Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision

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$\tau_{\mu^{\pm}}$ **measurements @ 1 – 10 part-per-million precision**

Motivation

MuLan: $\tau_{\mu^{+}}$

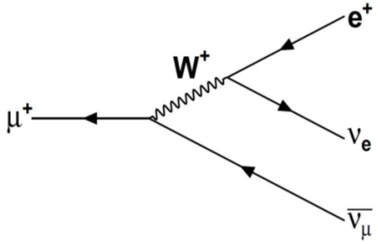
MuCap: μ^{-}**p capture**

Results & Summary

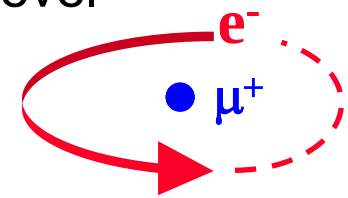
Plus: Final results on: Nucleon's Weak Induced Pseudoscalar Coupling

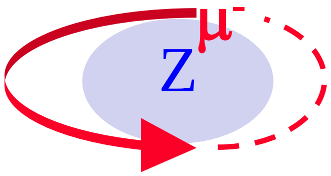
Positive and Negative Muon Lifetimes

- **Free muon** decay is a *pure weak* process …
  - determines G_μ, often called G_F



- **Muons stopped in matter:**
  - Positive muons decay "as if" free, or form atomic-bound muonium … with a lifetime shift expected at the ~ ppb level
  - → most precise Fermi Constant





  - Negative muons form "1-electron-like" muonic atoms and either decay ***or***, undergo nuclear capture.

$$\mu^- + p \rightarrow n + \nu_\mu$$

    - The decay and capture rates add; lifetime is "shorter"

$$\frac{\mathbf{1}}{\boldsymbol{\tau}} = \boldsymbol{\Lambda}_{\mathbf{total}} = \boldsymbol{\Lambda}_{\mathbf{decay}} + \boldsymbol{\Lambda}_{\mathbf{capture}}$$

⇧ ⇨ **Extract physics here**

τ_{μ^+}

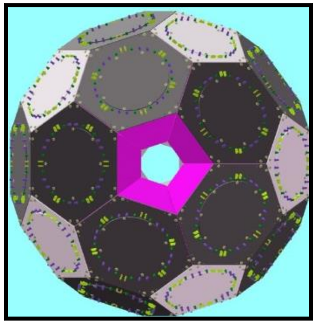
Positive muon lifetime motivation: Predictive power of the SM depends on well-measured input parameters

G_F

9 ppm

↓

0.6 ppm

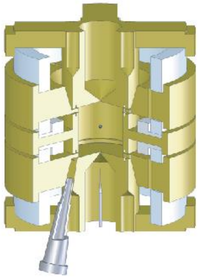


MuLan Collaboration
PRL **106**, 041803 (2011)

This work

α

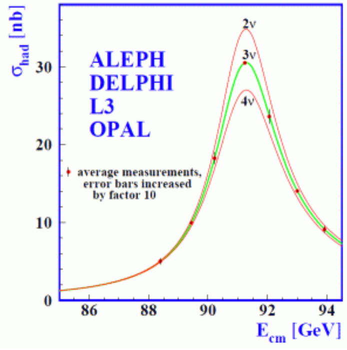
0.37 ppb



Hanneke, Fogwell, Gabrielse
PRL **100,** 120801 (2008)

M_Z

23 ppm

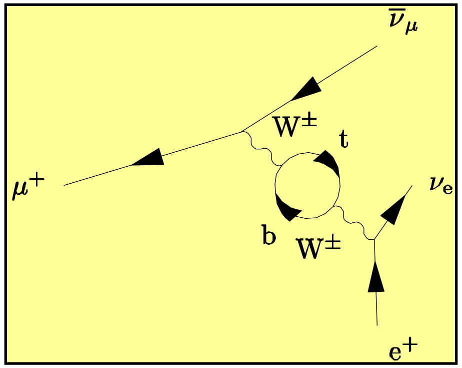


Phys.Rept.427:257-454,2006

$G_F = 1.166\ 378\ 8(7) \times 10^{-5}$ GeV^{-2} **$\alpha^{-1} = 137.035\ 999\ 084 \pm 0.000\ 000\ 051$** **$M_Z = 91.1875 \pm 0.0021$ GeV**

The Fermi constant is related to the electroweak gauge coupling *g* by

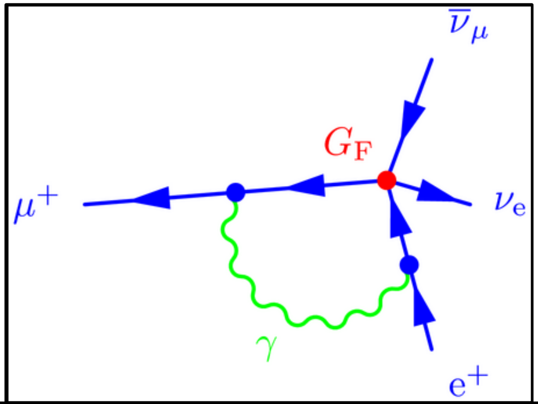
$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g^2}{8M_{\mathrm{W}}^2}\left(1 + \Delta r(m_{\mathrm{t}}, m_{\mathrm{H}}, \ldots)\right)$$



Contains all weak interaction loop corrections

In the Fermi theory, muon decay is a contact interaction

$$\frac{1}{\tau_{\mu^+}} = \frac{{G_{\mathrm{F}}}^2 m_\mu^5}{192\pi^3}$$



In 1999, van Ritbergen and Stuart completed full 2-loop QED corrections reducing the uncertainty in G_F from theory to < 0.3 ppm (it was the dominant error before)

$\tau_\mu+$

Example: connection from τ_μ to $\sin^2\theta_W$

- Momentum transfer $q^2 = (p_\mu - p_{\nu\mu})^2 = (p_e + p_{\nu e})^2 < m_\mu^2$ much smaller than M_W^2
- Thus, W propagator shrinks to a point and can be well approximated through a local four-fermion interaction,

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} = \frac{4\pi\alpha}{\sin^2\theta_W\, M_W^2} \equiv 4\sqrt{2}G_F$$

MuLan: $G_F = (1.166\,378\,8 \pm 0.000\,000\,7) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$

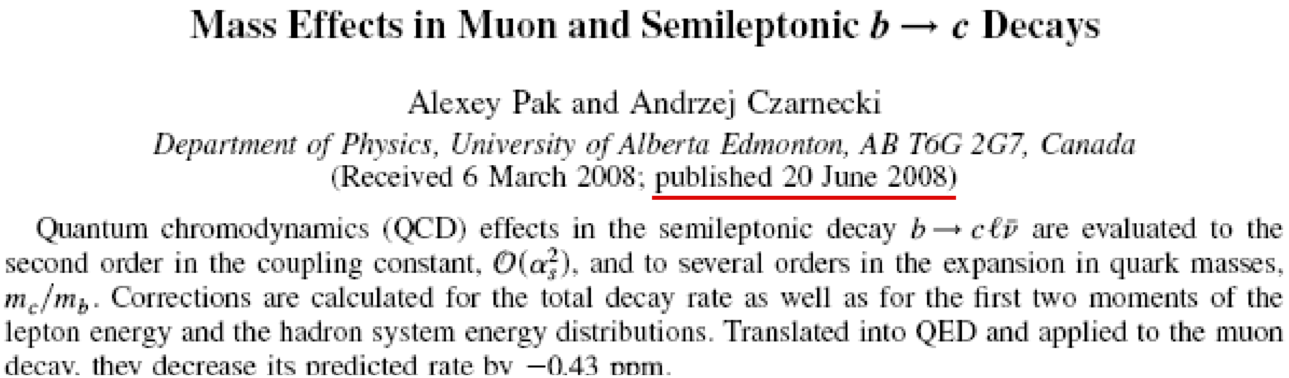
➡ $$\sin^2\theta_W = 0.215$$

(there are further quantum corrections here not included)

The push – pull of experiment and theory

- **Lifetime now largest uncertainty leads to 2 new experiments launched: MuLan & FAST**
  - **Both @ PSI, but very different techniques**
  - **Both aimed at "ppm" level G_F determinations**
  - **Both published intermediate results on small data samples**

- **Meanwhile, more theory updates …**



Mass Effects in Muon and Semileptonic $b \to c$ Decays

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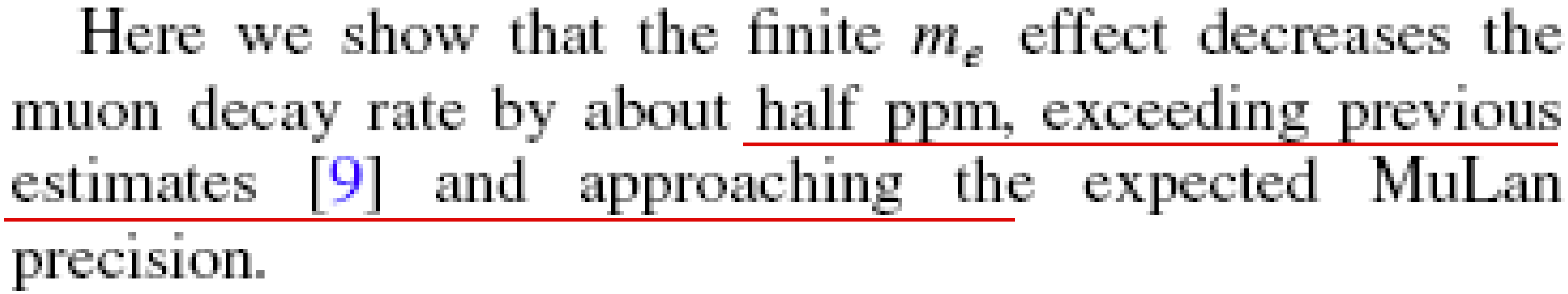
Department of Physics, University of Alberta Edmonton, AB T6G 2G7, Canada

(Received 6 March 2008; published 20 June 2008)

Quantum chromodynamics (QCD) effects in the semileptonic decay $b \to c\ell\bar{\nu}$ are evaluated to the second order in the coupling constant, $\mathcal{O}(\alpha_s^2)$, and to several orders in the expansion in quark masses, m_c/m_b. Corrections are calculated for the total decay rate as well as for the first two moments of the lepton energy and the hadron system energy distributions. Translated into QED and applied to the muon decay, they decrease its predicted rate by -0.43 ppm.

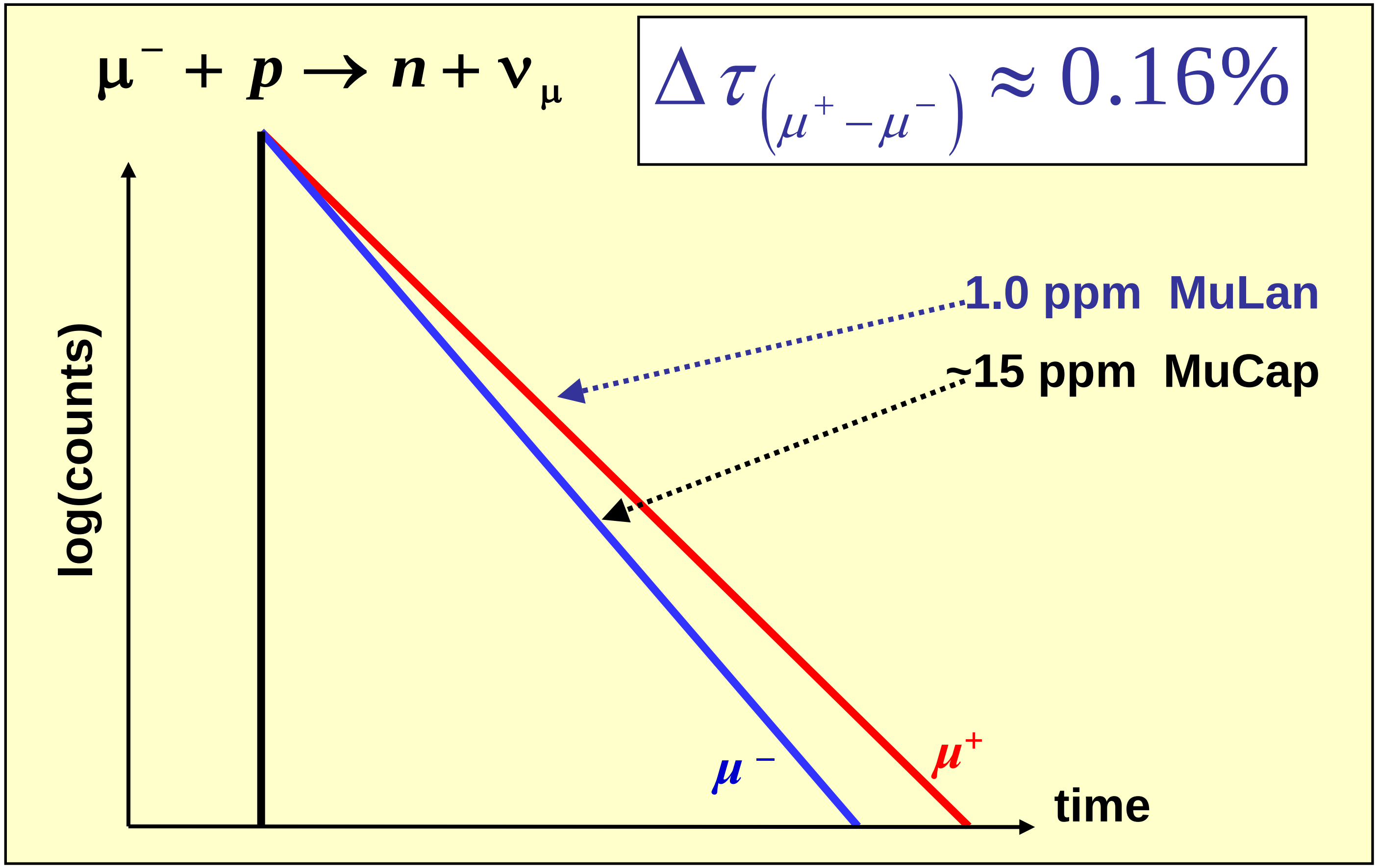


Here we show that the finite m_e effect decreases the muon decay rate by about half ppm, exceeding previous estimates [9] and approaching the expected MuLan precision.

