp, k_\perp'$, and k_z' fixed, with $k_\perp$ and $k_\perp'$ of order 300 MeV and $k_z' \sim E$ (say $k_z = 2E/3$). P is then measured as a function of k_z. In other words one looks at a fast forward π^- of fixed momentum, while varying the longitudinal momentum of the π^+. One wants a wide range of k_z, say from 500 MeV up to the beam momentum. Along with P, one should also measure $\rho(k_\perp, k_z, E)$ and $\rho(k_\perp', k_z', E)$ for use in the analysis discussed below.

If one uses the naive analogy to a gas then what one should do with the data is to compute the two particle correlation function $g(k_\perp, k_z, k_\perp', k_z', E)$:

$$g(k_\perp, k_z, k_\perp', k_z', E) = P(k_\perp, k_z, k_\perp', k_z', E) - \rho(k_\perp, k_z, E)\rho(k_\perp', k_z', E) \tag{21}$$

What is interesting is to see if this correlation function exhibits a correlation length, i.e. see if it goes to zero as k_z becomes small compared to k_z' and E. This is the first thing to investigate with the data. However, according to the theory of this paper it is the multiperipheral correlation function g_M (instead of g) that should have a correlation length. If g does not go to zero when k_z is small compared to k_z' and E, then it is worth looking for a correlation length in multiperipheral processes alone.

For k_z in the intermediate range (300 MeV $\ll k_z \ll$ E), both P and $\rho(k_\perp, k_z, E)$ should be free of diffractive contributions and hence differ only in normalization from P_M and ρ_M. Hence, if g_M shows a correlation length, one should have

$$P(k_\perp, k_z, k'_\perp, k'_z, E)/\rho(k_\perp, k_z, E) \simeq P_M/\rho_M \simeq \rho_M(k'_\perp, k'_z, E) \qquad (22)$$

There is no independent way to measure $\rho_M(k'_\perp, k'_z, E)$, so the content of this equation is that P/ρ should be independent of k_z when k_z is in the intermediate range. This suggests that one look for a correlation length by seeing if P/ρ does approach a constant for $k_z \ll E$. If so, the way to obtain a correlation length is to fit the departure of P/ρ from the constant to an exponential in the gas variable z, or else find the value of $\xi = |z-z'|$ above which P/ρ is constant.

It is quite possible that P/ρ will be constant for k_z in the intermediate range, but depart from this constant when k_z is small and P and ρ again have diffractive contributions.

For 25 GeV incident pions there is no intermediate range for k_z, because diffraction dissociation or isobar decays can produce pions at rest in the center-of-mass system. This will not be possible at NAL energies, unless diffraction dissociation involves intermediate states of mass > 2 GeV (cf. Franzini[2]).

The main aim in choosing experimental parameters is to have an intermediate range for k_z not contaminated with diffraction dissociation products. It is important for this reason not to let $k_\perp$ be much smaller than 300 MeV. The reason for this is that the invariant mass squared of the π^+ and π^-, for k_z in the intermediate region, is

$$s_{+-} \simeq k_\perp^2 k'_z / k_z \quad . \qquad (23)$$

If $k_\perp^2$ is small then s_{+-} will be small even if the π^+ is transverse in the center of mass (i.e. in the middle of the intermediate range). In practice one would like $s_{+-} > 4$ GeV^2 or so when the π^+ is transverse in the center of mass, to be well away from diffraction dissociation effects. One does not want to increase $k_\perp$ much beyond 300 MeV because then one gets into the tail of the transverse momentum distribution and out of the interesting region.[13]

Experiment 7: Test of Factorization in Multiperipheral Processes

In this experiment one measures $P(k_\perp, k_z, k_\perp', k_z', E)$ and $\rho(k_\perp', k_z', E)$ for very large fixed E in both πp and pp collisions. The momentum k' refers to a π^+ coming out at 90° in the center of mass, its momentum is held fixed. The momentum k refers to a π^- which is backwards in the lab system (as in the previous factorization experiment). Define

$$\rho_{M_{exp}}(k_\perp, k_z, E) = P(k_\perp, k_z, k_\perp', k_z', E)/\rho(k_\perp', k_z', E) \tag{24}$$

The purpose of this experiment is to see whether $\rho_{M_{exp}}(k_\perp, k_z, E)$ is the same for πp and pp collisions.

