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Measurement of the Decay Asymmetry Parameters in $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^+\pi^0$

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Abstract

We have measured the weak decay asymmetry parameters (α_{Λ_c}) for two Λ_c^+ decay modes. Our measurements are $\alpha_{\Lambda_c} = -0.94^{+0.21+0.12}_{-0.06-0.06}$ for the decay mode $\Lambda_c^+ \to \Lambda\pi^+$ and $\alpha_{\Lambda_c} = -0.45 \pm 0.31 \pm 0.06$ for the decay mode $\Lambda_c \to \Sigma^+\pi^0$. By combining these measurements with the previously measured decay rates, we have extracted the parity-violating and parity-conserving amplitudes. These amplitudes are used to test models of nonleptonic charmed baryon decay.

13.30Eg, 14.20Kp

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The weak decays of charmed baryons are more complex than those of charm mesons, since non-spectator effects like W-exchange and internal W-emission are significant. The measured differences in the lifetimes of Λ_c^+, Ξ_c^+, and Ξ_c^0 [1] and measurements of branching ratios in various exclusive decay modes such as $\Lambda_c^+ \to \Xi^0 K^+$ [2] and $\Lambda_c^+ \to \Sigma^+\pi^0$ [3] demonstrate this. Currently, the strong interaction effects in these decays can only be calculated using models, so it is critical to provide as much experimental guidance as possible. Unlike many of the observed charmed meson decays, the degree of parity violation in Λ_c^+ decays to a pseudoscalar and hyperon is an additional experimental observable.

For the decays we are studying, $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^+\pi^0$ [4], the parity violation is manifested by a polarization of the hyperon (Λ or Σ^+). Because the hyperon decay also violates parity, the hyperon polarization (helicity) can be measured by its decay angular distribution.

We measure the helicity angle Θ in the hyperon's rest frame between the proton's momentum and the direction opposite the Λ_c^+ momentum, as shown in Figure 1. The distribution for this angle is given by:

$$R = \frac{1}{2}(1 + \alpha_{\Lambda_c}\alpha\cos\Theta), \tag{1}$$

where α and α_{Λ_c}, the weak decay asymmetry parameters of the hyperon and of the Λ_c^+, are defined as:

$$\alpha = 2\frac{\kappa \mathrm{Re}(AB^*)}{|A|^2 + \kappa^2|B|^2}, \tag{2}$$

where $\kappa = p_f/(E_f + m_f)$ with p_f, E_f and m_f being the momentum, energy and mass of the final baryon. Here A is the parity-violating amplitude, and B is the parity-conserving amplitude in that particular decay. The hyperon asymmetry parameter α is known, and α_{Λ_c} will be our measurement.

The data were collected with the CLEO II detector at the Cornell Electron Storage Ring (CESR), which ran at and below the $\Upsilon(4S)$ resonance. The CLEO II detector is a solenoidal-magnet spectrometer and electromagnetic calorimeter. The central drift chamber measures a charged particle's momentum and its specific ionization, which is used for particle identification. The time-of-flight system provides additional particle identification information. The calorimeter consists of 7800 CsI(Tl) crystals located inside the magnet. It has high efficiency, fine segmentation, and excellent energy resolution, allowing us to reconstruct Σ^+ hyperons through their decay to $p\pi^0$. A complete description of the detector can be found elsewhere [5]. We employ a GEANT [6] based detector simulation for our Monte Carlo (MC).

The total integrated luminosity for the data sample is 1.9 fb^{-1}, corresponding to about two million $e^+e^- \to c\bar{c}$ events. The hadronic event selection requires at least three charged tracks, visible energy greater than 0.15 of the energy in the center of mass, and a distance less than 5.0 cm along the beam direction between the reconstructed primary vertex and the interaction point.

Parity violation in the $\Lambda_c^+ \to \Lambda\pi^+$ decay mode has been observed before [7,8]. The Λ candidates are reconstructed from their $p\pi^-$ decay mode which has a branching fraction of 64.1%. We search for a pair of oppositely charged tracks which intersect at a radial distance of greater than 1 mm from the primary vertex. The higher momentum particle is assumed to be the proton, and the specific ionization (dE/dx) measurement is required to be consistent with that hypothesis. A χ^2 combining the distance between the tracks in z at the intersection point and the extrapolated impact parameter of the Λ candidate with the primary vertex is required to be consistent with that of a Λ coming from the primary vertex.