τ_{μ^+}

τ_{μ^-}

Further motivation: Take difference between τ_{μ^+} and τ_{μ^-} in hydrogen to infer singlet capture rate Λ_S



$$\mu^- + p \rightarrow n + \nu_\mu$$

$$\boxed{\Delta\tau_{(\mu^+ - \mu^-)} \approx 0.16\%}$$

1.0 ppm MuLan

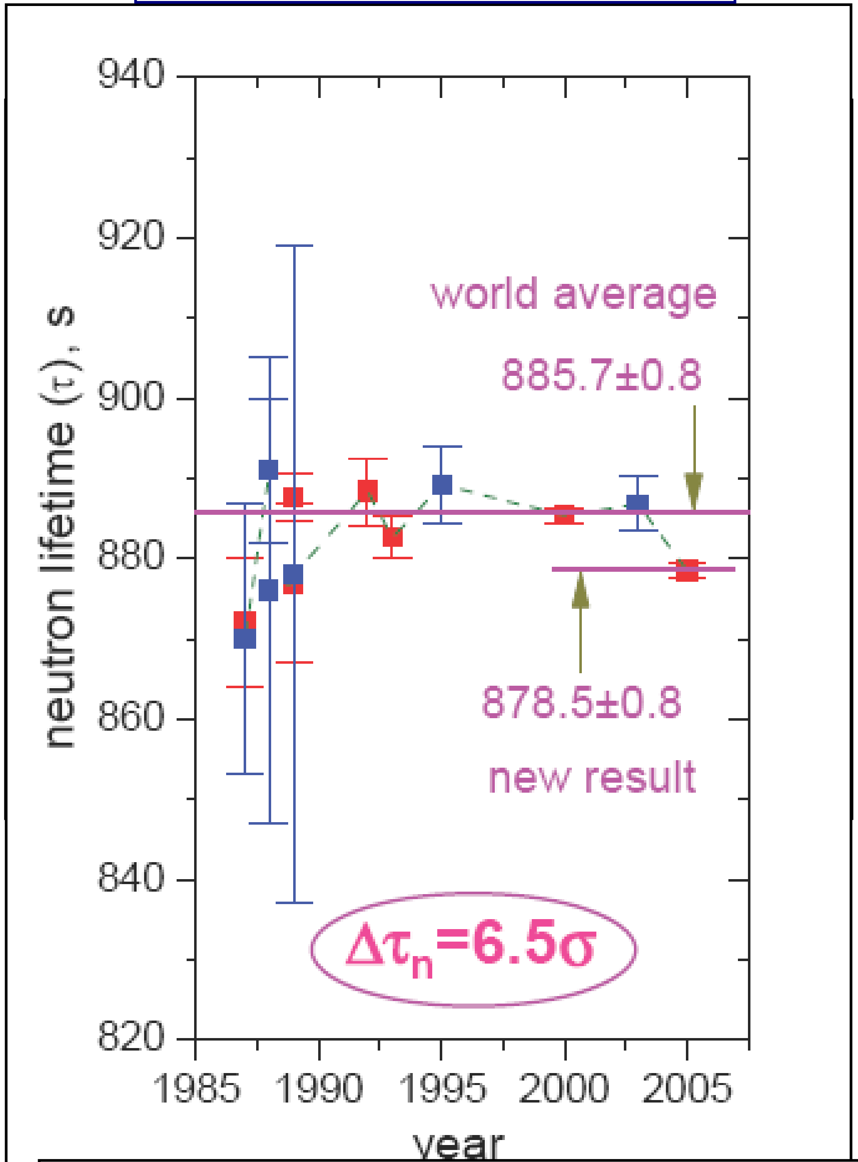
~15 ppm MuCap

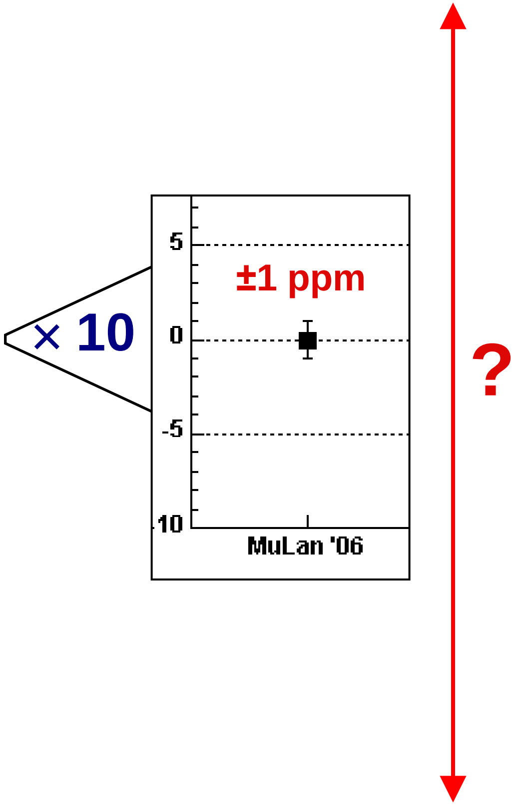
$$\Lambda_S = \Lambda_{\mu^-} - \Lambda_{\mu^+} = \frac{1}{\tau_{\mu^-}} - \frac{1}{\tau_{\mu^+}} \Rightarrow g_P$$

World avg $\delta\tau_\mu/\tau_\mu$ is 18 ppm, but is it right?

Precision
vs
Accuracy

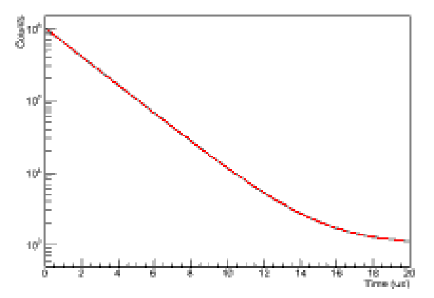
Lessons from History





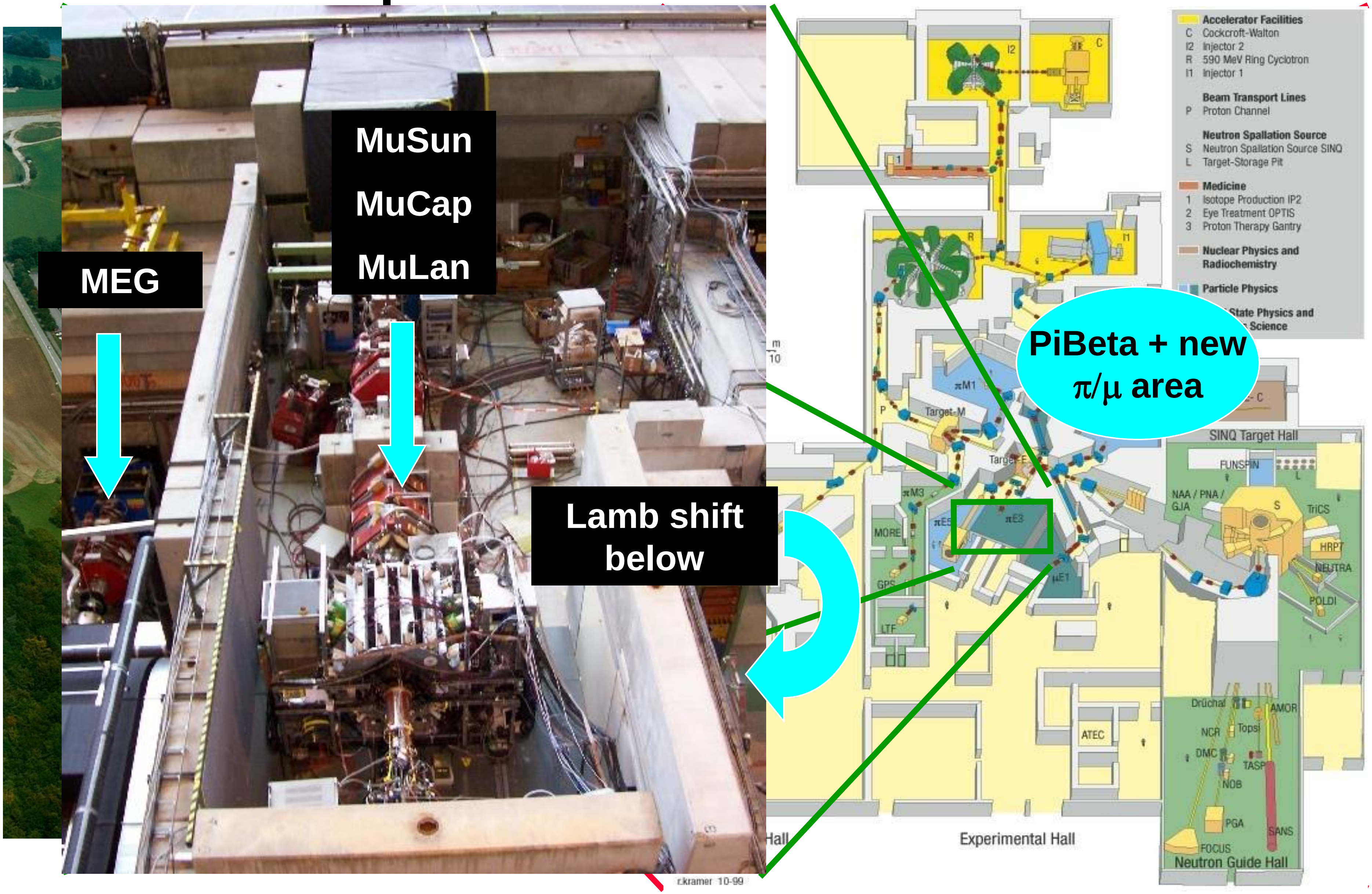
Goal of MuLan is 1 ppm.

The Experiments



- Generic design considerations
- MuLan τ_{μ^+}
- MuCap τ_{μ^-}

PSI: a 1.3 MW facility with many secondary muon beams. Example: πE3 beamline at PSI



MuSun

MuCap

MuLan

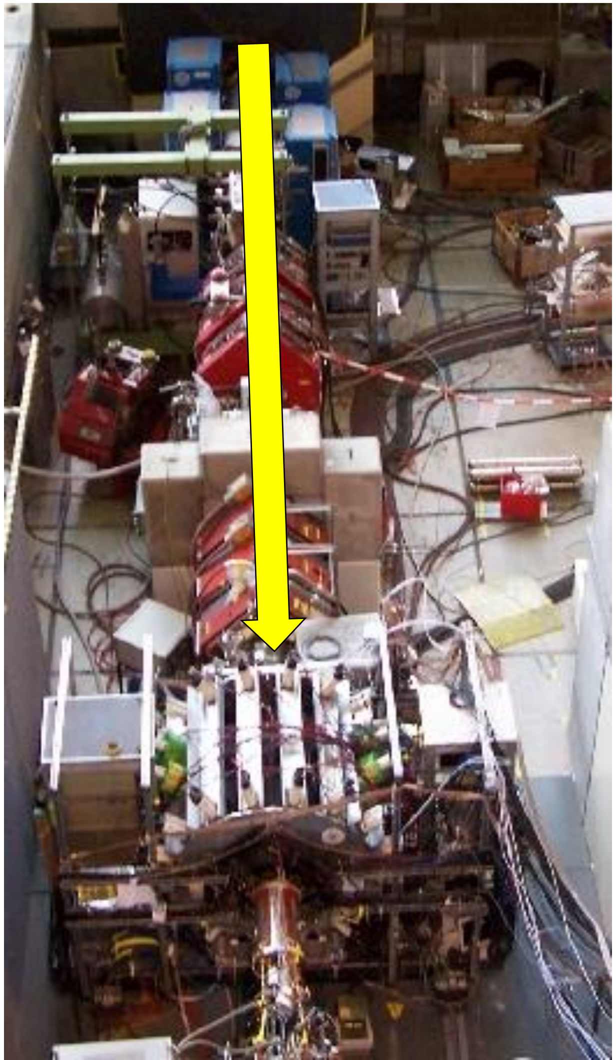
MEG

Lamb shift below

PiBeta + new π/μ area

Design Considerations
counts and *systematic* control

Need: **10^{12}** μ^+ decays (1 ppm) & **10^{10}** μ^- decays (10 ppm)



PSI:
- **2.2 mA protons @ 590 MeV**
- **πE3 low-energy muon beamline**
- **Time structured custom Kicker**

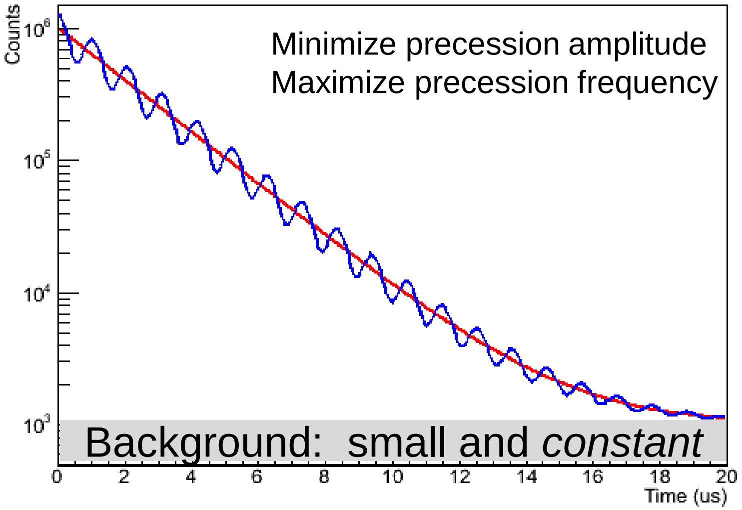
MuLan:
- ***~10^7 μ+ /s***
- **Beam-on / Beam-off periodic cycles**
- **Multiple decays per cycle**

MuCap:
- ***~10^5 μ^- /s***
- **Muon-on-demand**
- **1 measurement at a time**

Generic Design Considerations
counts and *systematic* control

τ_{μ^+}

- Gain stability
- Pileup $\propto e^{-2t/\tau}$
- Spin (μSR)
  - Symmetry; Dephasing

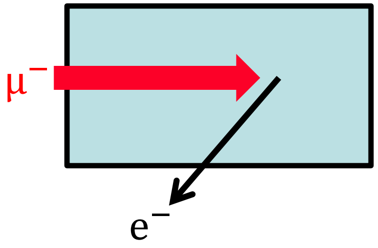


τ_{μ^-}

- Avoid impurities
- Fiducial volume
- Interference of μ & e tracks

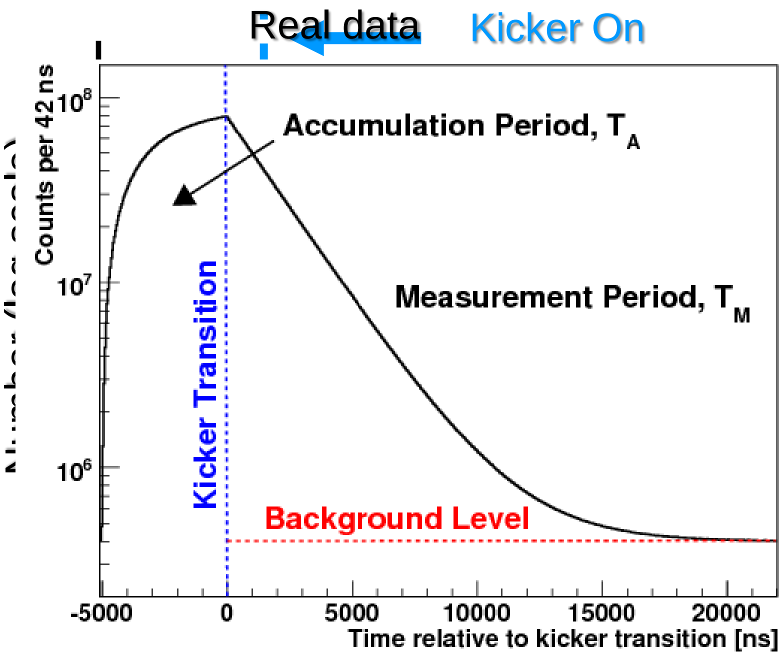
Capture rate $\propto Z^4$ → must stop in target