The only difference experimentally between this experiment and Experiment 3 is that instead of measuring the backwards pion spectrum for all collisions one is now measuring the backwards pion spectrum only for those collisions which emit a transverse π^+ in the center of mass as well as a backwards pion. In other words one is using the transverse pions as an event or beam monitor. This excludes diffractive events according to our theoretical picture; both P and ρ should be proportional to P_M and ρ_M and the ratios P/ρ should equal P_M/ρ_M. Furthermore the π^+ and π^- will be well separated in z so $P_M(k_\perp,k_z,k'_\perp,k'_z,E)$ should factor into $\rho_M(k_\perp,k_z,E)\rho_M(k'_\perp,k'_z,E)$. Hence one expects

$$\rho_{M_{exp}}(k_\perp,k_z,E) \simeq \rho_M(k_\perp,k_z,E) \qquad (25)$$

Hence comparing $\rho_{M_{exp}}(k_\perp,k_z,E)$ for πp and pp collisions is a test of factorization for multiperipheral processes alone. As in Experiment 3 one should make this test at several energies E so one can see whether violations of factorization decrease as E increases.

These seven experiments constitute the experimental program. If any of these experiments agrees well with the prediction, the result will be interesting whether or not one likes the diffractive plus multiperipheral picture of multiple production. Every one of the predictions cited can be wrong and it would be remarkable if they all were to be verified. If it turns out that diffractive events are not distinguishable from multiperipheral events, Experiment 7 may not be worth pursuing. There are many reasonable ways to modify the experiments proposed here but I hope that such modifications will be examined critically to determine if they achieve the objectives of these experiments as clearly and as simply as the specific experiments cited here.

Acknowledgements

When I first studied multiperipheralism I was much helped by conversations with Dr. A. Krzywicki. In the last year I have been helped and reinspired by several conversations with Professor R. P. Feynman. The discussion of two particle correlations described here resulted from conversations with Professor L. Brown and Drs. R. Peccei and D. Silverman at the Aspen Institute for Physics. I wish to thank also Professors S. Drell, J. Bjorken, W. Fraser, J. Peoples, and Dr. P. Roy for discussions.

Appendix. Short Range Forces and Bounded Transverse Momentum

In the text the hypothesis of short range forces in the Feynman gas was used without specifying it precisely. It was also pointed out that it is hard to reconcile the observed bounded transverse momentum of secondaries with one's intuitive picture of the Feynman gas if it has short range forces. The multiperipheral model suggests a definition of short range forces from which one predicts bounded transverse momenta: the purpose of this Appendix is to describe this definition and show how it leads to bounded transverse momenta.

An intuitive picture of short range forces would be that the dependence of $\sigma_n(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\ldots,\underset{\sim}{r}_n,\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ on one of the $\underset{\sim}{r}$'s, say $\underset{\sim}{r}_1$, would be affected only by other $\underset{\sim}{r}$'s which are near $\underset{\sim}{r}_1$. A quantitative way of saying this is that if one changes $\underset{\sim}{r}_1$ to $\underset{\sim}{r}_1'$ the exact value of $\underset{\sim}{r}_2$ does not matter if $|\underset{\sim}{r}_1-\underset{\sim}{r}_2|$ is large, i.e.