Candidates within 5 MeV/c^2 of the known Λ mass are combined with other positively charged tracks in the event. Combinatoric backgrounds are reduced by requiring that $\Lambda\pi^+$ combinations have $x_p > 0.5$, where $x_p = p/p_{max}$ and $p_{max} = \sqrt{E_{beam}^2 - m_{\Lambda_c^+}^2}$, and that the π^+ must be within 90° of the Λ_c^+ candidate's direction. The distribution is fitted to a 3$^{\rm rd}$ order Chebyshev polynomial and a Gaussian, fixed to the MC width of 7.9 MeV/c^2, plus a box-shaped function obtained from the MC to model the reflection from the decay $\Lambda_c^+ \to \Sigma^0\pi^+$, where $\Sigma^0 \to \Lambda\gamma$ and the γ is ignored. We observe 414 ± 30 events.

We divide the data into four $\cos\Theta$ bins and fit the $\Lambda\pi^+$ invariant mass distributions with the mean and the width fixed to that of the overall fit. Since the asymmetry of $\Lambda_c^+ \to \Sigma^0\pi^+$ can be different from that of $\Lambda_c^+ \to \Lambda\pi^+$ the relative size of the $\Lambda_c^+ \to \Sigma^0\pi^+$ reflection to the $\Lambda_c^+ \to \Lambda\pi^+$ signal is not fixed. The invariant mass distributions can be seen in Figure 2. A clear decrease in the number of $\Lambda\pi^+$ events occurs as $\cos\Theta$ increases.

We calculate the efficiency for each $\cos\Theta$ bin. The yields are then efficiency corrected, normalized, and plotted as a function of $\cos\Theta$, as shown in Figure 3. The slope is -0.30 ± 0.07, with a confidence level of 98%. Knowing that $\alpha_\Lambda = 0.64 \pm 0.01$, we obtain $\alpha_{\Lambda_c} = -0.94 {}^{+\,0.21}_{-\,0.06}$. The errors are statistical only, and they are asymmetric because α_{Λ_c} cannot be smaller than -1.0.

We have estimated a variety of systematic errors. The first is due to imperfect modeling of the efficiency. The MC predicted efficiency as a function of $\cos\Theta$ varies by a small amount, due to the loss of efficiency for low momentum pions from Λ decay, which curl up in the magnetic field. We have used pions from D^{*+} decays to study the accuracy of our modeling of the efficiency for low momentum pions. We also have varied the assumed fragmentation function of the Λ_c^+'s in our MC, since this changes the momentum distribution of the pions from Λ decay. We have varied the fitting procedure for finding the $\Lambda\pi^+$ yield by varying the background shape and by either modeling the $\Sigma^0\pi^+$ bump or excluding the region. Combining all sources of systematic error in quadrature, we find a total systematic error on α_{Λ_c} of ± 0.12 dominated by the fit to the $\Lambda\pi^+$ invariant mass distributions. Our result is consistent with previous measurements by CLEO [7] and ARGUS [8] of $-1.0^{+0.4}_{-0.0}$ and -0.96 ± 0.42 respectively.

We search for Σ^+ candidates in the $p\pi^0$ decay mode where the π^0 subsequently decays to $\gamma\gamma$. A more detailed description of our Σ^+ reconstruction technique can be found elsewhere [3]. Proton candidates are identified by their ionization loss in the central drift chamber and/or their time-of-flight. The Σ^+ is relatively long lived ($c\tau = 2.40$ cm), and decays a measurable distance from the primary interaction vertex. Hence protons from the Σ^+ decay have large impact parameters with respect to the primary vertex. We require that the proton impact parameter in the (r, ϕ) plane be greater than 0.6 mm.