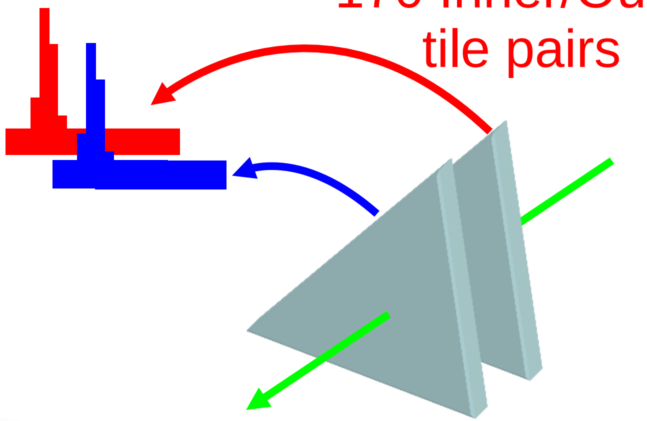
a TPC specific effect



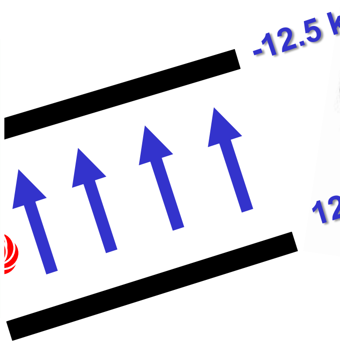
τ_{μ^+}

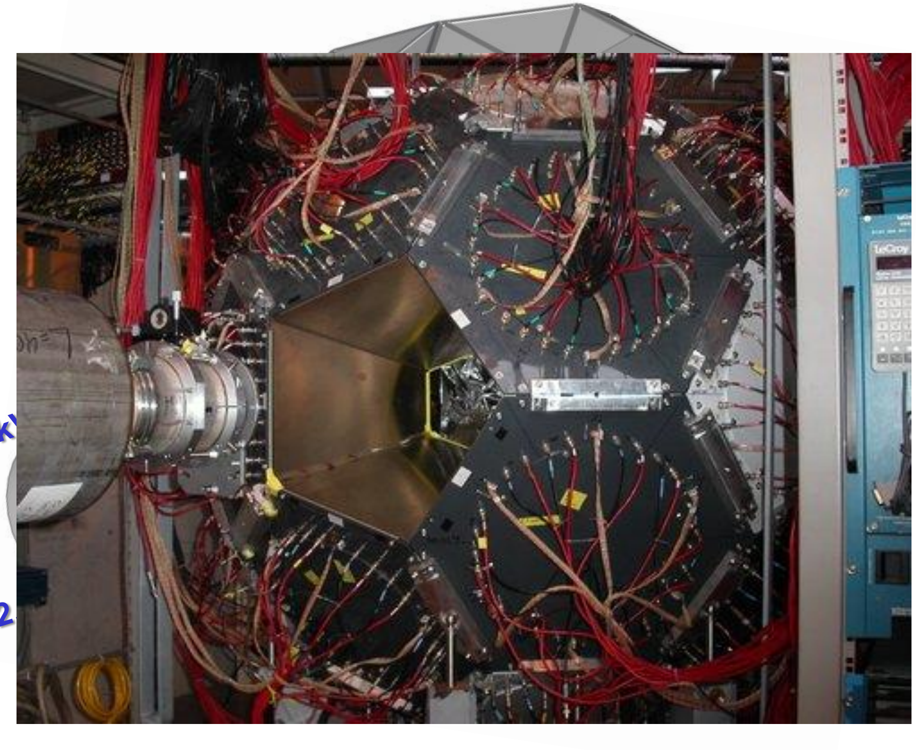
The MuLan experimental concept…




450 MHz WaveForm Digitization

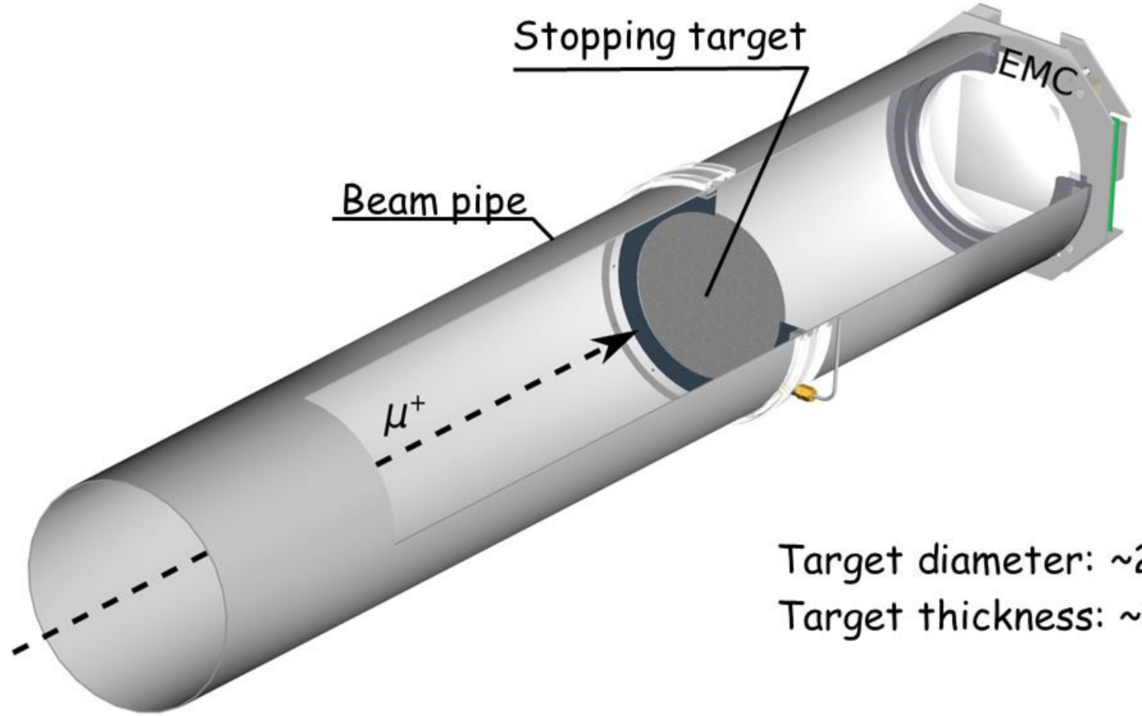
170 Inner/Outer tile pairs





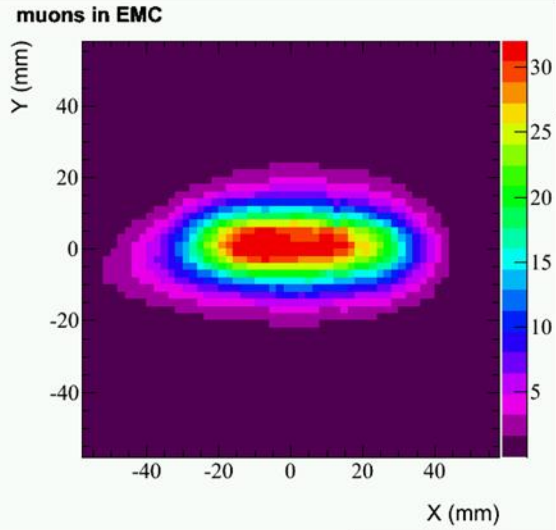
Detector has symmetric design around stops

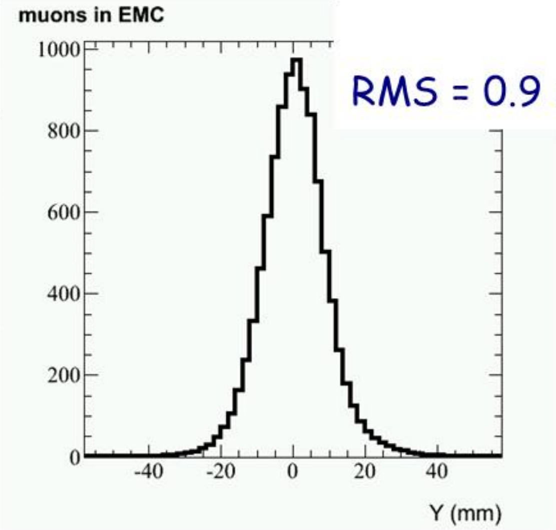
Stopping Target



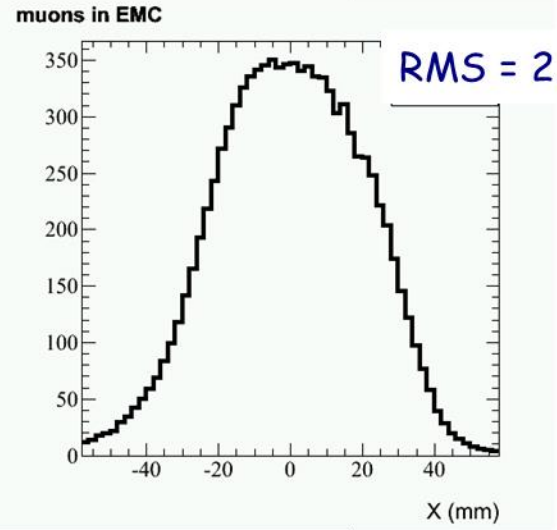
Target diameter: ~20 cm
Target thickness: ~0.5 mm

Stopping Target

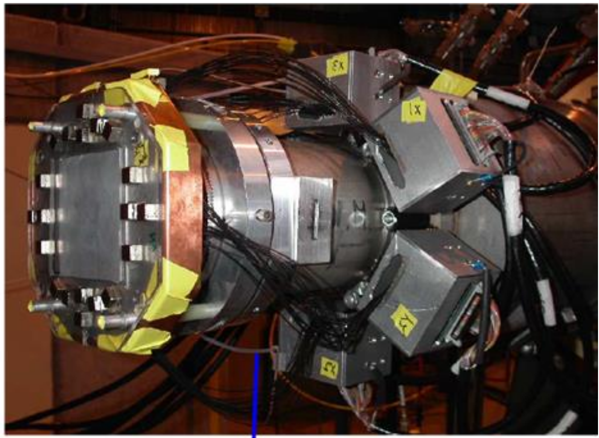


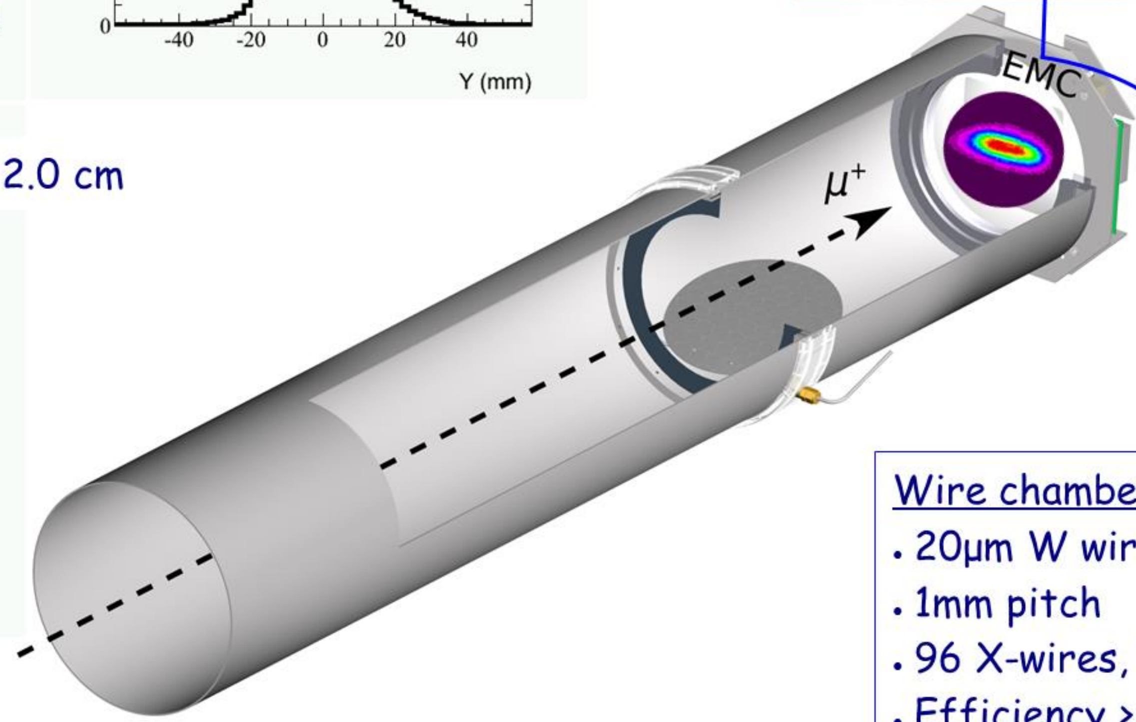


RMS = 0.9 cm



RMS = 2.0 cm

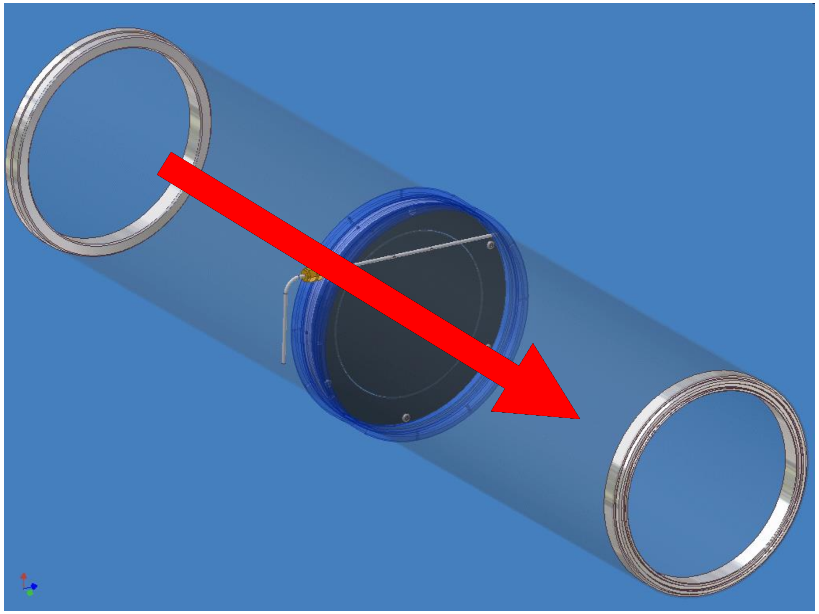




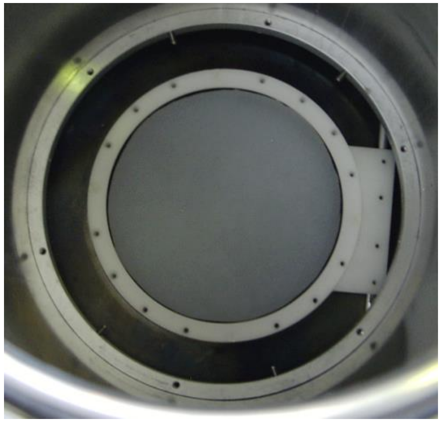
<u>Wire chamber</u>

- 20μm W wires
- 1mm pitch
- 96 X-wires, 96 Y-wires
- Efficiency > 95%

Stopping targets selected to control spin

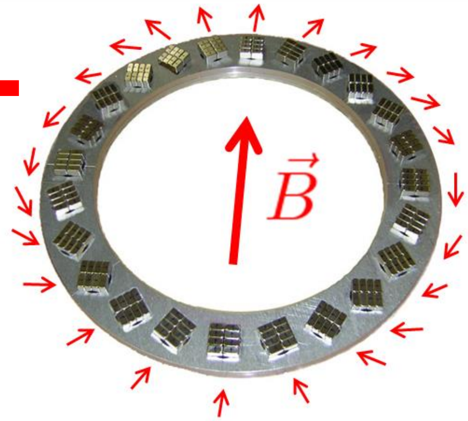


Quartz



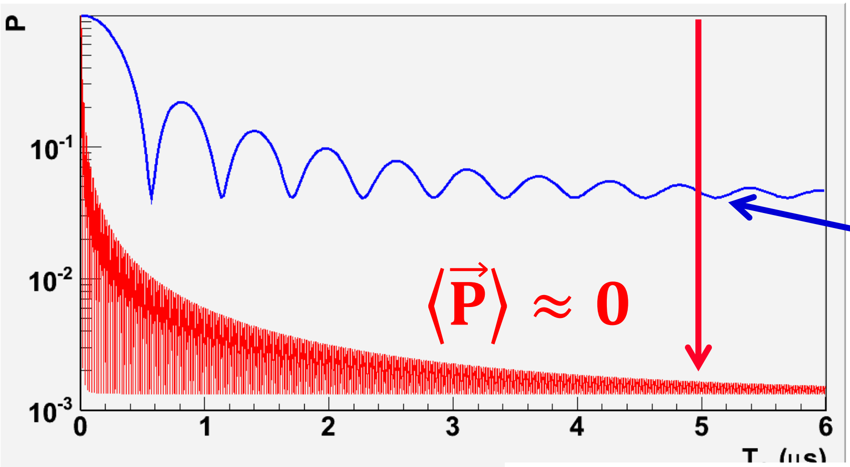
+

Halbach Array



AK-3*

- Internal 0.5 T transverse field
- Precess rapidly
- Dephase owing to different arrival times



