Single Iteration Tuning for Multicell RF Cavities for Cornell ERL

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A method for tuning multicell RF cavities was devised, accurate enough to achieve 99% field flatness with a final frequency within 50 kHz of the target RF frequency in a single iteration. The method is based on a model of capacitively coupled LC oscillating circuits and tuning the cavity is based on the mathematical predictions of tuning the LC circuit model. The electric field amplitude in each cell in the RF cavity is calculated from the frequency shift induced in each cell for all N_c modes by a metal bead. This data along with the resonant frequencies of the modes are fed into the LC model, and proper tuning shifts are calculated for each cell.

I. INTRODUCTION

Cornell University has proposed the construction of an Energy Recovery Linac (ERL) prototype where the main Linac will be 5 7-cell 1.3 GHz niobium superconducting RF cavities. It is important to the efficient operation of the prototype to have an even and perfectly timed accelerating RF cavity. In order to achieve the energy recovery goals and minimize loses, the RF accelerating cavities must be perfectly calibrated prior to installation into the ERL. Tuning is divided into two steps: homogenizing the field across all cells, then tuning the cavity as a whole to the proper final desired frequency. These two frequency shifts will be implemented in a single tuning of the cavity.

II. MEASURING RF CAVITIES

In order to test for field flatness, a setup for field measuring must be created. The setup must be able to determine through direct measurement or calculation the maximum electric field in each cell for all $\mathbf{TM_{010}}$ modes. However, directly measuring absolute field strength is too difficult so another technique must be devised. A tiny volume ΔV is chosen on the axis such that

$$|\overrightarrow{B}| \approx 0 \tag{1}$$

for the vanishing magnetic field and the electric field is approximately uniform, so that

$$|\overrightarrow{E}| \approx const. \tag{2}$$

Thus, adapting an equation from [Sla 50] yields

$$\frac{f'}{f} = 1 + \frac{1}{U}\int_{\Delta V} -\frac{\epsilon}{2}|\overrightarrow{E}|^2 \equiv 1 + \frac{\delta f}{f} \tag{3}$$

where U is the total energy stored in the system and ΔV is the volume of the bead. There would be a second integral over the energy density of the magnetic field as well, but is zero on the axis and left out. This equation implies that there is a correlation between a measured shift in resonant frequency of the cavity and the difference in the displaced electric

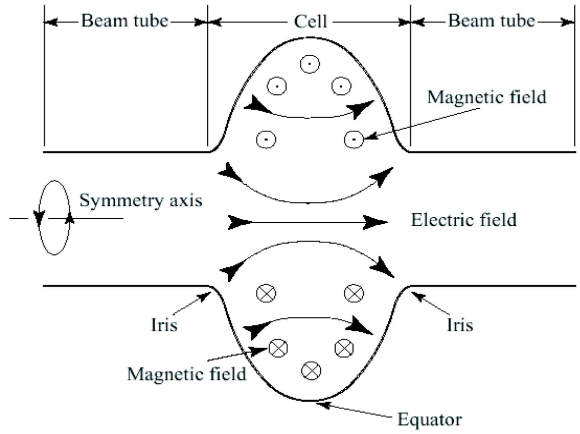

FIG. 1: The electric and magnetic fields inside of a single RF cell

and magnetic fields weighted to the respective ϵ and μ values. Fortunately, in the $\mathbf{TM_{010}}$ modes, the magnetic field vanishes and the electric field peaks as you approach the axis (see figure 1).

Using the fact that the electric field is approximately constant over the volume, the integral in equation (3) becomes

$$\int_{\Delta V} \frac{\epsilon_o}{2} |\overrightarrow{E}|^2 \approx \frac{\epsilon_o}{2} |\overrightarrow{E}|^2 \Delta V \tag{4}$$

Substituting equation (4) into equation (3), it follows that

$$\frac{f'}{f} \approx 1 - \frac{\epsilon_o}{2U} |\overrightarrow{E}|^2 \Delta V \tag{5}$$

which can be re-arranged to

$$f' - f \approx -f \frac{\epsilon_o}{2U} |\overrightarrow{E}|^2 \Delta V \tag{6}$$

or

$$\delta f \propto |\overrightarrow{E}|^2 \tag{7}$$

This means that perturbing the cell at the axis will perturb only the electric field and therefore show up as a change the resonant frequency of the cavity. Thus, by measuring the relative frequency shifts δf of the resonant frequency of a cavity while perturbing each cell uniformly, the relative electric fields can be calculated.