$$\frac{\sigma_n(\underset{\sim}{r}_1',\underset{\sim}{r}_2,\ldots,\underset{\sim}{r}_n,\underset{\sim}{s}_1,\underset{\sim}{s}_2)}{\sigma_n(\underset{\sim}{r}_1,\underset{\sim}{r}_2,\ldots,\underset{\sim}{r}_n,\underset{\sim}{s}_1,\underset{\sim}{s}_2)} = \frac{\sigma_n(\underset{\sim}{r}_1',\underset{\sim}{r}_2',\ldots,\underset{\sim}{r}_n,\underset{\sim}{s}_1,\underset{\sim}{s}_2)}{\sigma_n(\underset{\sim}{r}_1,\underset{\sim}{r}_2',\ldots,\underset{\sim}{r}_n,\underset{\sim}{s}_1,\underset{\sim}{s}_2)} \tag{A.1}$$

in a limit in which $|\underset{\sim}{r}_2-\underset{\sim}{r}_1| \to \infty$ while $|\underset{\sim}{r}_1-\underset{\sim}{r}_1'|$ and $|\underset{\sim}{r}_2-\underset{\sim}{r}_2'|$ are held fixed. But this is not the prediction of the multiperipheral model. In the multiperipheral model one must first introduce a set of momentum transfer variables. If $k_1,\ldots,k_n$ are the four-momenta of the secondaries, then one defines

$$\begin{aligned} q_1 &= p_1 - k_1 \\ q_2 &= q_1 - k_2 \\ &\vdots \\ q_{n-1} &= q_{n-2} - k_{n-1} = k_n - p_2 \end{aligned} \tag{A.2}$$

(see Fig. 3).

In the model of multiple meson exchanges, the k_i are the momenta of secondaries in order of emission from the multiperipheral chain (Fig. 3). If one writes the cross section for emitting n secondaries as a function $\sigma_n(q_1,\ldots,q_{n-1},p_1,p_2)$ of the n-1 momentum transfers, then the multiperipheral cross section factorizes into functions depending on neighboring q_i (i.e. q_1 and q_2, or q_2 and q_3, etc.). The principal source of this dependence on neighboring q's is the kinematic restriction that $(q_i - q_{i-1})^2 = m_i^2$ where m_i is the mass of the i^{th} secondary; the propagators of the multiperipheral chain depend only on a single q_i. If the vertices of the chain are not point-like, they also depend on neighboring q_i.

The first generalization from the multiperipheral model is that the short range force idea should be stated in terms of momentum transfers q_i instead of secondary momenta k_i. But there are still problems. The multiperipheral model predicts a factorization in terms of an ordering of the q_i rather than in terms of separation of q_i from q_j. That is, the dependence of σ_n on q_i in the multiperipheral model is independent of the values of q_{i+2}, q_{i-2}, etc., regardless of whether q_{i+2} or q_{i-2} is close to q_i or not, and dependent on $q_{i\pm1}$ even if $(q_i - q_{i\pm1})^2$ is large. In summary the σ_n's of the multiperipheral model look more like a random walk distribution function than a gas distribution with short range forces.

An analysis of the multiperipheral model shows that on the average the q_i tend to order themselves so that $q_{io} > q_{i+1o}$ and $k_{iz} > k_{i+1z}$; the highest energy transfers occur at the end of the chain associated with the incident particle, and the secondaries with the highest longitudinal momentum are emitted near this end of the chair. Also the quantities $(q_i - q_{i-j})^2$ tend to increase as j increases, so on the average the factorization of the multiperipheral is similar to a short range force hypothesis. This suggests that the way to generalize the multiperipheral model is as follows. First, one defines an ordering of the secondary momenta, namely, they should be numbered so that $k_{1z} \geq k_{2z} \geq \cdots \geq k_{nz}$. (An alternative is to order in terms of the z variables: $z_1 \geq z_2 \geq z_3 \ldots$). Then one defines $q_1,\ldots,q_{n-1}$ by Eqs. (A.2). Then one supposes that the dependence of $\sigma_n(q_1,\ldots,q_{n-1},p_1,p_2)$ on q_i is determined only by those q_j for which $(q_j - q_i)^2$ is small. Since

$s_{ij} = (q_i - q_j)^2$ is the total mass squared of the secondaries numbered i+1 to j (or j+1 to i if $j < i$), it is reasonable to suppose a dependence on s_{ij} when s_{ij} is in the resonance region; the crucial assumption is that there will be no appreciable dependence on s_{ij} when s_{ij} is beyond the region of the principal resonances.