The π^0 candidates are formed by pairing energy clusters in the calorimeter that are not matched to charged tracks, have an energy of at least 30 MeV, and have at least one of the clusters in the highest resolution portion of the calorimeter ($|\cos\theta| < 0.71$), where θ is the angle with respect to the beam axis. The photon momenta are then adjusted by a kinematic fit constrained by the known π^0 mass and by the assumption that the photons originate at the primary vertex. Candidates with momentum greater than 100 MeV/c are kept.

The Σ^+ candidates are identified with an iterative method that finds an estimated decay point and calculates the Σ^+ four-momentum assuming the decay occurred there. The π^0's are refit assuming the photons originate at this decay vertex. This procedure improves our Σ^+ mass resolution and signal-to-background ratio. We choose those $p\pi^0$ combinations within 15 MeV/c^2 of the nominal Σ^+ mass to be our Σ^+ candidates.

These Σ^+ candidates are combined with π^0's. We require the $\Sigma^+\pi^0$ candidates to have $x_p > 0.5$ and that the angle between the π^0 and the Λ_c^+ momentum vectors in the lab frame be less than 90°. This is identical to the procedure followed for the $\Lambda\pi^+$ case. We also require the π^0s from the Λ_c^+ to have mo4mentum above 500 MeV/c to further suppress combinatoric backgrounds. The distribution is fitted to a 3rd order Chebyshev polynomial and a Gaussian, fixed to the MC width of 20.8 MeV/c^2. We observe 89 ± 14 events.

As in the $\Lambda\pi^+$ case, we divide the data into four $\cos\Theta$ bins. The four different invariant mass distributions can be seen in Figure 4. An increase in the number of $\Sigma^+\pi^0$ events occurs as $\cos\Theta$ increases.

The normalized, efficiency-corrected $\Sigma^+\pi^0$ yield as a function of $\cos\Theta$ is plotted in Figure 5. We fit the plot to a straight line and find a slope of 0.22 ± 0.15 with a confidence level of 28%. The Σ^+ asymmetry is large, $\alpha_{\Sigma^+} = -0.98 \pm 0.015$. We therefore obtain $\alpha_{\Lambda_c} = -0.45 \pm 0.31$, where the error is statistical only.

We have considered systematic errors similar to those in the $\Lambda\pi^+$ case. The efficiency for the π^0 coming from the Σ^+ changes very slowly with momentum, since the electromagnetic calorimeter is efficient for photons above 30 MeV/c. Hence the efficiency is flat as a function of $\cos\Theta$. The fitting of the $\Sigma^+\pi^0$ invariant mass distributions is less sensitive to the background shape since there are no reflections from other particles. The total systematic error on α_{Λ_c} is ± 0.06 which is significantly smaller than in the $\Lambda\pi^+$ case.

Based on factorization, Bjorken [9] first predicted α_{Λ_c} to be -1 for $\Lambda_c^+ \to \Lambda\pi^+$. Since then, the two-body baryon-pseudoscalar decays of charmed baryons have been studied by a number of authors. Theoretical models predict the parity-conserving and parity-violating amplitudes, and from these the asymmetry and the decay rate can be calculated. Therefore, a useful model should correctly predict both the asymmetry and the decay rate. Since both $B(\Lambda\pi^+)/B(pK^-\pi^+)$ [10] and $B(\Sigma^+\pi^0)/B(pK^-\pi^+)$ [3] have been measured, we can make simultaneous comparison, which is shown in table I. The predictions of Żenczykowski agree best with the complete set of data. They are within 1σ for all quantities except the decay width of $\Sigma^+\pi^0$, which is within 2σ. Three of the five models have a strong disagreement with the $\Sigma^+\pi^0$ asymmetry, while are in better agreement with the other quantities. The predictions of Uppal, Verma and Khanna are quite good for asymmetries but poor for decay widths.

If a model fails to predict the decay width and asymmetry correctly, it can be instructive to compare its predictions of the $s-$wave and $p-$wave amplitudes, A and B , with those found from the experiment. We ignore any final-state interactions as do the theoretical models. Since A and B always appear quadratically we will get two sets of solutions. It is necessary to measure the angle between the initial and final baryon spins to discriminate between these solutions, which we are unable to do. The results are shown in Table II.