III. BEADPULL METHOD OF FIELD MEASURING

To find the maximum electric fields in each cell, $\overrightarrow{E} \approx \sqrt{(|\delta f|)}$ is traced as a function of position z along the axis of the cavity. To uniformly perturb all cells of a cavity to measure the relative frequency shifts, a metal bead was placed on a nylon string ($\epsilon = 3.5$ at 1 MHz) and run along the axis of the cavity. The cavity was oriented horizontally with the tension on the nylon string so that sag by the string was less than 1 mm and therefore insignificant to desired measurements. An HP-8753c Network Analyzer was connected to

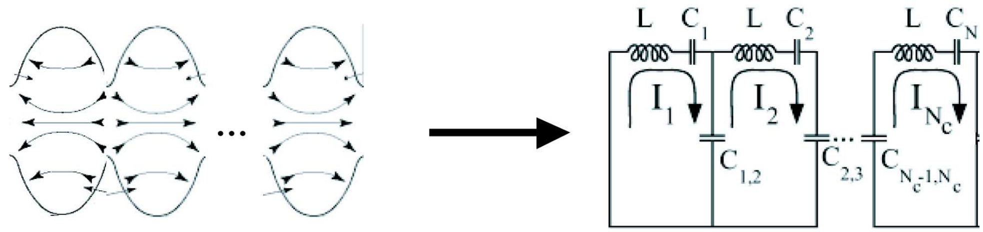

FIG. 2: Coupled LC oscillators circuit model of RF cavity

feedthroughs mounted on copper endplates at the cavities ends with a 1 inch hole concentric to the axis of the cavity with antennae extending into the cavity. The network analyzer traced the frequency shift of the frequency of the cavity with respect to the unperturbed cavity's resonant frequency, all in the phase mode. The bead was brass and chosen in size such that the resonant frequency of the whole cavity shifted no more than 80kHz. The bead was driven by an Mdrive17 stepping motor from IMS. The setup was computer controlled and automated so that the user could leave the room during measuring and not disturb the cavity. A Labview program was written based on a DESY program that set up the network analyzer, then cycled through reading the frequency shift from the network analyzer, plotting the frequency shift and the electric field, then stepping the motor. The program was designed so that virtually every parameter could be controlled via the Labview console, such as span, step size for motor, etc. A step size resolution of 1 mm was used with the bead starting at the edge of the flange and stepping all the way through the cavity. This process was run for all $\mathbf{TM_{010}}$ modes in the cavity, equal to the number of cells in the cavity. This data was then used to determine how much the cavity should be tuned to achieve field flatness at the correct frequency.

IV. MATHEMATICAL TUNING

Note: model and ensuing mathematics closely follow those initially discussed by [Sek 90] and later by [Liepe 2001]

The N_c coupled oscillators of an RF cavity are often modeled as N_c capacitively coupled LC oscillators. Each oscillator has its own periodicity, each oscillator affects only the oscillator on each side proportional to its coupling constant, and there are N_c first order modes of harmonic oscillation for N_c cells, all similar to an N_c cell RF cavity (see figure 2).