There is another part to any short-range force hypothesis, namely, the dependence of $\sigma_n(q_1 \ldots q_{n-1}, p_1, p_2)$ on q_i must not be changed if one changes the total number of secondaries but leaves unchanged the q_j near q_i (in a gas analogy this is saying the distribution function for particles in one region cannot be changed if one adds or subtracts particles in a far-away region). This is also true in the multiperipheral model and should be demanded for any generalization from it. A quantitative statement of this requirement will not be given.

As far as the properties of one and two particle spectra are concerned, a short range force hypothesis in terms of the q_i is as good as a short range force hypothesis in terms of the k_i themselves. The reason is that $k_i = q_i - q_{i-1}$ and q_i and q_{i-1} are close variables; if k_i and k_j are well separated the variables q_i and q_{i-1} tend also to be well separated from q_j and q_{j-1}, and as a result the particles of momentum k_i and k_j tend to be uncorrelated. One also assumes uncorrelated dependence on p_1 or p_2 and q_j if $(p_1 - q_j)^2$ or $(p_2 + q_j)^2$ is large; this then leads to factorization of the dependence of spectra on the secondary momenta k and the incident momenta when the secondary is (in the gas analogy) well away from the walls. These results can be shown in detail for the multiperipheral model itself using the multiperipheral integral equation of Amati, Fubini, and Stanghellini[10]; to show that scaling laws hold for a generalized short range force picture quantitatively is more difficult but it seems reasonable that they should still hold.

Finally, using the momentum transfer variables q_i it is easy to bound the transverse momenta of secondaries. All that is necessary is to bound the momentum transfers q_i^2. The reason that this bounds the transverse components $k_{i\perp}$ of k_i is the following. First, one shows that the 2-vector (q_{io},q_{iz}) is spacelike (except possibly for i near 1 or n). The reason is simple. Consider the two-vectors (p_{1o},p_{1z}), (p_{2o},p_{2z}), (k_{1o},k_{1z}), $\cdots$ (k_{no},k_{nz}). These are all timelike two-vectors, with masses $\mu_1,\mu_2,m_1,\ldots,m_n$. If the two-vector momentum transfer from p_1 to $k_1+\ldots+k_i$ is timelike also it acts like a final state particle in one of two ways: either in the sense of splitting p_1 into $q_i+k_1+\ldots+k_i$ or in splitting p_2 into $-q_i+k_{i+1}+\ldots+k_n$. For the first splitting to be possible the mass μ_1 of p_1 must exceed the sum of the masses m_j of $k_1 \ldots k_i$ but since $m_j \geq m_j$ this sum usually exceeds μ_1 (except for special cases when i is small and the outgoing particles have lower rest masses than the incident particle). Likewise it is unlikely for the sum of m_j from $j = i+1$ to $j = n$ to be less than μ_2. Hence (q_{io},q_{iz}) is spacelike. But this means $q_{i\perp}^2 < |q_i^2|$, so if $|q_i^2|$ is bounded then $q_{i\perp}^2$ is also; since $k_i = q_i - q_{i-1}$, $k_{i\perp}^2$ is bounded also.

This argument breaks down if the incident particles have large perpendicular momenta, for then the two-vectors (p_{1o}, p_{1z}) and (p_{2o}, p_{2z}) have masses $\mu_{1\perp}$ and $\mu_{2\perp}$ which are large and it is quite easy for (q_{io}, q_{iz}) to be timelike. In summary, all links of the multiperipheral chain and its generalizations know the direction of the momentum of the incident particle; however the magnitude of this momentum is forgotten through a random walk effect as one goes many links away from the incident particle.

