

# Determination of ERL optics with Undulators

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Cornell University is currently investigating the feasibility and benefits of building an Energy Recovery Linear Accelerator. One possible layout is shown in Fig.1. The ERL will use undulators (series of dipole magnets with alternating poles), to produce high quality x-ray beams. Part of this investigation is optimizing the electron/positron bunches to make sure that the required beam parameters can be achieved. In order to manipulate the beams and achieve the proposed results you need to alter the magnet strengths within the ERL. Here I will investigate whether it's possible to take into account the undulators' effects on the particles' optics and still achieve the desired beam parameters. We will show that it is possible to conform the beam to the specifications all the way throughout the ERL. Computer simulation software and computer optimizers were used to help us attain our goals.

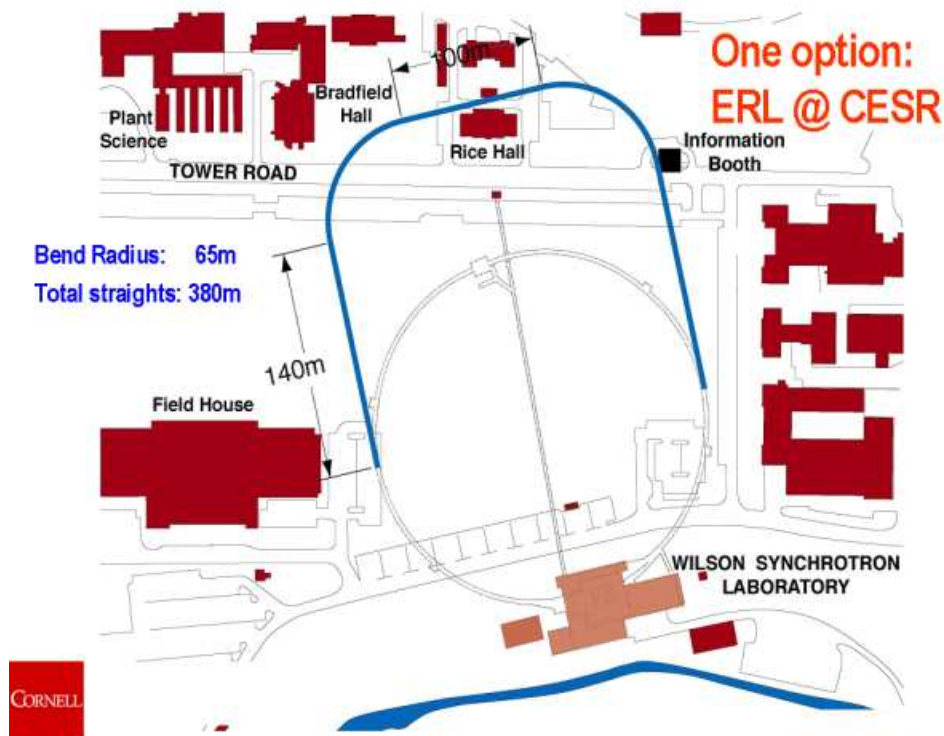


FIG. 1: One of the proposed construction sites for the ERL, using existing infrastructure of CESR

## I. INTRODUCTION

The benefit of building an ERL is it's ability to produce low electron emittance and higher brilliance x-rays. These x-rays will be used by the CHESS facility at Cornell for research in

various areas. Currently Cornell has an  $e^+$ ,  $e^-$  storage ring in operation, CESR. Although CESR through synchrotron radiation can produce x-rays for the CHESS laboratory facility experiments, their parameters are inferior to what an ERL could provide. Being able to produce intense high brilliance x-rays will allow a new realm of research to open up and to fine tune current studies. An ERL, like a storage ring, achieves the intense x-ray beams by means of undulators. However in the case of an ERL the electrons only travel through the device once so that small emittances of an electron source can be used, while in a storage ring they travel through each undulator for millions of turns. In spite of this, as in a storage ring, the properties of the beam have to be checked through the ERL to make sure that the beams have the required sizes and bunch-length in each undulator.