In addition, this model has an ω^j for the entire circuit for each mode which corresponds to the resonant frequencies ω^j of the N_c modes in the RF cavity. Furthermore, the relative electric field in each cavity for each mode corresponds to the current I_n^j in the n^{th} cell for the j^{th} mode. Kirchhoff's loop rule was used to solve for the currents, which yielded the following equations with impedance values

$$V_L + V_{C_1} + V_{C_{1,2}} = (i\omega^j L + (\frac{1}{i\omega^j C_1}) + (\frac{1}{i\omega^j C_{1,2}}))I_1 - (\frac{1}{i\omega^j C_{1,2}})I_2 \quad (8)$$

$$\begin{aligned} V_L + V_{C_n} + V_{C_{n-1,n}} + V_{C_{n,n+1}} &= -(\frac{1}{i\omega^j C_{n-1,n}})I_{n-1} + (i\omega^j L + (\frac{1}{i\omega^j C_n}) + (\frac{1}{i\omega^j C_{n-1,n}}) + (\frac{1}{i\omega^j C_{n,n+1}}))I_n \\ &\quad -(\frac{1}{i\omega^j C_{n,n+1}})I_{n+1} \end{aligned} \quad (9)$$

$$V_L + V_{C_{N_c}} + V_{C_{N_c-1,N_c}} = -(\frac{1}{i\omega^j C_{N_c-1,N_c}})I_{N_c-1} + (i\omega^j L + (\frac{1}{i\omega^j C_{N_c}}) + (\frac{1}{i\omega^j C_{N_c-1,N_c}}))I_{N_c} \quad (10)$$

multiplying equations (8), (9), and (10) by $i\omega_o^2\omega^j C_n$ and setting

$$LC = \frac{1}{\omega_o^2}, \frac{C}{C_n} = 1 + \delta_n, \frac{C}{C_{n,n+1}} = k_{n,n+1} \quad (11)$$

Equations (8), (9), and (10) become

$$\omega_o^2((1 + \delta_1 + k_{1,2})I_1^j - (k_{1,2})I_2^j)) = (\omega^j)^2 I_1^j \quad (12)$$

$$\omega_o^2(-(k_{n-1,n})I_{n-1}^j + (1 + \delta_n + k_{n-1,n} + k_{n,n+1})I_n^j - (k_{n,n+1})I_{n+1}^j) = (\omega^j)^2 I_n^j \quad (13)$$

$$\omega_o^2(-(k_{N_c-1,N_c})I_{N_c-1}^j) + (1 + \delta_{N_c} + k_{N_c-1,N_c})I_{N_c}^j) = (\omega^j)^2 I_{N_c}^j \quad (14)$$

for the n^{th} cell of the j^{th} mode. This can be better expressed by the matrix equation

$$\omega_o^2 \mathbf{A} I^j = (\omega^j)^2 I^j \quad (15)$$

where A is the tridiagonal matrix

$$\mathbf{A} = \begin{bmatrix} 1+\delta_1+k_{1,2} & -k_{1,2} & 0 & \cdots & 0 \\ -k_{2,1} & 1+\delta_2+k_{2,1}+k_{2,3} & -k_{2,3} & \cdots & 0 \\ 0 & -k_{3,2} & 1+\delta_3+k_{3,2}+k_{3,4} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & -k_{N_c-1,N_c} \\ 0 & 0 & \cdots & -k_{N_c,N_c-1} & 1+\delta_1+k_{N_c,N_c-1} \end{bmatrix} \quad (16)$$

which is defined by the physical constraints and geometry of the cavity and is constant unless the cavity changes. In addition the current vector

$$I_n^j = \begin{bmatrix} I_1^j \\ I_2^j \\ \vdots \\ I_{N_c}^j \end{bmatrix} \quad (17)$$

is the relative current in the n^{th} oscillator in the j^{th} mode of the circuit, corresponding to the relative electric field strengths in the n^{th} cell in the j^{th} $\mathbf{TM_{010}}$ mode of the cavity. In the model, ω^j represents the resonant frequency of the j^{th} mode of the circuit and cavity.

Furthermore, the $\omega_o^2\mathbf{A}$ matrix has many mathematical implications critical to the model. For example, equation (15) is an eigenvalue matrix equation with ω^j as the eigenvalues and I_n^j as the corresponding j^{th} column eigenvector .