References

1. See, e.g., O. Czyzewski in Proceedings of the 14th International Conference on High Energy Physics, Vienna, p. 377.
2. For recent experimental studies of multiple production, see the talks of P. Franzini, M. Koshiba, W. D. Walker, and R. Panvini in High Energy Collisions, ed. by C. N. Yang *et al.* (Gordon and Breach, New York, 1969). See also J. L. Day *et al.*, Phys. Rev. Letters **23**, 1055 (1969); D. B. Smith *et al.*, *ibid.* **23**, 1064 (1969).
3. For some current theoretical work on multiple production, see T. T. Chou and C. N. Yang, Phys. Rev. Letters **25**, 1072 (1970); J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. **188**, 2159 (1970); R. P. Feynman in High Energy Collisions (Ref. 2); G. F. Chew, T. Rogers, and D. R. Snider, UCRL-19457, preprint (1970); N. F. Bali, L. S. Brown, R. D. Peccei, and A. Pignotti, Phys. Letters **33B**, 175 (1970).
4. It is part of the Feynman picture (Ref. 3) that one should distinguish multiperipheral events from diffractive events.
5. For recent work on bounds, see R. J. Eden, invited talk at the Wisconsin Conference "Expectations for Particle Reactions at the New Accelerators" (April 1970).
6. I shall use the term "diffractive processes" to include both diffraction and diffraction dissociation. The original reference on diffraction dissociation is M. Good and W. Walker, Phys. Rev. **120**, 1857 (1960).

7. The ideas of this paper differ in spirit from the limiting fragmentation hypothesis of Benecke et al. (Ref. 3) in that we distinguish two types of processes (diffractive and multiperipheral) contributing strongly to the total cross section. Some of the predictions discussed here agree with predictions of Benecke et al.

8. See also T. T. Chou and C. N. Yang (Ref. 3).

9. J. Finkelstein and K. Kajantie, Phys. Letters 26B, 305 (1968); G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

10. D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962). For a review see S. Fubini in Strong Interactions and High Energy Physics, ed. R. G. Moorhouse (Plenum Press, New York, 1964), p. 259. For another review of the scaling laws see K. Wilson, Acta Phys. Austr. 17, 37 (1963). For an attempt to check the scaling laws against cosmic ray data see O. Czyzewski and A. Krzywicki, Nuovo Cimento 30, 603 (1963).

11. R. P. Feynman, private communication.

12. The function σ_n has been normalized in such a way that no factors of 2π occur in Eq. (11); also the incident particle is assumed to be highly relativistic so the velocity factor is a constant and included in the normalization.

13. As noted earlier, the trouble with looking at small cross sections is that they may be affected by small effects which are negligible in large cross sections.

14. It is hard to understand, given the hypothesis of "short range forces", how $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ can depend even on $k_\perp$. This problem arises because the separation of $k_\perp$ into k_z and $k_\perp$ is defined in terms of the direction of the momentum of the incident particle, whereas the hypothesis of short range forces suggests that the density at $\underset{\sim}{r}$ should be independent of the momentum of the incident particle. The resolution of this problem is that the definition of "short range forces" that is suggested by the multiperipheral model has some subtle features that permit $\rho(\underset{\sim}{r},\underset{\sim}{s}_1,\underset{\sim}{s}_2)$ to depend on the directions but not the magnitudes of $\underset{\sim}{s}_1$ and $\underset{\sim}{s}_2$. This is explained in the Appendix.

15. I apologize for having used x both as the x coordinate of $\underset{\sim}{r}$ and as Feynman's inelasticity variable; from now on x will appear explicitly only as an inelasticity.

16. The distribution in n for multiperipheral processes has been predicted to be a Poisson distribution; see, e.g., G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

17. This explanation is found in, for example, C. N. Yang in High Energy Collisions (Ref. 2). For an experimental curve see S. Drell, invited paper presented to Conference on "Expectations for Particle Reactions at the New Accelerators" (Madison, April 1970).

18. For a review of double Pomeron exchange see E. L. Berger, invited paper at the Irvine Regge Pole Conference (Dec. 1969). For a test of double Pomeron exchange see R. Lipes, G. Zweig, and W. Robertson, Phys. Rev. Letters 22, 433 (1969).