## II. ENERGY RECOVERY LINEAR ACCELERATOR (ERL)

The energy recovery scheme is a relatively new proposed method of operation for a particle accelerator. Like the name implies it is able to recover the energy from used electron/positron beams. In doing so it saves electricity and allows for higher currents. Electrons aren't dumped with high energies thus re-using this energy saves power and money. Furthermore, the problem of finding a sufficient and safe method of dumping very high beam powers is reduced.

The anticipated method for retrieving the energy from the used bunches involves sending these bunches around a return loop once, extracting their energy on a second turn through the linac, then dumping the beams. While the particle bunches are sent around the ERL they need to have the desired properties in each undulator. Once they have been used, they are sent through the linac once more. However, this time the electron/positron beams are 180 degrees out of phase with the accelerating fields. The fields will then decelerate the beams which transfers their energy to the ERL cavities to be used to accelerate the next bunches that pass through the accelerating cavities.

This method of energy recovery allows better manipulation of beam properties. As opposed to storage rings, which let the particles circulate over and over in the storage ring until they reach an equilibrium beam size, the ERL will only let them circle once. Storage rings most often do not produce the optimal desired beam configurations.

The custom method for producing the desired coherent, high emittance x-rays is using undulators within the ERL. Undulators, like wiggler magnets, have the property of wiggling the beams when they pass through them. The beams follow a snake-like trajectory as shown in Fig.2 while going through these types of magnets.

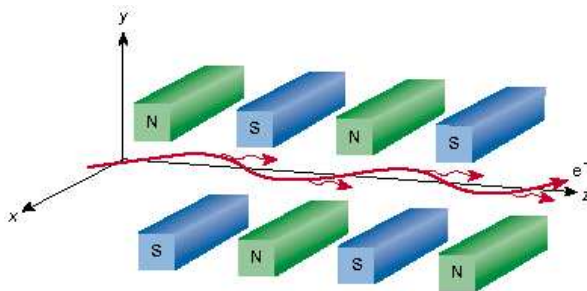


FIG. 2: Snake-like particle trajectory as it goes through an undulator

Since the electrons follow a bent trajectory they emit radiation. This radiation opens at an angle inversely proportional to the relativistic  $\gamma$  of the particles. Hence, if the particles "wobble" more than this characteristic angle each time they are bent they produce a sweeping light cone. Such a situation is produced by wiggler magnets. In an undulator the wobbling effect is not as intense. The particles wiggle less than the light cone opening, thus the light cones of all bends interfere as seen in Fig.3.

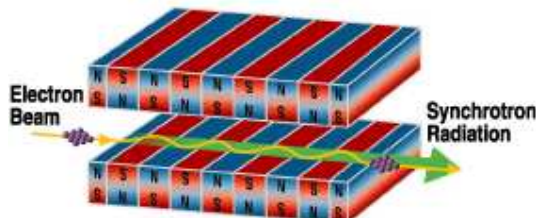


FIG. 3: Coherent radiation emitted by wiggling particles bunch through undulator

In order to maximize the results from the undulators the beams need to be sent in with the appropriate parameters. The constraints include:

- a)  $\beta_x$ . Used to describe the beam envelope, i.e. width of its horizontal cross-section. The envelope is proportional to the square root of  $\beta_x$ [1].
- b)  $\beta_y$ . Corresponding to  $\beta_x$ , but in the vertical direction.
- c)  $\alpha_x$ . Defined as  $-\frac{1}{2}d\beta_x/dz$ .
- d)  $\alpha_y$ . Corresponding to  $\alpha_x$ , but in the vertical direction.
- e)  $\eta$ . The deviation from the ideal trajectory for particles with energy deviations, called dispersion.
- f)  $\eta'$ .  $d\eta/dz$ , called dispersion prime.
- g) Time of flight (R56). This is related to how compressed along the direction of motion the bunch of particles is, i.e. to the length of the bunch.

All of these constraints are calculated using first order equations. Furthermore, we use second order equations to satisfy constraints as:

- h) Second order  $\eta$
- i) Second order  $\eta'$
- j) Second order Time of Flight (T566)

### III. FITTING

Constraints and the aforementioned parameters all have to be satisfied in the middle of each undulator since it is there where it is most crucial to have the desired beam properties. The current proposed version of the ERL at Cornell University incorporates 7 undulators. They are arranged in a mirror symmetric fashion. Coming from the east we have a 2m undulator, then two 5m undulators. Then there is the longest and most important undulator of 25m length. Finally, there are the two 5m undulators followed by the 2m undulator, all shown in Fig. 4.