Cheng and Tseng's calculation agrees well on 3 of the 4 amplitudes in the first set. It only fails on A in the $\Sigma^+\pi^0$ case, where it has the wrong sign. Both Cheng and Tseng, and Xu and Kamal claim that A in $\Sigma^+\pi^0$ is sensitive to the on-shell correction.

In conclusion, we have made the first measurement of the decay asymmetry in $\Lambda_c^+ \to \Sigma^+\pi^0$ and an improved measurement of the decay asymmetry in $\Lambda_c^+ \to \Lambda\pi^+$. Most model predictions agree with the $\Lambda_c^+ \to \Lambda\pi^+$ measurements, while the $\Lambda_c^+ \to \Sigma^+\pi^0$ asymmetry measurement seems to be the hardest to predict.

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FIGURES

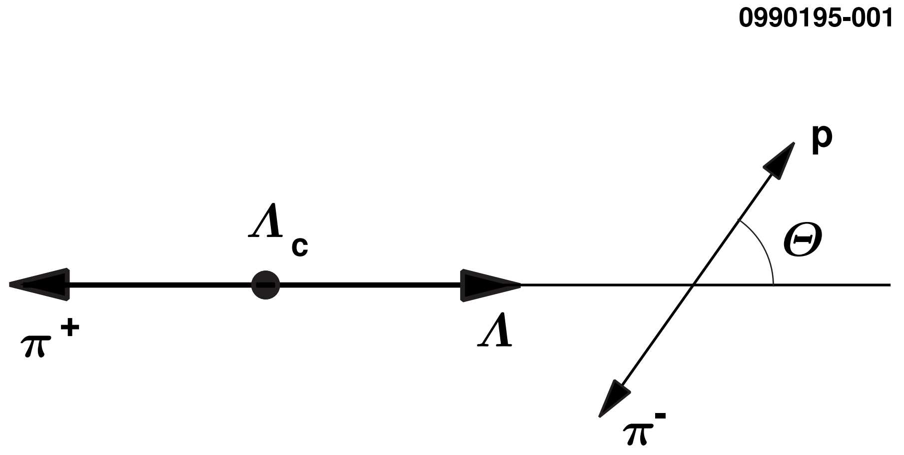

FIG. 1. Definition of the angle Θ in $\Lambda\pi^+$ decay

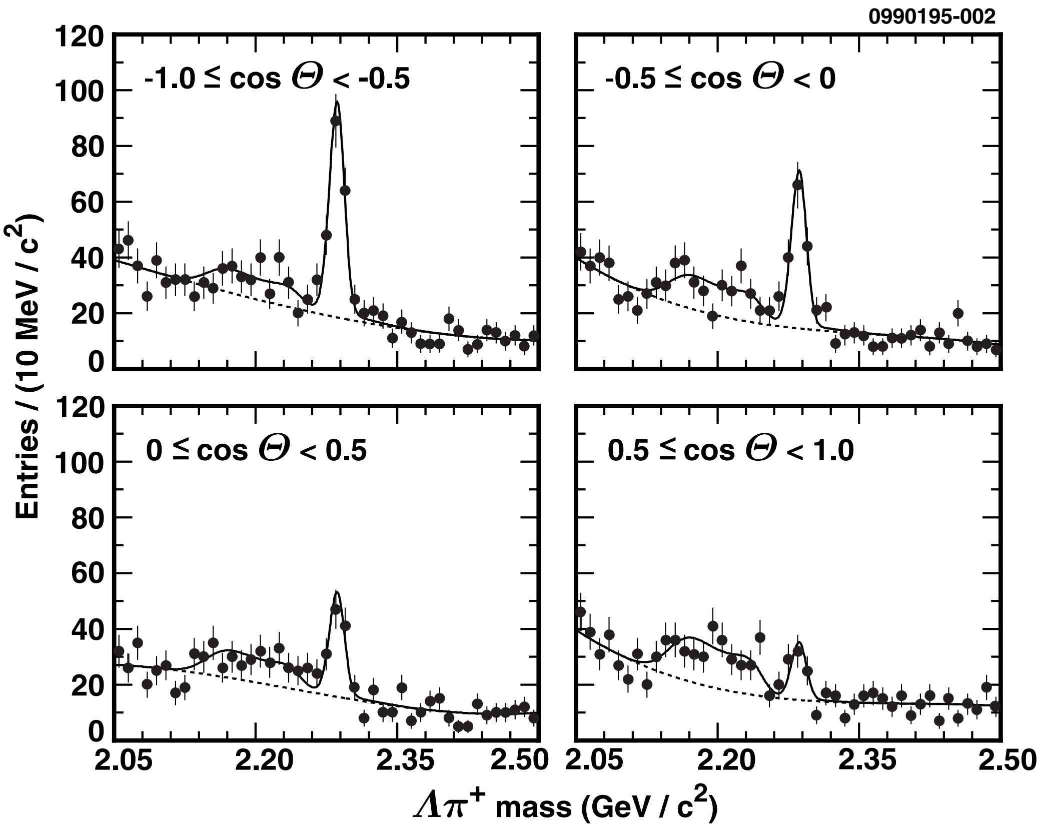

FIG. 2. $\Lambda\pi^+$ Invariant mass distribution in four $\cos\Theta$ bins

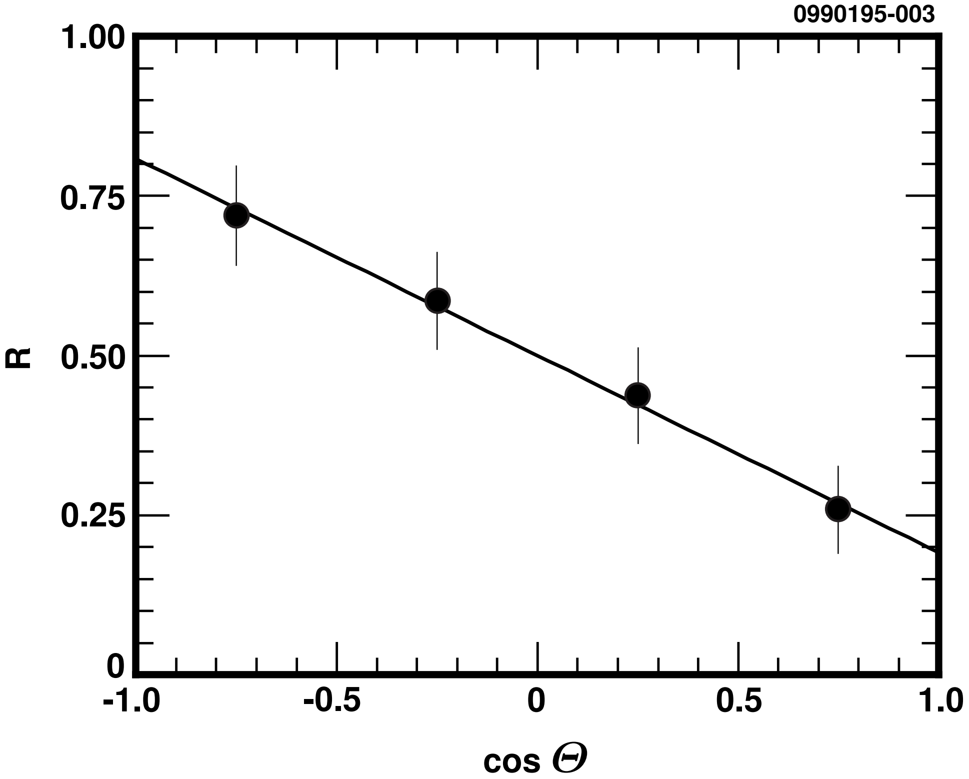

FIG. 3. Efficiency corrected yields vs $\cos\Theta$ for decay $\Lambda_c^+ \to \Lambda\pi^+$