On a physical level, it seems evident that by changing the relative shapes of the cells (thereby changing the frequency of each cell), there should exist a solution such that the field inside of all cells in the π mode have the same magnitude. In addition, it seems evident that by tuning all cells the same amount, the resonant frequency of the cavity could be changed *without* changing the relative field strengths. Mathematically, this is achieved through changing the $\omega_o^2\mathbf{A}$ matrix such that the eigenvalues are the desired frequencies of the model and the eigenvectors are the corresponding relative currents in each oscillator.

According to the model, this is achieved by changing the relative capacitances C_n of each oscillator, the variable that gives rise to the different I_n for each cell. Therefore, there exists a matrix $\mathbf{P}$ such that

$$\omega_o^2(\mathbf{A}+\mathbf{P})I^j_{tuned} = (\omega^j_{tuned})^2 I^j_{tuned} \tag{18}$$

where

$$\mathbf{P} = \begin{bmatrix} P_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & P_{2,2} & 0 & \cdots & 0 \\ 0 & 0 & P_{3,3} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & P_{N_c,N_c} \end{bmatrix}, I_{tuned} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ \vdots \\ (-1)^{N_c-1} \end{bmatrix} \tag{19}$$

is a diagonal matrix representing the amount to tune each capacitor so that the ideal currents and frequencies are calculated.

V. PHYSICAL TUNING

Since all of the modeling was done using relative measurements, but the cavity must have an absolute characteristics, a method must be devised to translate the mathematical change into a physical number. The easiest way is to tune cell by cell, which is the same as tuning element by element of the $\mathbf{P}$ matrix. Thus, setting all values of the $\mathbf{P}$ matrix to 0 except $P_{1,1}$ is like mathematically tuning just the first cavity the exact amount it should be. Adding the new $\mathbf{P}$ matrix to the $\mathbf{A}$ matrix, then calculating the eigenvalues of the $\omega_o^2(\mathbf{A}+\mathbf{P})$ matrix will return the absolute values of the modes after the first cell is correctly tuned. Due to changing conditions in the lab, the frequency shift from the measured frequency of the π mode and the calculation is used to tune the cell. The first cell is then tuned until the π mode has shifted the same amount that the model says it should in order for cell 1 to be in tune. The process is repeated for cell two. All values but the first two on the diagonal in the $\mathbf{P}$ matrix are set to zero, and the eigenvalues of this $\omega_o^2(\mathbf{A}+\mathbf{P})$ matrix are calculated. The π mode frequency is then subtracted from the final frequency after just cell one is in tune, yielding the frequency shift the π mode undergoes while tuning the second cell with the first cell already in tune. The process is then carried out for all cells.

Referring back to equation (3), a frequency shift can be induced as with the bead by changing only the electric field of a cell. If such a change were permanent, the frequency shift of the cavity would be permanent. Tuning the cells works on this premise. Cells are deformed by crushing or pulling the sides of the cell. This greatly changes the electric field across the cell, but barely changes the vanishing magnetic field by the iris of the cell. The cells are tuned in order that the math tells us. The tuning mechanism consists of two steel plates, approximately 16 inches square, 1/2 inch thick with 3 3/4 inch holes cut in the middle to match the outer diameter of the cavity's iris. The plates were tapered down to approximately 1/4 inch thick at the edge of where the hole was cut, starting approximately four inches away from the edge. This allowed the plates to fit between the cells snugly and conform to the elliptical shape of the cell it was tuning. The plates were cut in half, one of each half mounted vertically on a track on a heavy duty two way vice. The cavity was placed so that the plates were on either side of the cell desired to be tuned. The other half