Figure Captions

Fig. 1. Double Regge pole exchange graph for process $p + p \rightarrow p + p + \pi^+ + \pi^-$. The exchanges can be π exchange or any combination of Pomeron and ρ exchange. The invariants s_1 and s_2 are the invariant mass squared for the Regge poles I and II respectively.

Fig. 2 Sketch of the Feynman gas showing walls at ζ_1 and ζ_2 and the bound on $|\underset{\sim}{r}_\perp|$. There can be some leakage beyond the walls, as noted in the text. μ_1 and μ_2 are the incident particle masses.

Fig. 3 Multiperipheral chain.

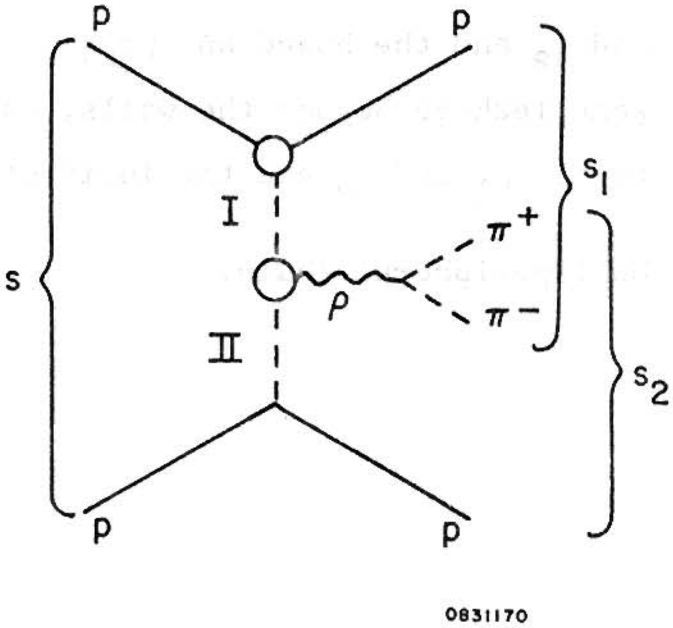


Fig. 1

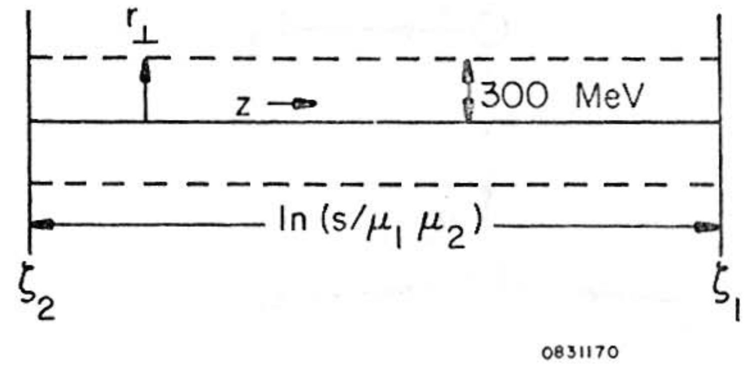
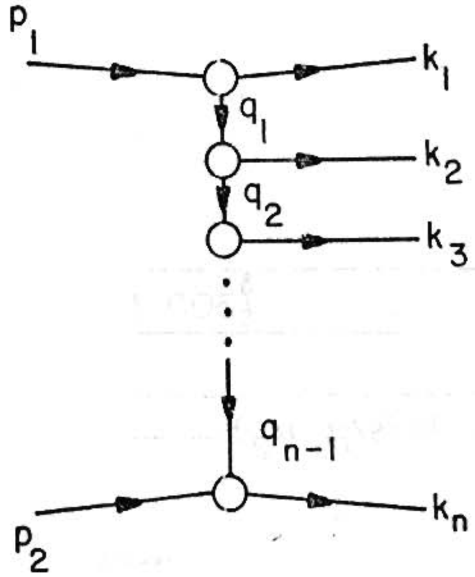


Fig. 2



0831170

Fig. 3