Start Accumulation

Start Measurement

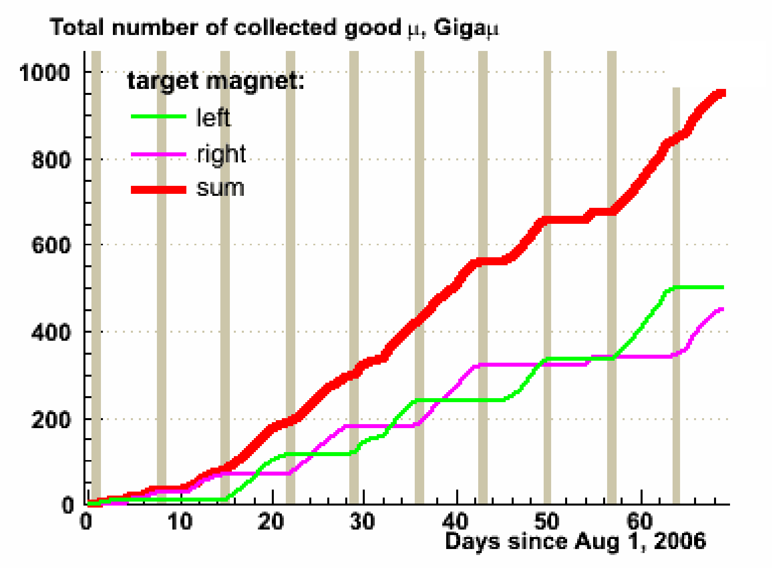
Quartz

- Form muonium 90%
- Precession period few ns
- Control 10% free muon spins by symmetry of detector

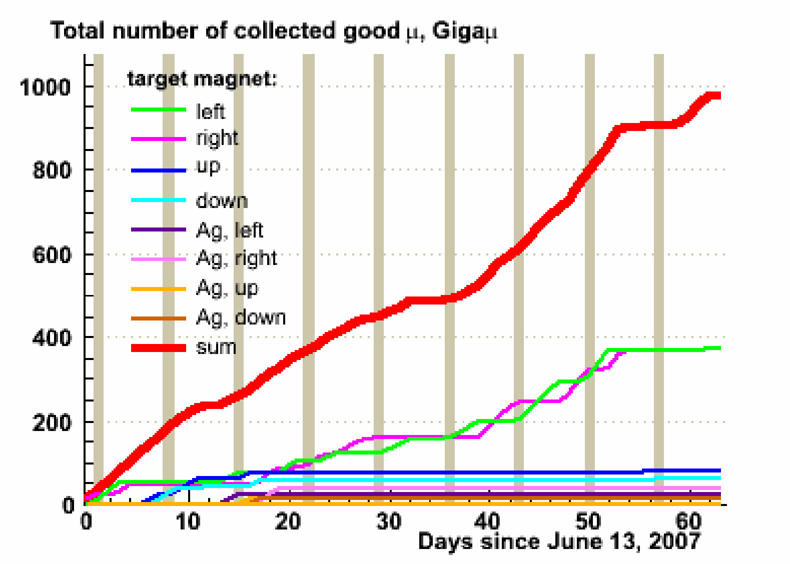
*Arnokrome-3 (~28% chromium, ~8% cobalt, ~64% iron)

MuLan collected two datasets, each containing 10^{12} muon decays

Ferromagnetic Target, 2006



Quartz Target, 2007



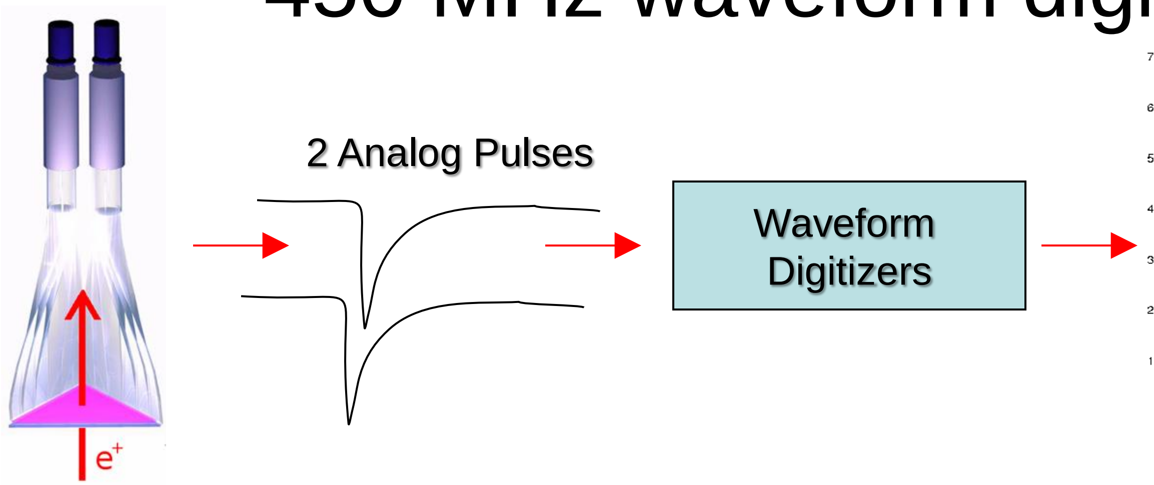
- Two (very different) data sets
  - Different blinded clock frequencies used
  - Revealed only after all analyses of both data sets completed
  - Most systematic errors are common

The detector is composed of 20 hexagon and 10 pentagon sections, forming a truncated icosahedron.

Each section contains either 6 or 5 tile elements

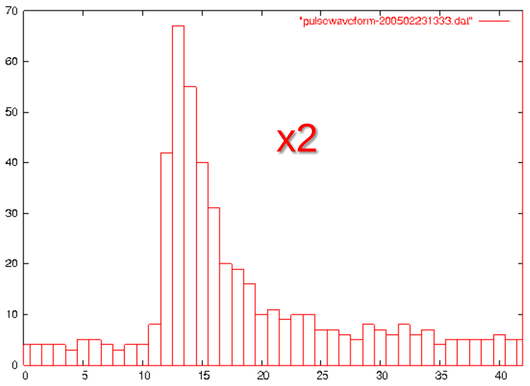
Each element is made from two independent scintillator tiles with light guides and photomultiplier tubes.

170 scintillator tile pairs readout using 450 MHz waveform digitizers.

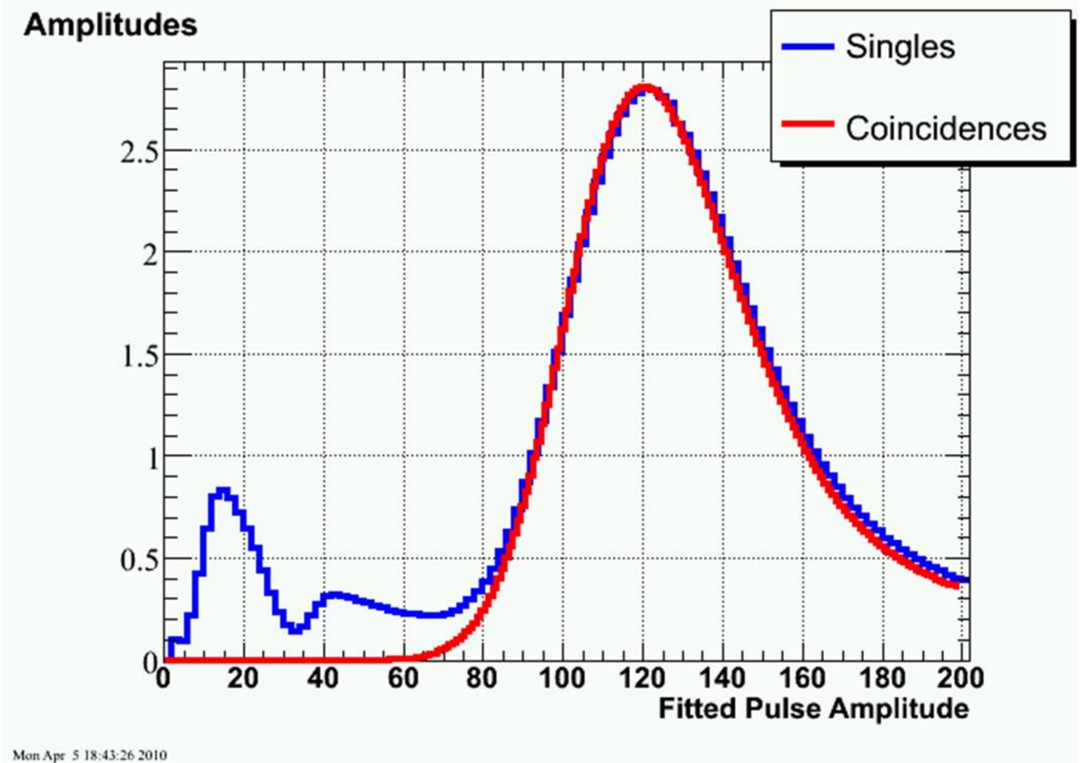


2 Analog Pulses

Waveform Digitizers



1 clock tick = 2.2 ns

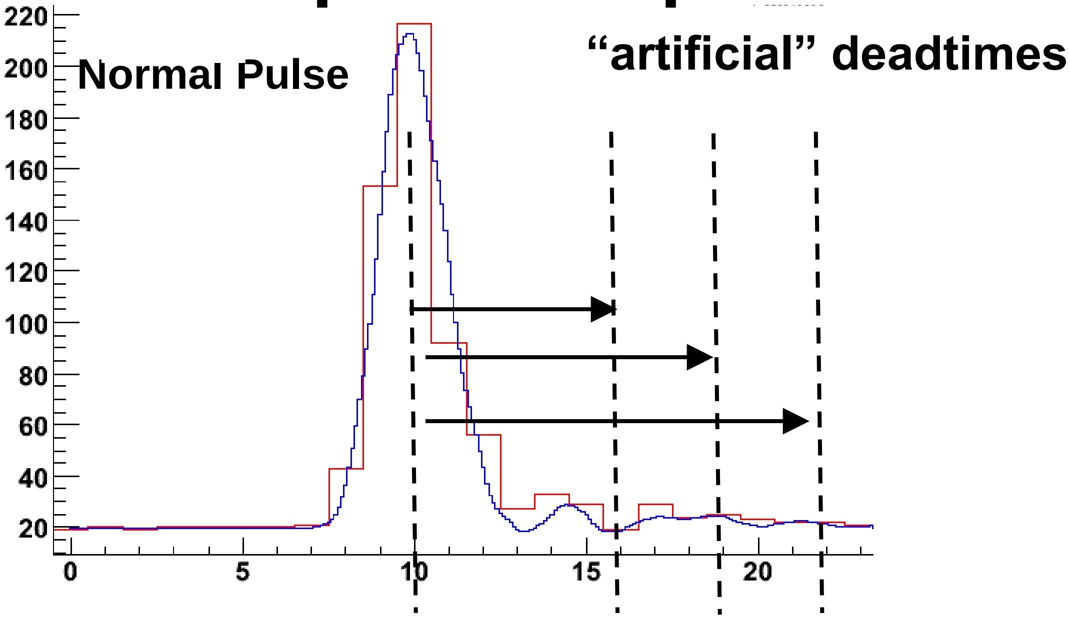


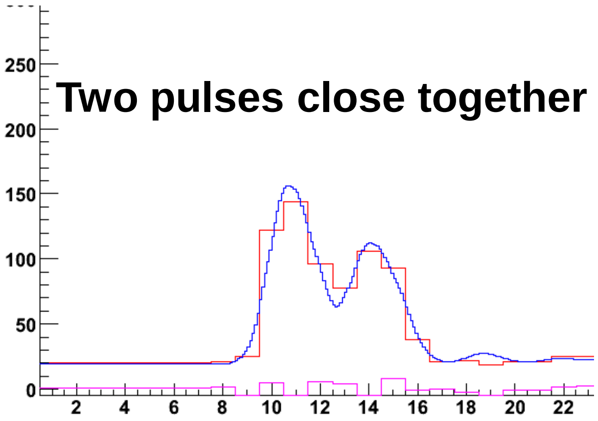
Raw waveforms for 170 inner and outer scintillators are fit using calibrated pulse templates