I chose to split the arc into the 8 pieces of Fig.4. The 8 sections are:

- 1) Start - 2m undulator
- 2) 2m undulator - 5m undulator

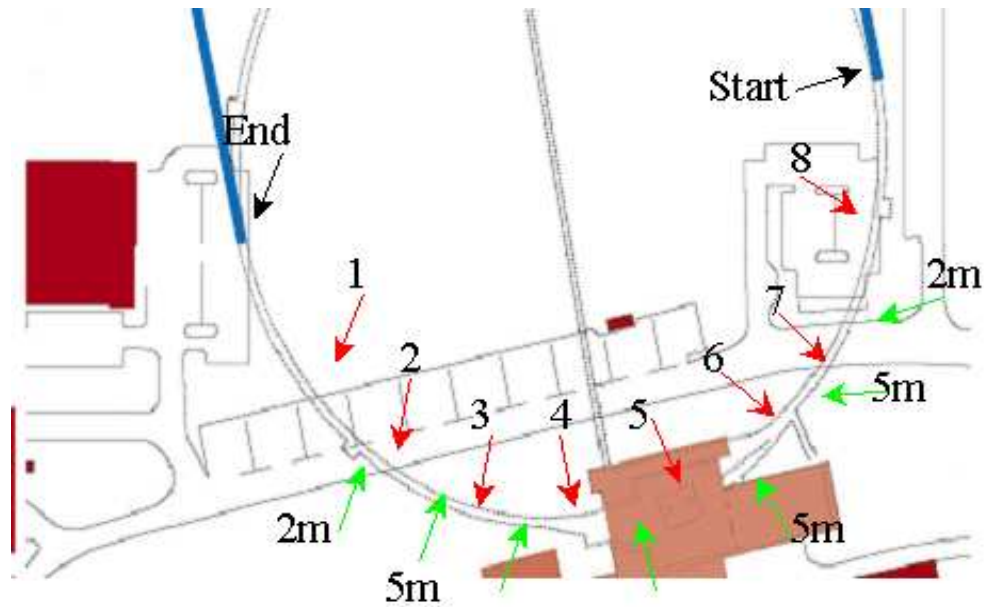


FIG. 4: Green arrows indicate positions of the undulators in the ERL. Red arrows indicate fit sections.

- 3) 5m undulator - 5m undulator
- 4) 5m undulator - 25m undulator
- 5) 25m undulator - 5m undulator
- 6) 5m undulator - 5m undulator
- 7) 5m undulator - 2m undulator
- 8) 2m undulator - End

Within each piece, except the last, I match the first conditions of the six types (a-f). This is done using the quadrupoles available in each section. All except the first and last sections have exactly as many quadrupoles as constraints. The second order constraints are taken care of by using six available sextupoles. Sequentially in order these fits are:

- a) Fit section 1
- b) Fit section 2
- c) Fit section 3
- d) Fit section 4
- e) Use section 1 to fit the time of flight (constraint g) in the middle of the 25m undulator
- f) Fit section 5
- g) Fit section 6
- h) Fit section 7
- i) Fit section 8 and time of flight (constraint g)
- j) Fit second order constraints to the 25m undulator, using the sextupoles in section 1
- k) Fit second order constraints to the end, using the sextupoles in section 8

We are concerned with the time of flight constraint (constraint g) only in the middle of the arc and at the end. Therefore, after completing the first four fits, I use again the first section to fix the time of flight in the middle of the 25m undulator since the first section has extra magnets. I proceed to fit the last four sections and I also use the last section to fix the time of flight at the end of the arc, again because it has more magnets. Finally, after these 9 fit routines I proceed to fit the second order optics with the six sextupoles. The first

three are used to fix the three second order constraints to the middle of the 25m undulator, the last three sextupole magnets fix these constraints at the end of the arc.