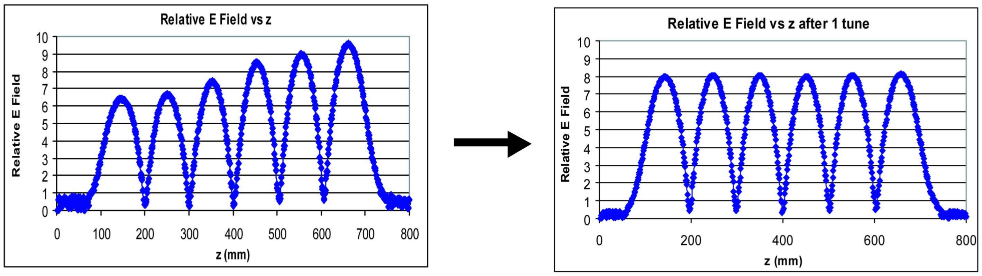

FIG. 3: The relative electric field strength calculated from beadpulls in arbitrary units along the axis (l) before the tuning (r) after the tuning

of both plates were placed to match up with their counterparts on the track, hugging the iris on either side of the cell to be tuned. The plates were then moved together crushing the cavity or apart pulling the cavity, to lower or raise the frequency, respectively.

This process was used to tune a 1.5 GHz 6 cell niobium cavity. Note that the Cornell ERL proposal wishes to have 1.3 GHz cavities, but this experiment was a development of a process to be used on the future RF cavities. The beadpull frequency shift for all six $\mathbf{TM_{010}}$ were recorded and converted to relative field strength. This data along with the resonant frequencies were fed into a matlab program which performed all the aforementioned mathematics and output the frequency shift to be observed by the π mode while tuning each cell in order. The tuning of the cavity was done using aforementioned tuning apparatus to stretch and crush the cells. Due to the nature of niobium which has an elastic stretch, the cells must be over tuned so that the cavity will relax back to the desired frequency. The final frequencies of the π mode after tuning each cell were all within 3 kHz of the actual shift determined by matlab.

VI. RESULTS AND CONCLUSIONS

The flatness of the cavity went from 83% to 99% with a single iteration using the formula

$$fieldflatness = \frac{\overline{(peaks)}}{peak_{max}} \tag{20}$$

In addition, the resonant frequency of the cavity went from 1,499.863 MHz (off by 154 kHz) to 1,500.050 MHz (off by 50 kHz). This minute discrepancy can be accounted for by minute thermal shifts between the time of tuning and the final measurement taken.

Similarly, when the cavity is in the linac, it will be at $\sim$ 2K. The temperature change induces a drastic change in resonant frequency of the cavity, but since the niobium is homogeneous and symmetrical, it will shrink evenly and all of the relative dimensions will be the same in the cavity. Thus, the relative field strength does *not* change but the resonant frequency *does* change. The goal for this experiment was a warm resonance tuning exercise, further research must be done to find out how much the resonant frequency of the cavity will change upon cooldown. Once this number is determined, it must be added to the 17 kHz of the nylon string and added on to 1.300000 GHz. This will be the input to the matlab program as the final desired frequency of the cavity so that when placed in the machine in LHe, the cavity should have a flat field at exactly 1.3 GHz.

Since resonant frequencies of the cavity change in the lab and inside the cryostat, a cold tuning system is to be used in conjunction with the RF cavities. This will be modeled after the ones used at the Tesla Test Facility (TTF) which consists of a mechanical device that stretches the whole cavity uniformly within the cryostat. All cold tuning is done within the elastic limit of niobium and has a range of a few hundred kHz. Thus, field flatness must be perfect at room temperature, but the resonant frequency may vary from the goal number of 1.3 GHz (+ nylon string shift + cooldown shift) as lab conditions change because it can be made up for in the cryostat.

The goal was met to achieve 95% field flatness within $\sim$ 50kHz in a single iteration. Accuracy should improve as the setup becomes permanent and the tuner becomes trained. The setup was merely a prototype and with more accurate devices and controlled lab conditions (especially during measuring) accuracy should improve. The greatest source of error in the process is still the human error in tuning, especially due to the elastic nature of the niobium. Even having a perfect mathematical model and resolution greater than 1 kHz does not help because overtuning the cell by about 100 kHz to relax back perfectly to a value $\pm$ 1 kHz requires better lab training for a tuner.

VII. ACKNOWLEDGEMENTS

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