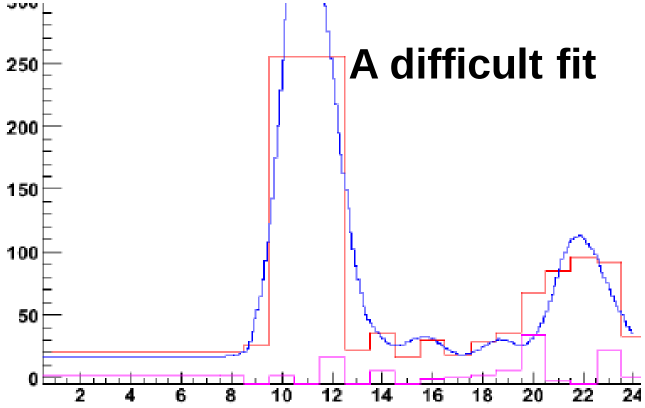
>2 x 10^{12} decays

130 TB data

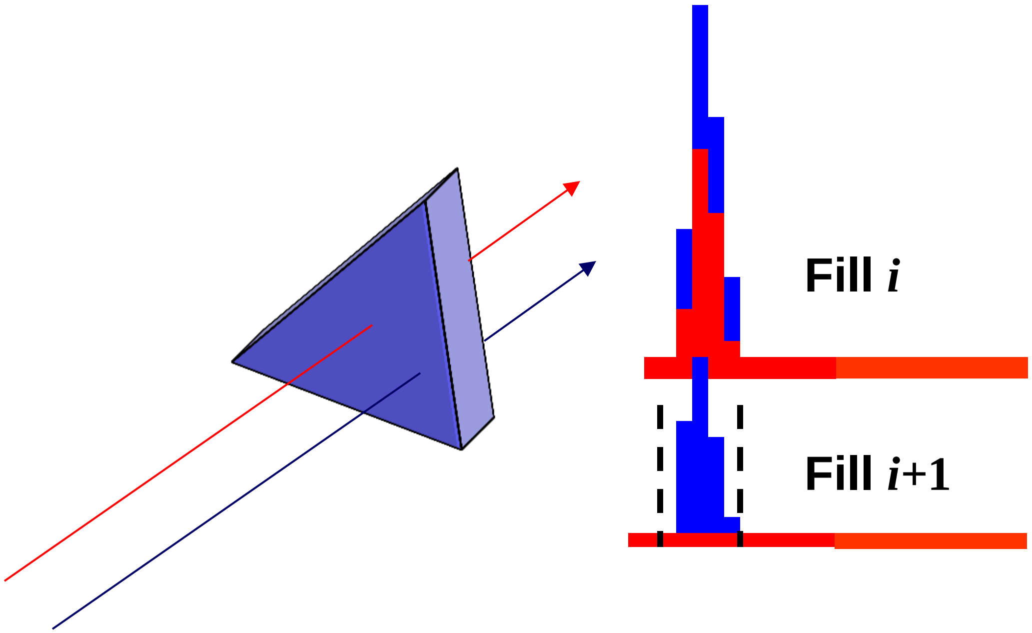
at NCSA



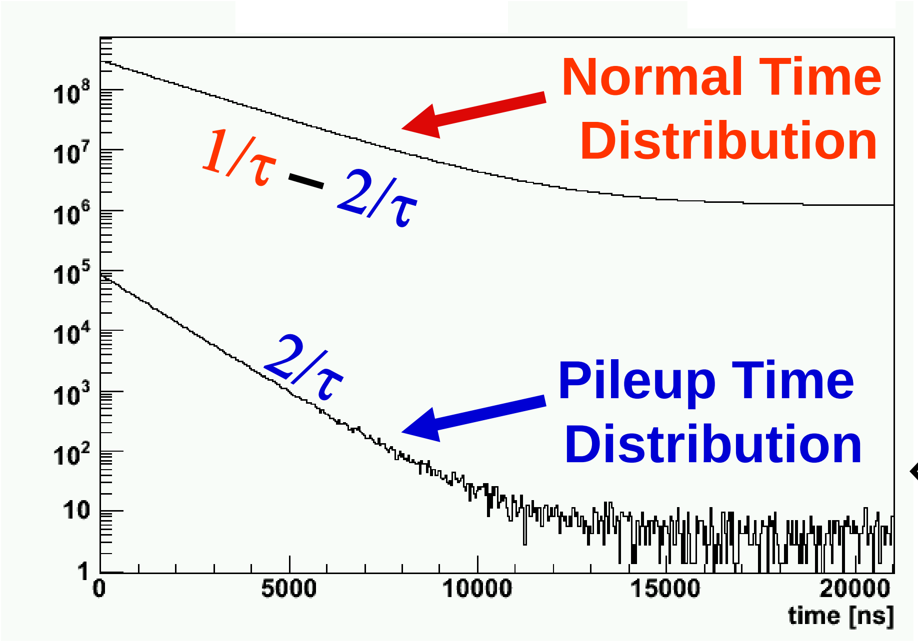




Leading order pileup is a ~5x10^{-4} effect, yet …

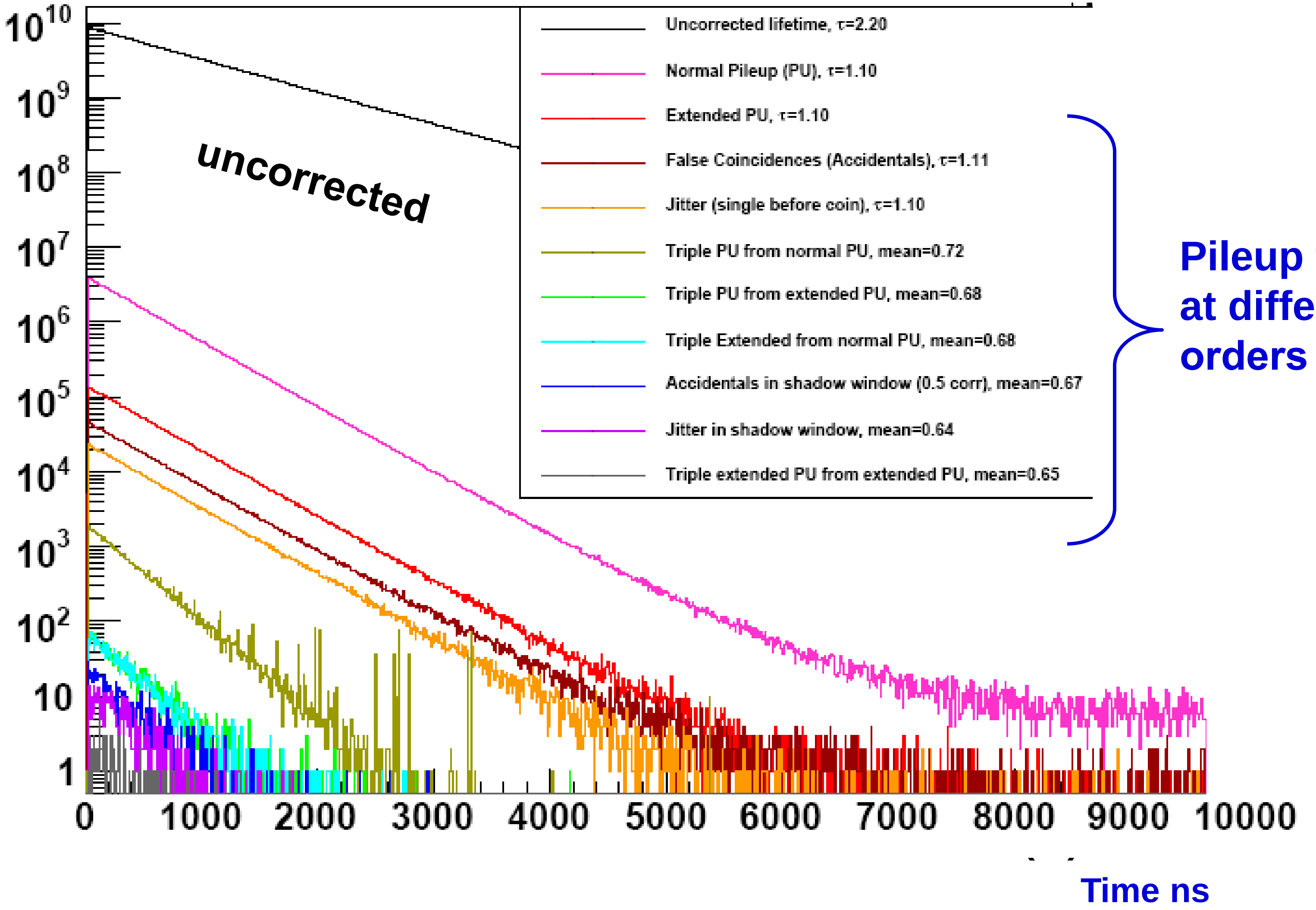


- Statistically reconstruct pileup time distribution
- Fit corrected distribution



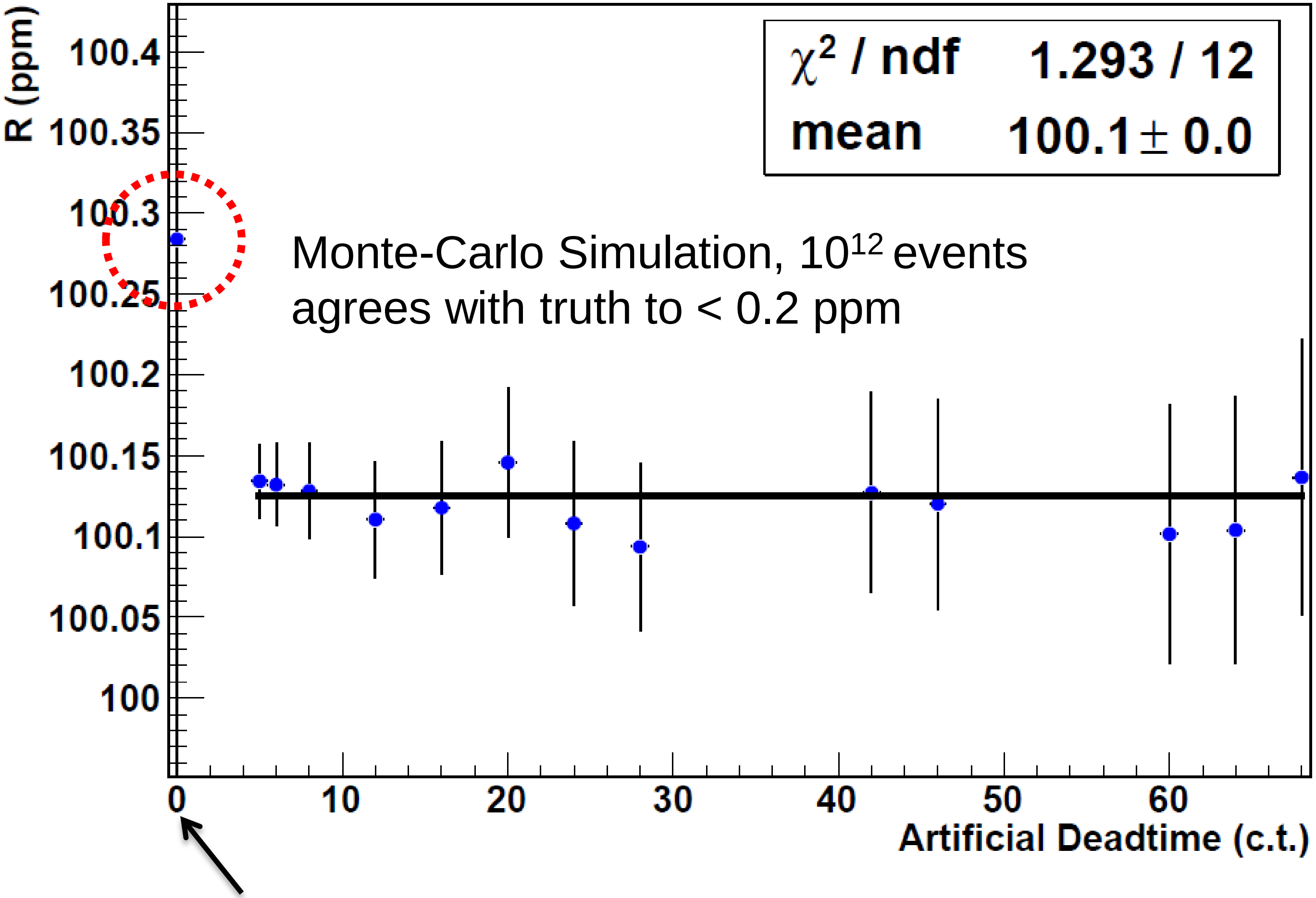
This is only the 1st order effect

Pileup to sub-ppm requires higher-order terms



Pileup terms at different orders …

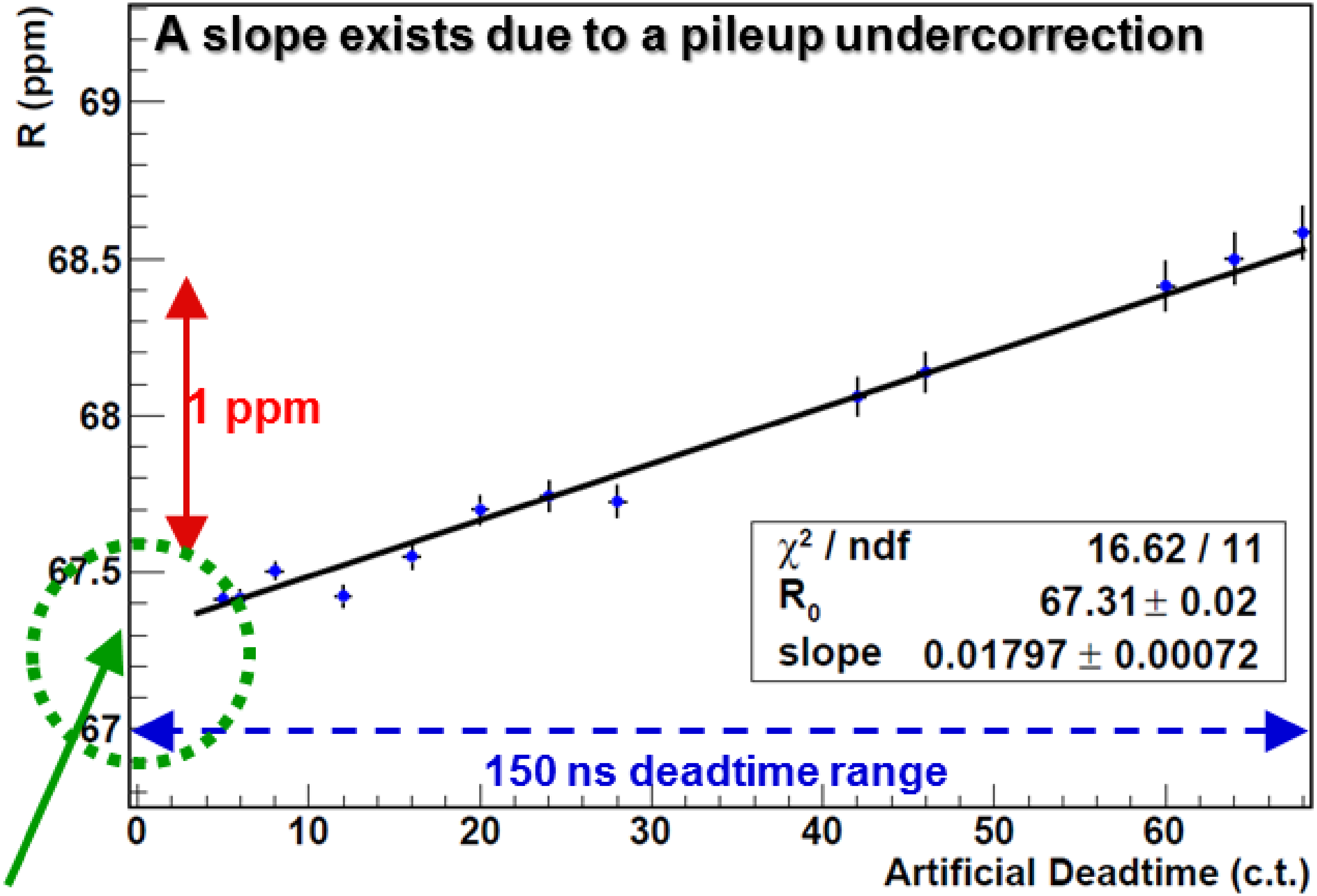
The pileup corrections were tested with Monte-Carlo.