#### IV. ACTUAL CALCULATIONS

Once the above procedure has been completed, first only taking into account the first order optics and no effect of the undulators, it produced a good solution for the specific optimization. After that the the second order optics constraints were added and everything was refit. One this procedure was completed the undulators and their linear and non-linear effects were added. They were added two at a time, symmetrically, starting from the outside working in. Each time while adding the undulators the magnet strengths were refit both for the first and the second order optics. Finally after adding the long 25m undulator the final magnet strengths were obtained.

In adding the undulators we neglected to specify an appropriate number of sections into which the device is cut during optics calculations. BMAD, based on CERN's MAD, and developed further by the Cornell University Physics Department[3], is used for ERL simulation purposes. The specifications of the ERL are stored in a lattice file, including all magnets, their strengths, parameters and the final setup of the ERL. In the undulator's case, the program splits the magnets up into little sections, since the undulators are made up of 100 to 300 alternating polarity dipoles. For each section it calculates a magnetic field and uses that field to determine how the beam is moving through that part of the undulator.

The undulators were chosen to be split into about five times the number of dipoles they contained. Since within each complete dipole the particle undergoes a whole wavelength wiggle, splitting the dipole into five pieces produces precise data without taking too long to compute the fields. The optimization process with and without the undulators was performed in the same order as described above.

#### V. RESULTS

As can be seen from Fig.5 the original quadrupole  $K$  values, that I started my optimization process with which are used to calculate the field of the magnets using eq. 1[2], differ very little from the new  $K$  values.

$$K \equiv \frac{dB_y}{dx} / (B\rho) + \rho^{-2}, B\rho \equiv p/e \quad (1)$$

The quadrupoles that were most affected were the ones around the middle (positions 39-42), close to the 25m undulator. They changed by about 3.5 to 8%.

Also the sextupole  $K$  values remained sufficiently small, within a range of  $0.02/m^3$  -  $-0.00005/m^3$  as can be seen in Fig.6, then all the second order optics constraints are satisfied perfectly.

The  $\beta_x$  and  $\beta_y$  function values were within the acceptable limits, usually below 100m as shown in Fig.7. In Figures 7,8,9,10 the 7 undulators are located at 113m, 133m, 155m, 188m, 220m, 242m, and 262m. In the case of the beta function values there is a small problem point towards the end where  $\beta_x$  rises to about 130m. Running another fit loop on the last part of the arc, where the problematic beta and the extra quadrupoles exist, can most likely rectify this problem.