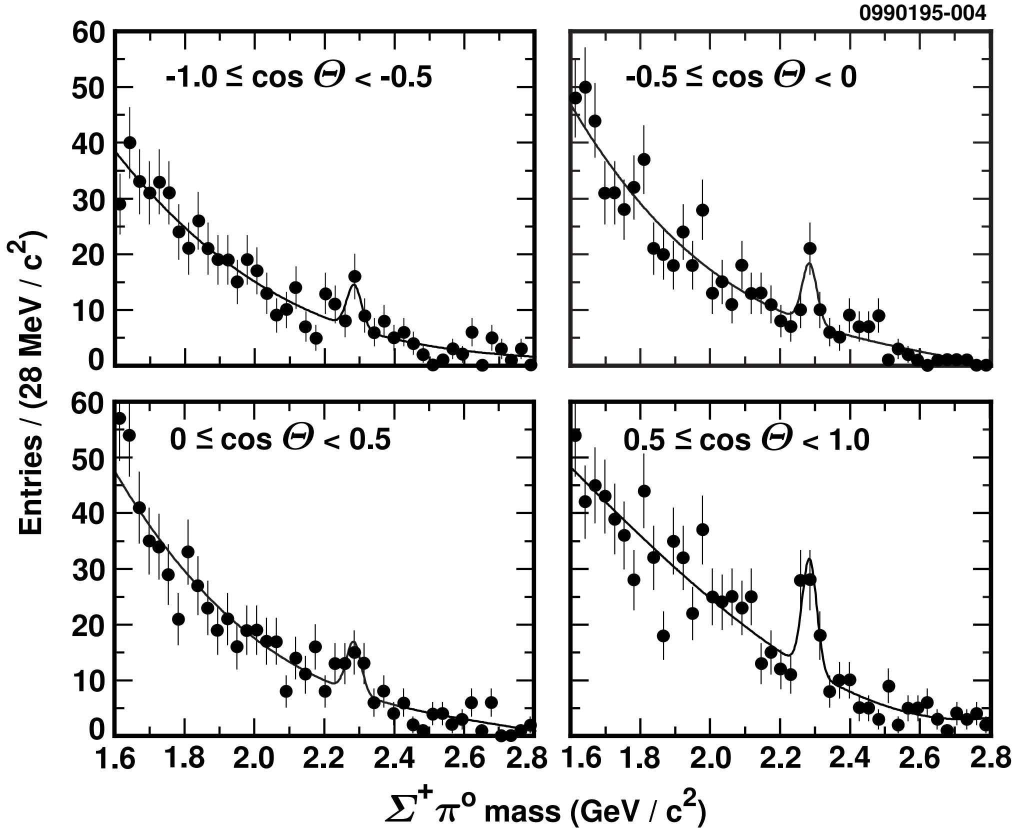

FIG. 4. $\Sigma^+\pi^0$ Invariant mass distribution in four $\cos\Theta$ bins

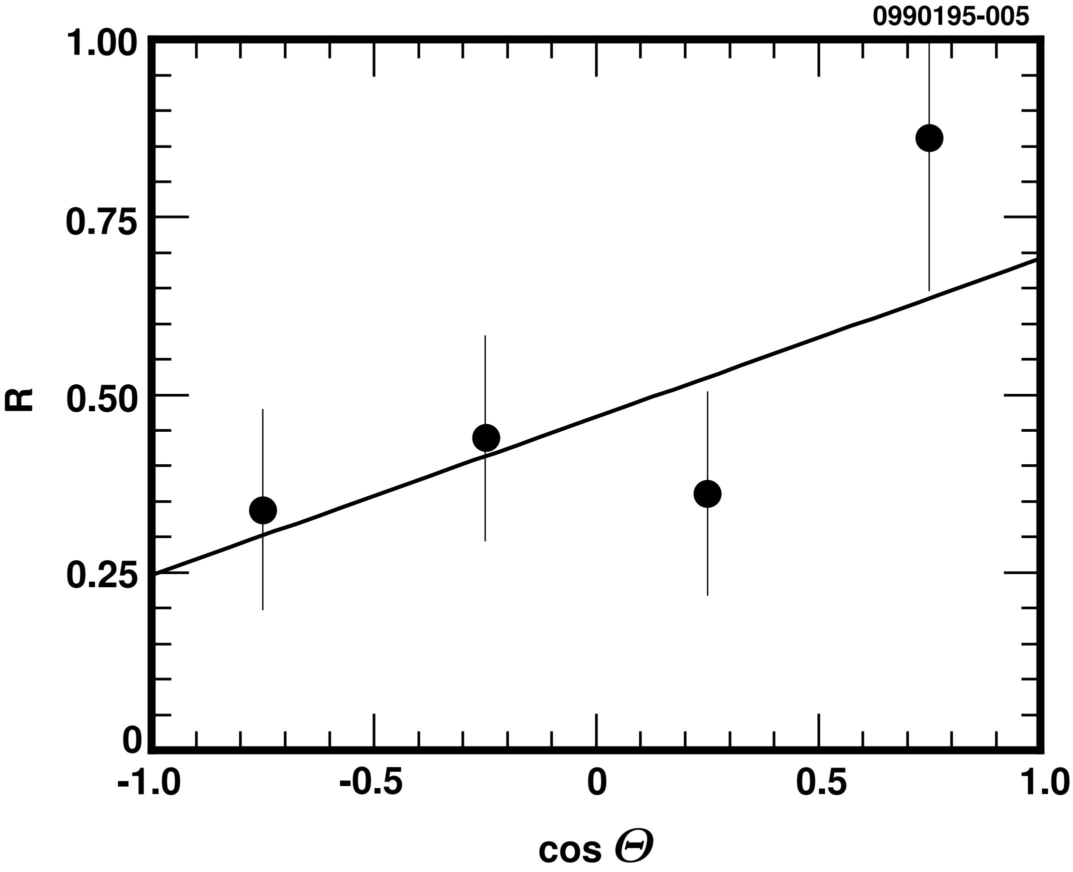

FIG. 5. Efficiency corrected yields vs $\cos\Theta$ for decay $\Lambda_c^+ \rightarrow \Sigma^+\pi^0$

TABLES

	$\Lambda_c^+ \to \Lambda\pi^+$		$\Lambda_c^+ \to \Sigma^+\pi^0$	
	$\Gamma(10^{11}\mathrm{s}^{-1})$	α	$\Gamma(10^{11}\mathrm{s}^{-1})$	α
CLEO	0.40 ± 0.11	$-0.94\ {}^{+\ 0.21}_{-\ 0.06}\ {}^{+\ 0.12}_{-\ 0.06}$	0.44 ± 0.12	$-0.45 \pm 0.31 \pm 0.06$
Xu & Kamal [12]	0.81	-0.67	0.17	0.91
Cheng & Tseng [13]	0.46	-0.96	0.38	0.83
Körner & Krämer [14]	0.37	-0.70	0.16	0.71
Uppal, Verma & Khanna [15]	1.17	-0.85	1.22	-0.32
Żenczykowski [16]	0.31	-0.86	0.23	-0.76

TABLE I. Comparison of experimental decay widths and asymmetries with model predictions. Experimental widths are derived from CLEO relative branching ratios and