Monte-Carlo Simulation, 10^{12} events agrees with truth to < 0.2 ppm

1.19 ppm statistical uncertainty

Final deadtime corrected lifetime



A slope exists due to a pileup undercorrection

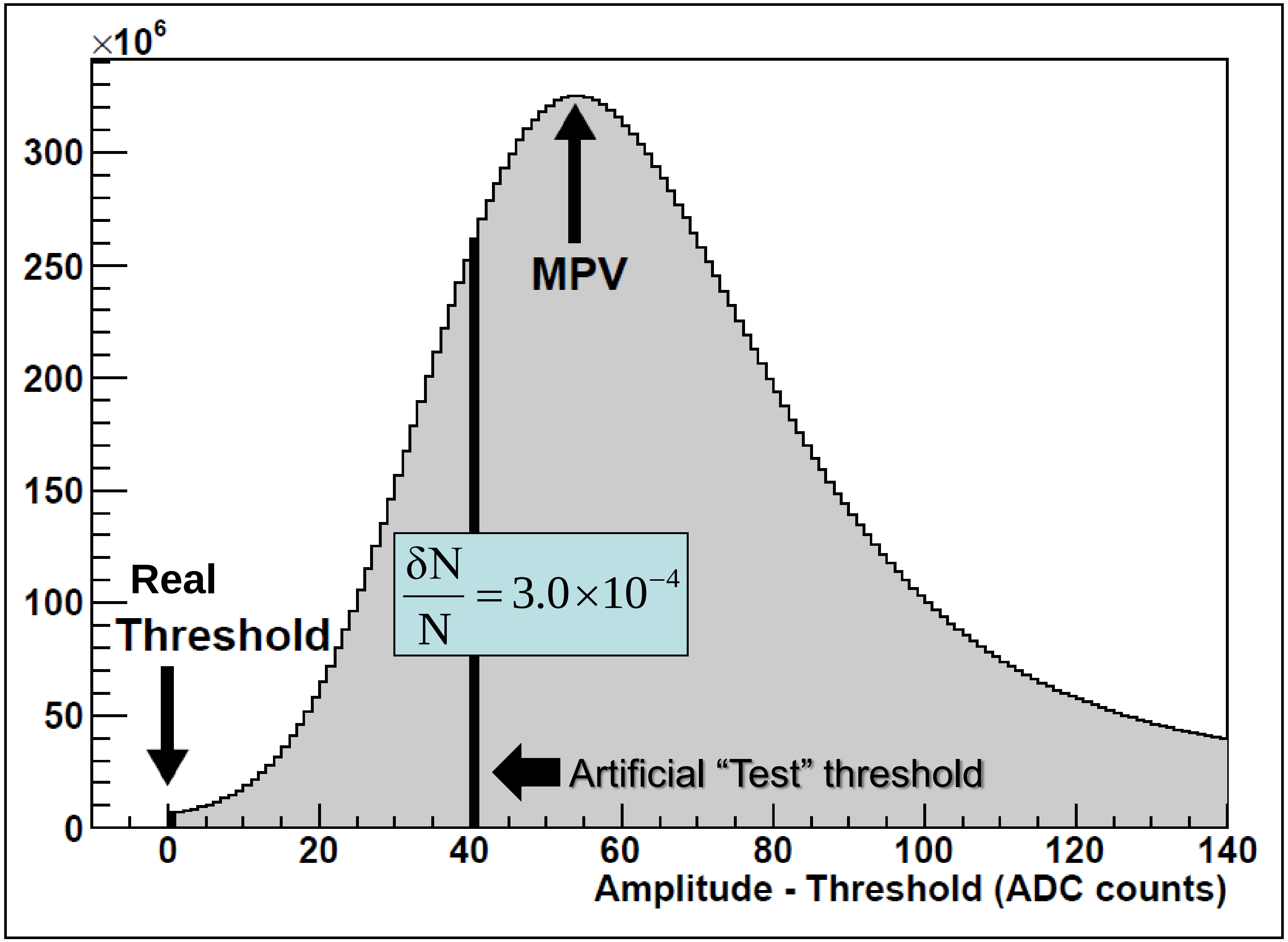
1 ppm

150 ns deadtime range

χ^2 / ndf	16.62 / 11
R_0	67.31 ± 0.02
slope	0.01797 ± 0.00072

Extrapolation to 0 deadtime is correct answer

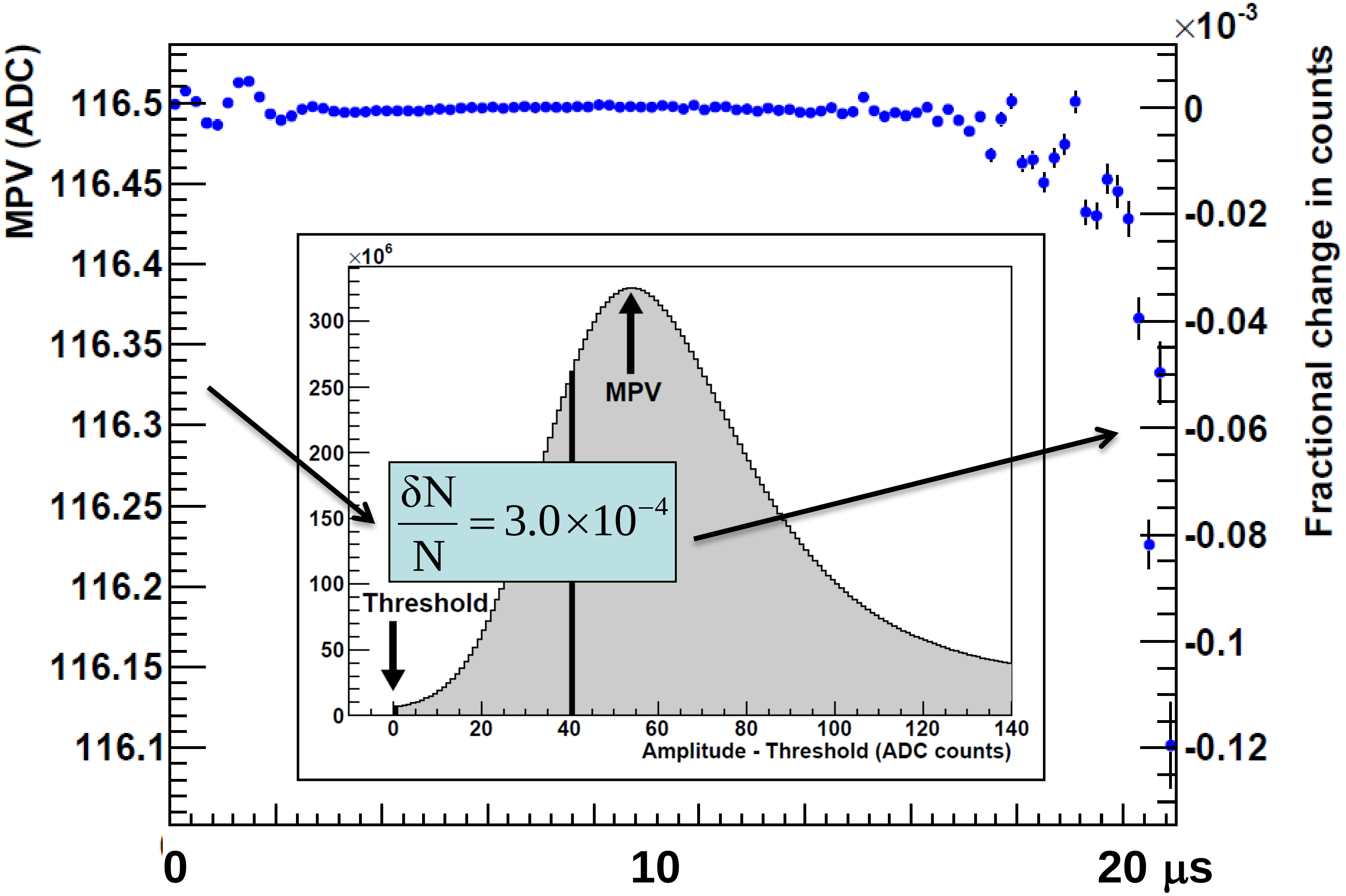
Gain variation vs. time is derived from the stability of the peak (MPV) of the fit to pulse distribution



$$\frac{\delta N}{N} = 3.0 \times 10^{-4}$$

Gain variation vs. time is derived from the stability of the peak (MPV) of the fit to pulse distribution

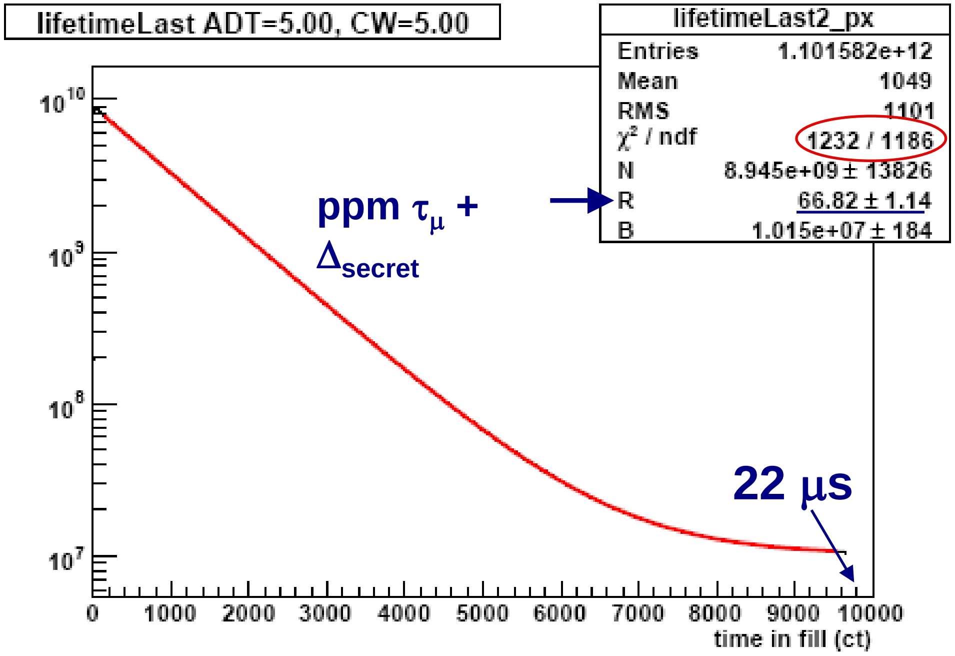
If MPV moves, implies greater or fewer hits will be over threshold



$$\frac{\delta N}{N} = 3.0\times10^{-4}$$

Gain(t) is PMT type dependent. Carefully studied and reduced to 0.25 ppm uncertainty. Gain correction gives a 0.5 ppm shift in result vs uncorrected

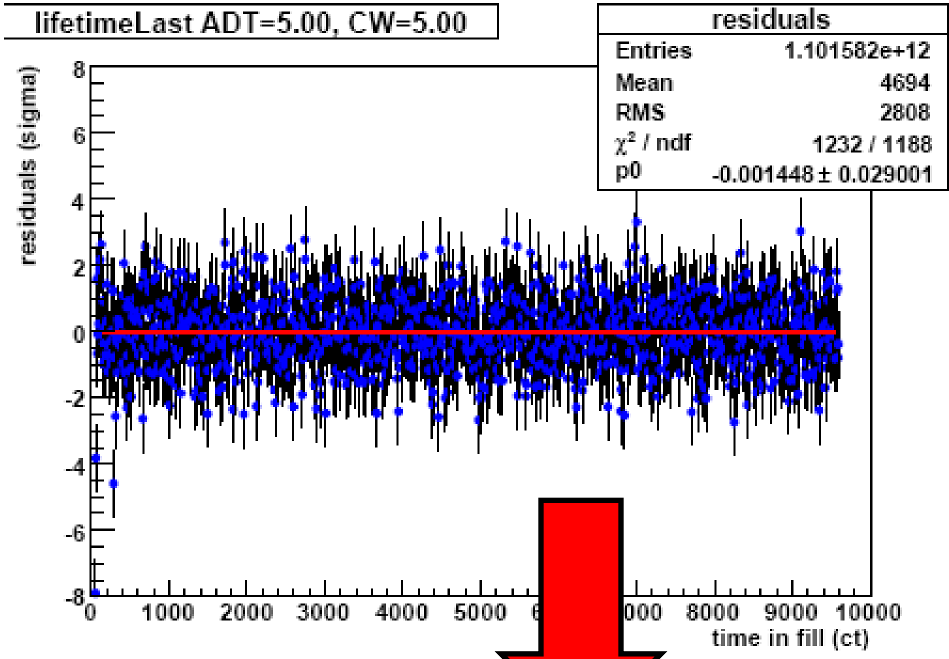
τ_{μ^+} MuLan fit of 30,000 AK-3 pileup-corrected runs



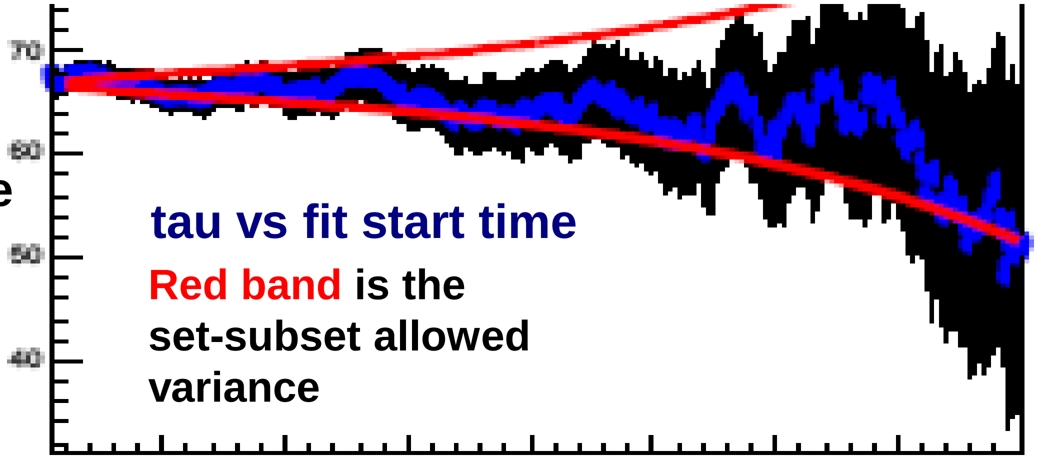
lifetimeLast2_px	
Entries	1.101582e+12
Mean	1049
RMS	1101
χ^2 / ndf	1232 / 1186
N	8.945e+09 ± 13826
R	66.82 ± 1.14
B	1.015e+07 ± 184

ppm τ_μ + Δ_{secret}

22 μs



residuals	
Entries	1.101582e+12
Mean	4694
RMS	2808
χ^2 / ndf	1232 / 1188
p0	-0.001448 ± 0.029001