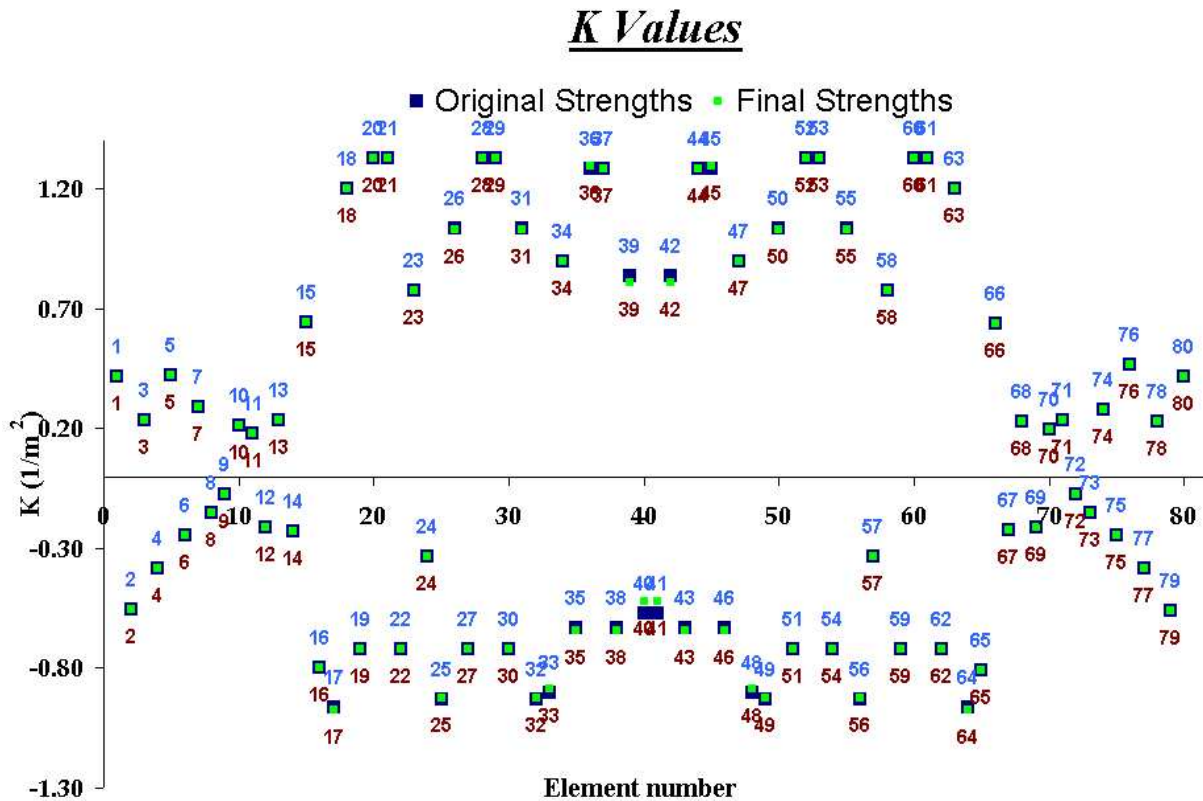


FIG. 5: Original and final quadrupole K values

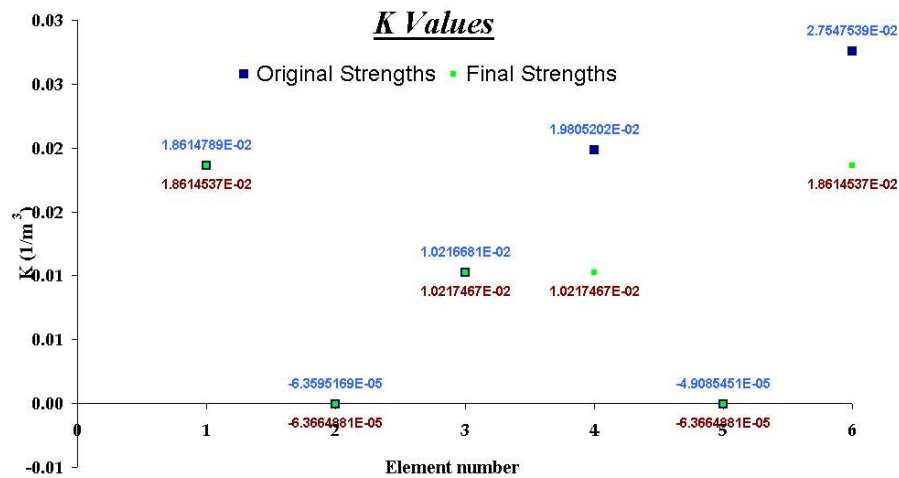


FIG. 6: Original and final sextupole K values

The  $\alpha_x$ ,  $\alpha_y$ ,  $\eta$  and  $\eta'$  constraints were all satisfied to a high degree. Within the center of each undulator it was required that these constraints be zero as shown in Figs. 8, 9,10.

Also, the time of flight constraint was satisfied very well, see Fig.10. By optimizing it within the first section of the arc to the middle produced a desired value of -0.22437m. For the last section of the arc the required time of flight,  $R56 = 0$ , was achieved.