PDG 94 values of the Λ_c^+ lifetime and $pK^-\pi^+$ branching fraction. Theoretical predictions that were published as branching fractions have been converted to decay widths using Λ_c^+ lifetime.

	$\Lambda_c^+ \to \Lambda\pi^+$		$\Lambda_c^+ \to \Sigma^+\pi^0$	
	A	B	A	B
CLEO II	$-3.0\ {}^{+0.8}_{-1.2}$	$12.7\ {}^{+2.7}_{-2.5}$	$1.3\ {}^{+0.9}_{-1.1}$	$-17.3\ {}^{+2.3}_{-2.9}$
	$-4.3\ {}^{+0.8}_{-0.9}$	$8.9\ {}^{+3.4}_{-2.4}$	$5.4\ {}^{+0.9}_{-0.7}$	$-4.1\ {}^{+3.4}_{-3.0}$
Xu & Kamal [12]	-2.7	20.8	-2.9	-6.0
Cheng & Tseng [13]	-3.5	13.2	-2.4	-14.6
Körner & Krämer [17]	-1.9	13.9	-1.3	-9.9

TABLE II. Comparison of the measurements of the s and p wave amplitudes, A and B, in $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda_c^+ \to \Sigma^+\pi^0$ with theoretical predictions. The amplitudes have all been converted to common units of $G_F V_{cs} V_{ud} \times 10^{-2}$ GeV^2.