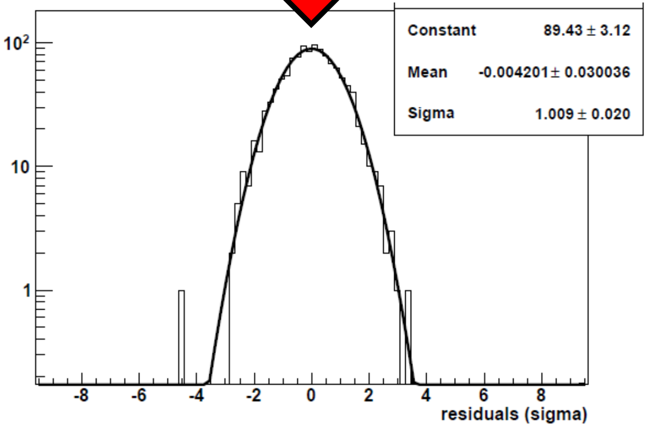
Relative τ (ppm)

tau vs fit start time

Red band is the set-subset allowed variance

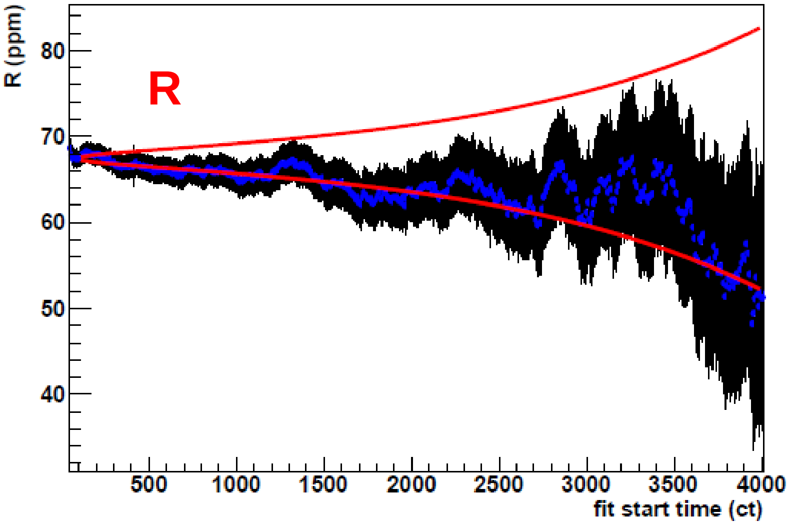
0

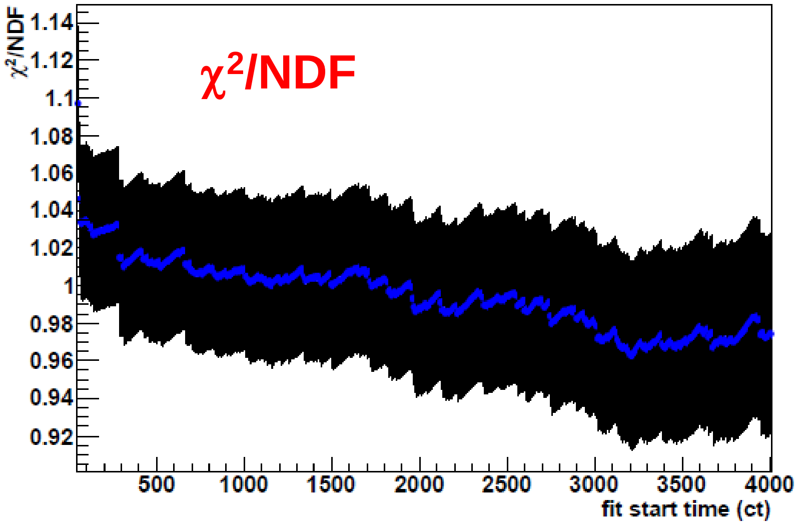
9 μs

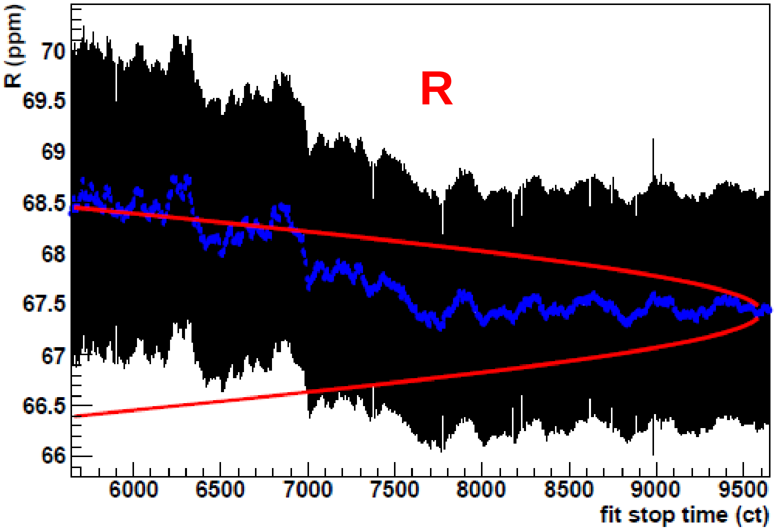


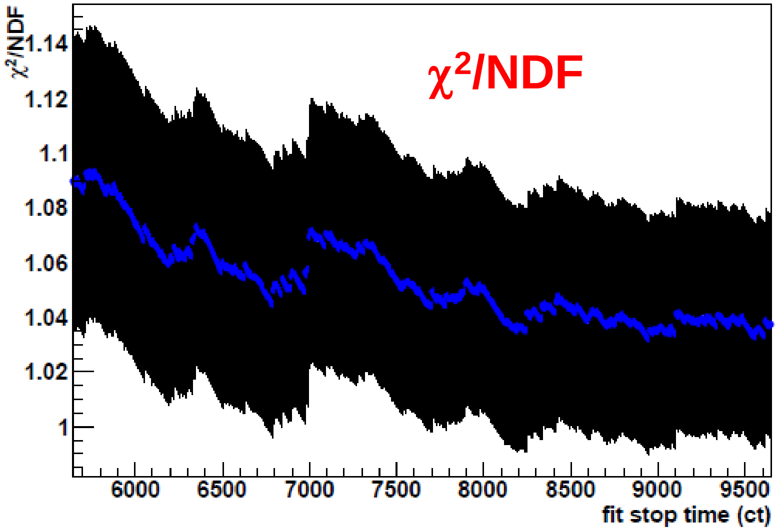
Constant	89.43 ± 3.12
Mean	-0.004201 ± 0.030036
Sigma	1.009 ± 0.020

Varying the fit start and stop time shows good self-consistency.

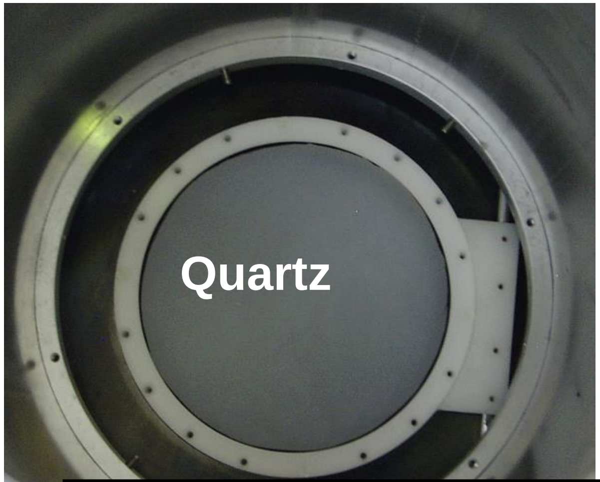


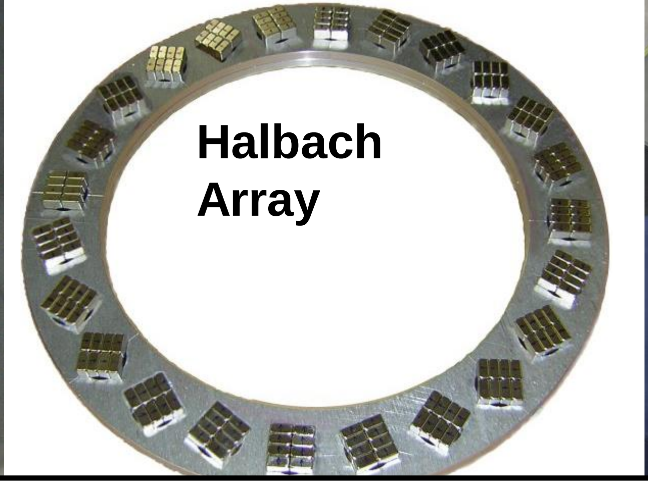


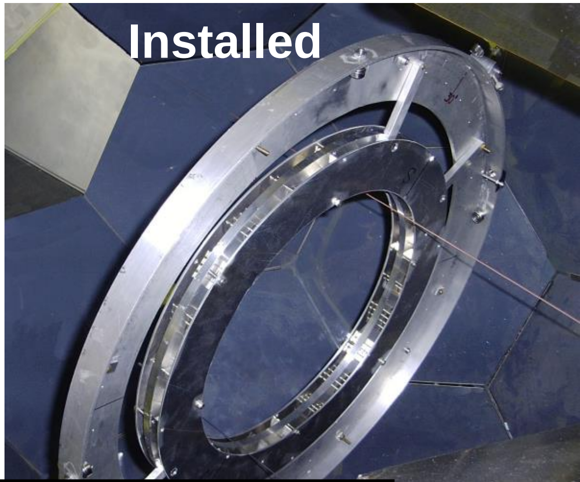


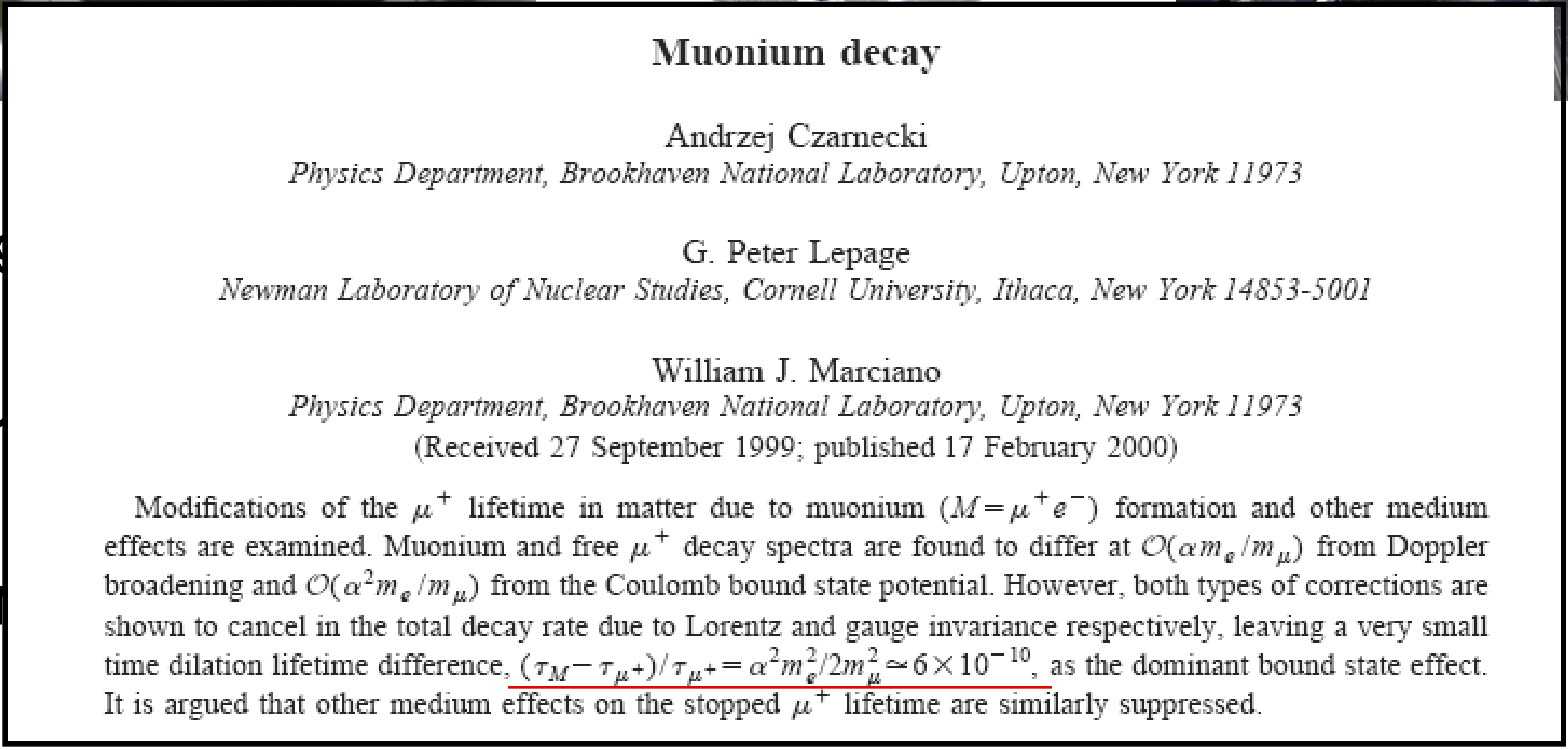


Crystal quartz is really different. This was meant to challenge the otherwise "easy" AK3 target









Muonium decay

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(Received 27 September 1999; published 17 February 2000)

Modifications of the μ^+ lifetime in matter due to muonium ($M=\mu^+e^-$) formation and other medium effects are examined. Muonium and free μ^+ decay spectra are found to differ at $\mathcal{O}(\alpha m_e/m_\mu)$ from Doppler broadening and $\mathcal{O}(\alpha^2 m_e/m_\mu)$ from the Coulomb bound state potential. However, both types of corrections are shown to cancel in the total decay rate due to Lorentz and gauge invariance respectively, leaving a very small time dilation lifetime difference, $\underline{(\tau_M-\tau_{\mu^+})/\tau_{\mu^+}=\alpha^2 m_e^2/2m_\mu^2 \simeq 6\times 10^{-10}}$, as the dominant bound state effect. It is argued that other medium effects on the stopped μ^+ lifetime are similarly suppressed.

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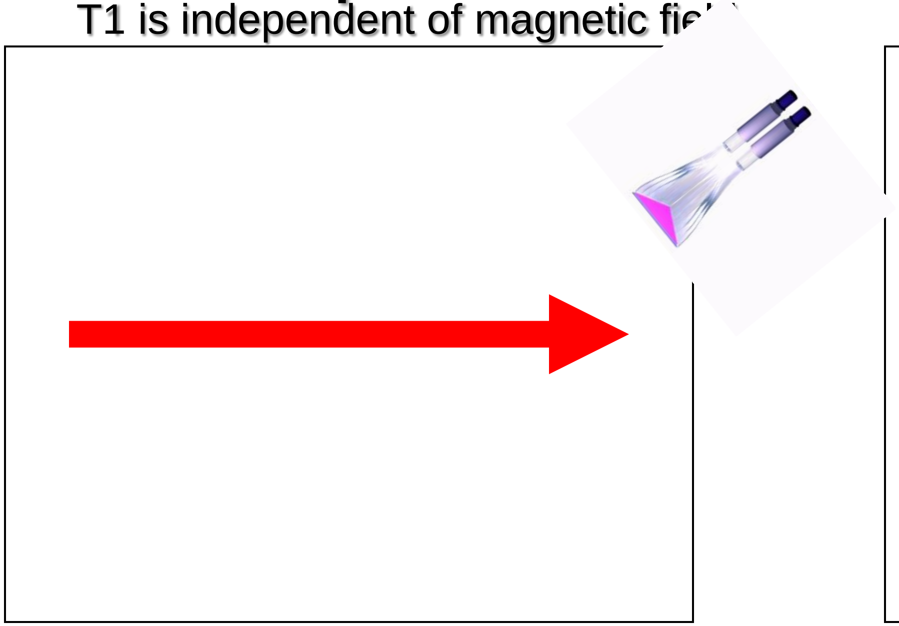
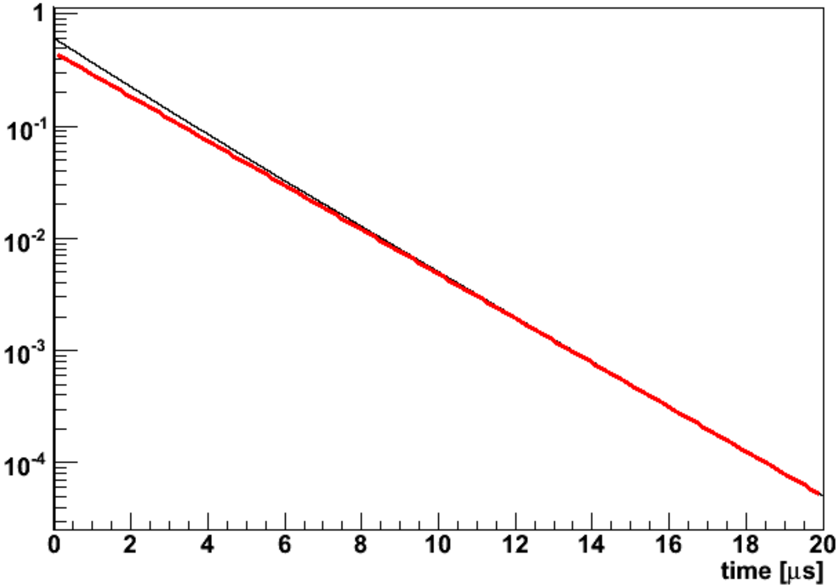
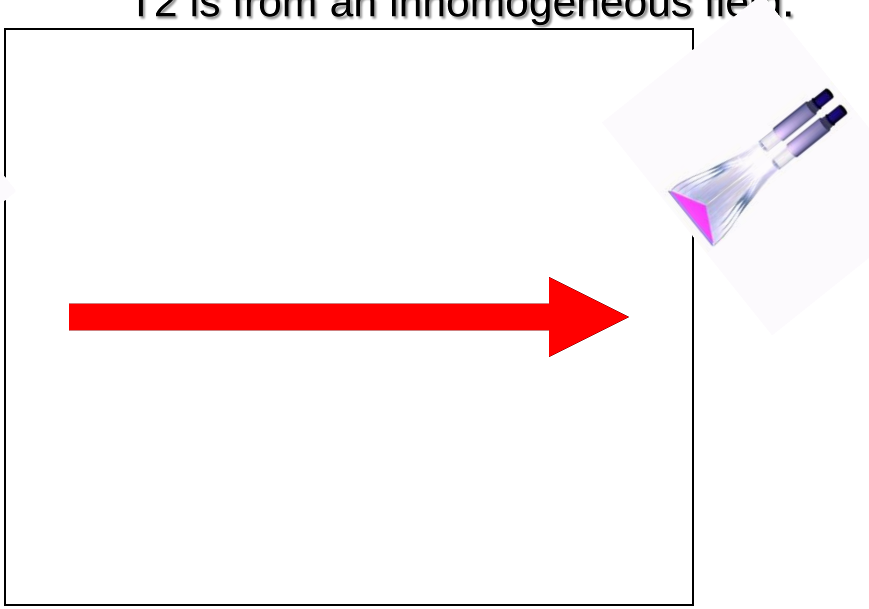
μSR relaxation results in a reduction of the polarization magnitude.