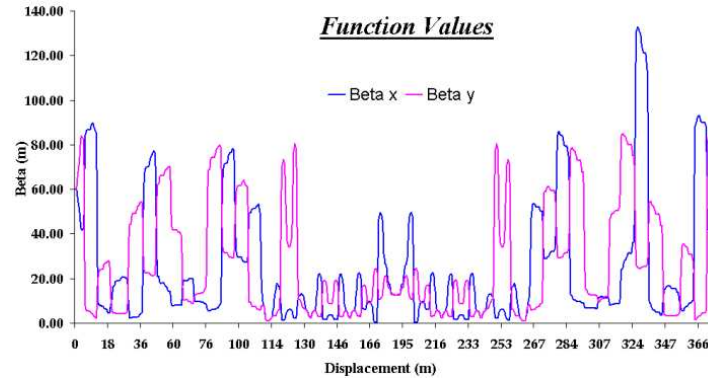


FIG. 7: Beta function values after optimization

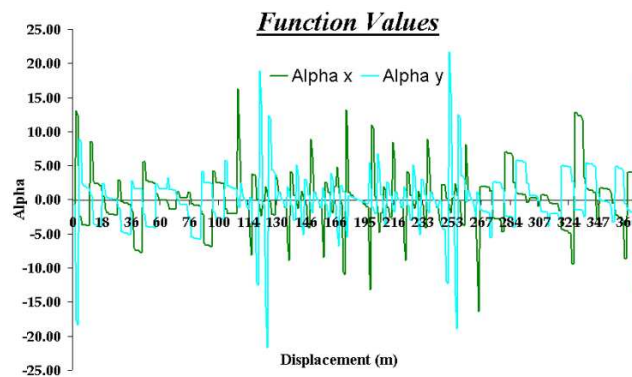


FIG. 8: Alpha function values after optimization

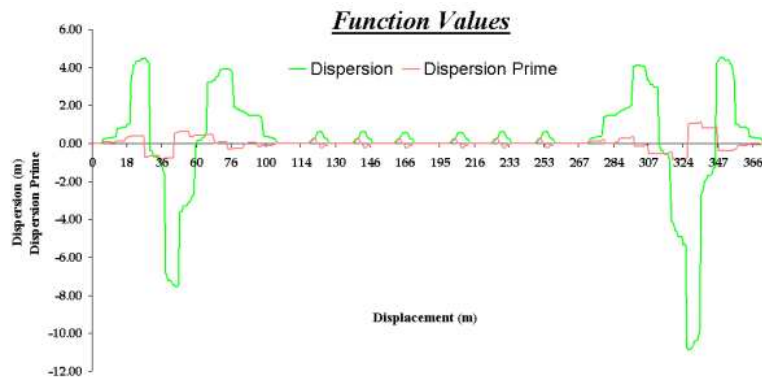


FIG. 9: Dispersion and dispersion prime function values after optimization

## VI. CONCLUSIONS

After all optimization has been completed, one may introduce a few more fit sections within the first and last part of the arc to better fit constraints within the arc. As an example one could try to reduce the beta values towards the end of the arc. Even these preliminary results now are promising, since they show that the perturbations that undulators produce in the first and second order optics can easily be compensated by moderate changes

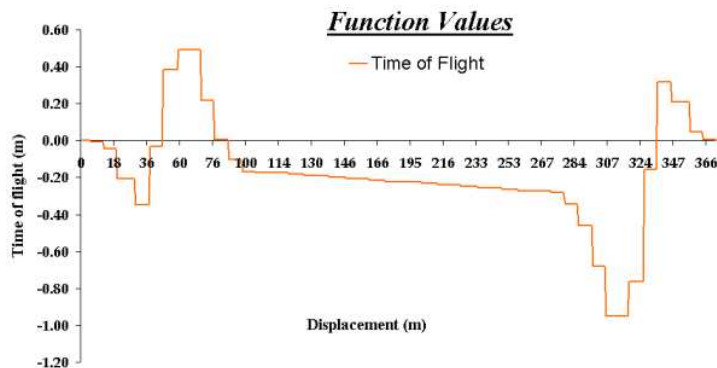


FIG. 10: Time of flight function values after optimization

to quadrupole and sextupole strengths.

## VII. ACKNOWLEDGMENTS

I would firstly like to thank Giovanni Bonvicini for giving me the opportunity to partake in such a great experience. Thanks to Rich Galik and his behind the scenes team for making this a great summer. David Sagan was great in introducing me to the BMAD programming scheme and helped out a lot with debugging. Finally many thanks to my mentor Georg Hoffstaetter who has been there the whole way. From helping me understand the project to guiding me through it and finally overseeing the results. This summer REU experience was made possible by the NSF REU grant PHY-0101649 and the research cooperative agreement PHY-9809799.

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