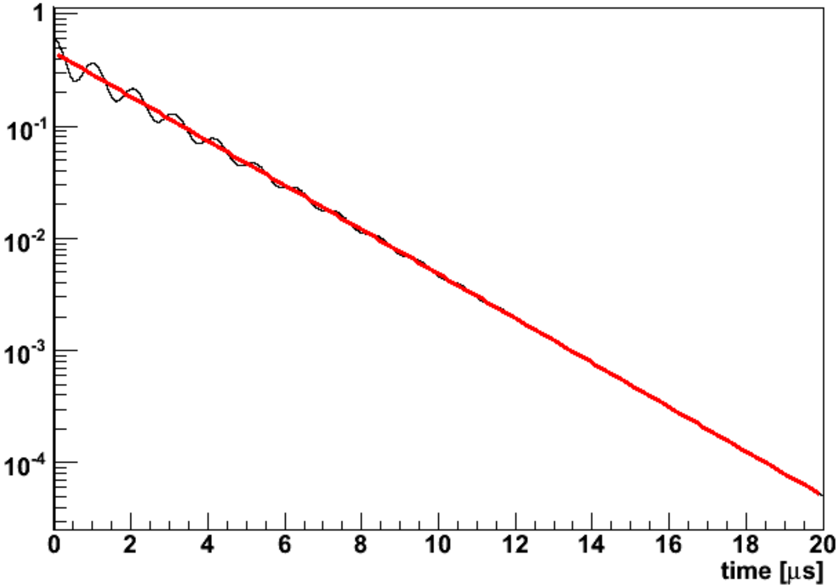
T1 is independent of magnetic fie





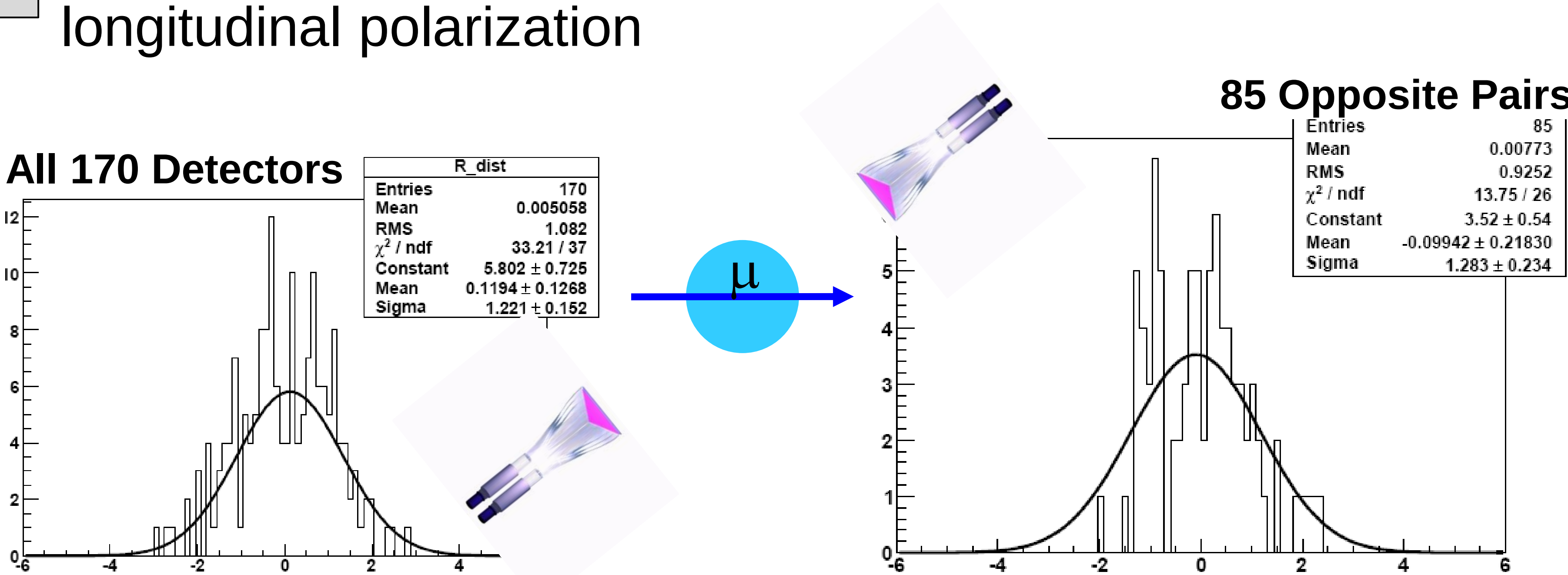
T2 is from an inhomogeneous field.

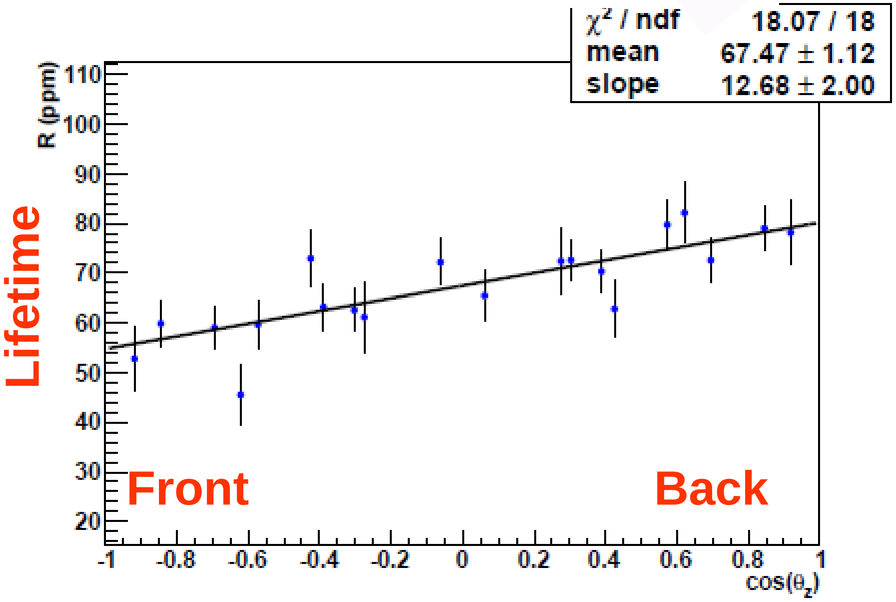


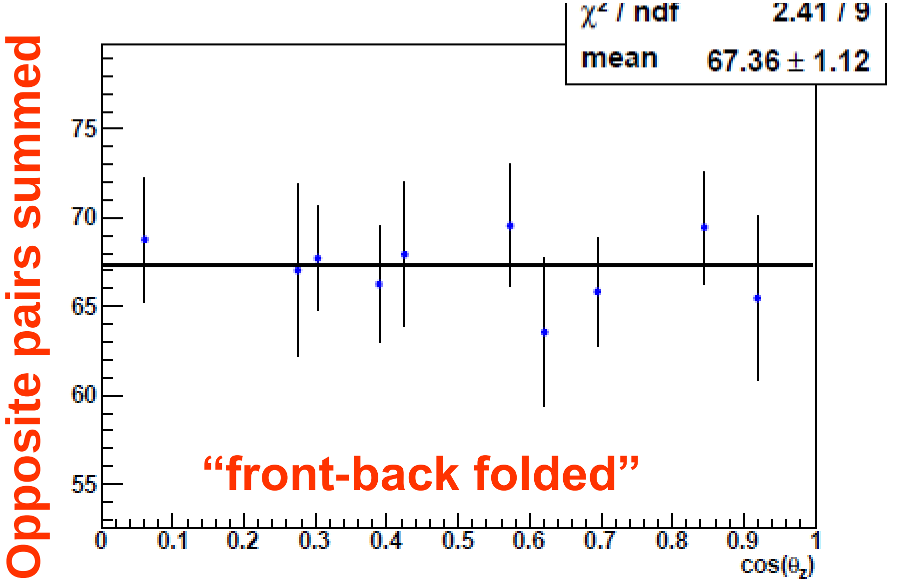


τ_{μ^+}

A small asymmetry exists front / back owing to residual longitudinal polarization

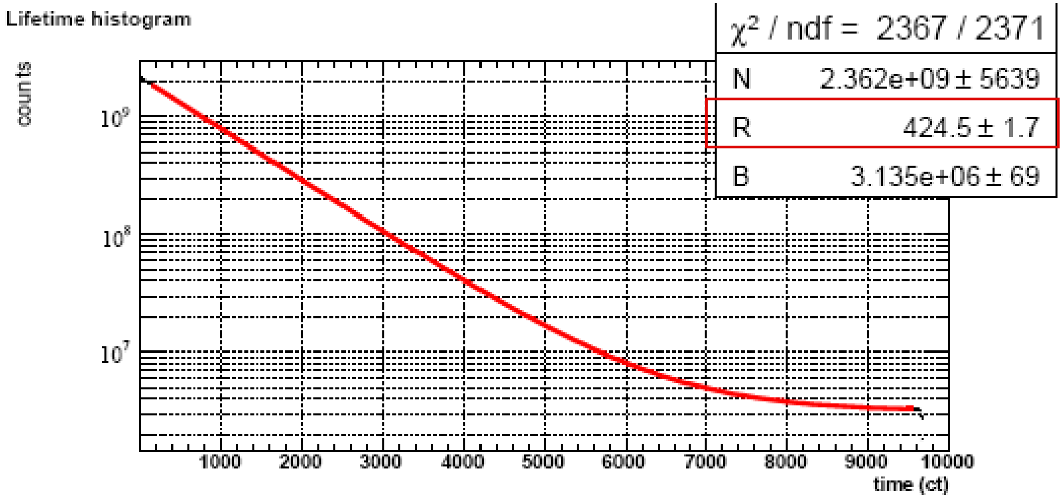


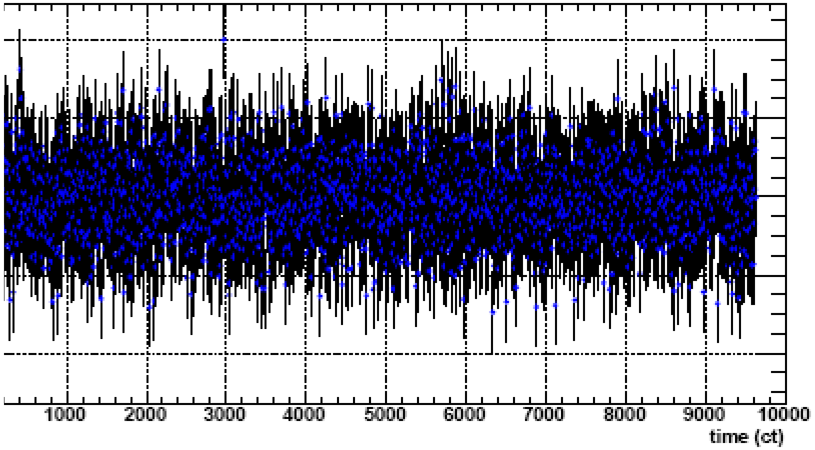


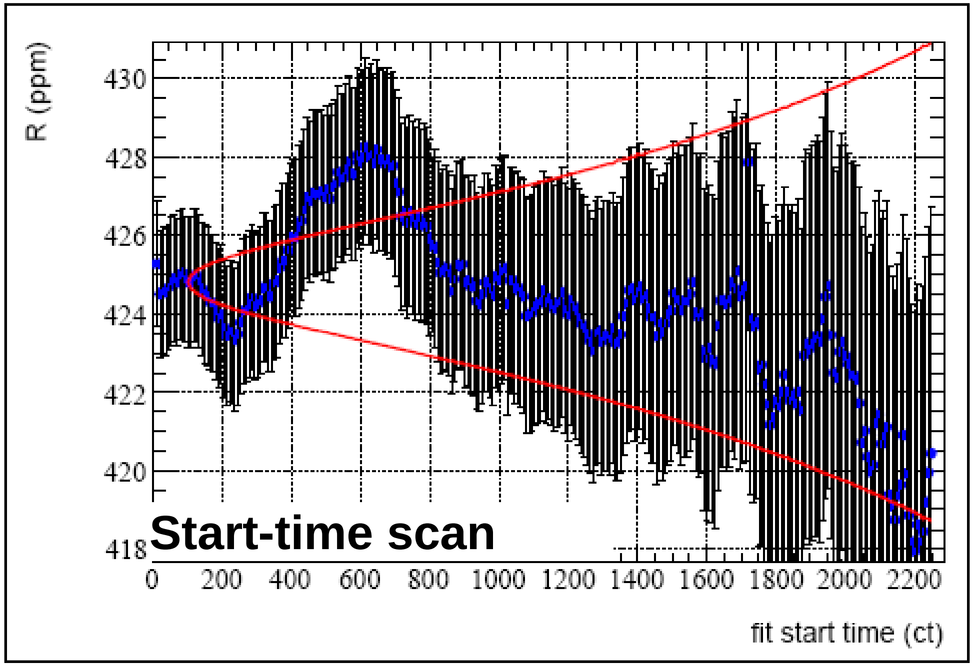


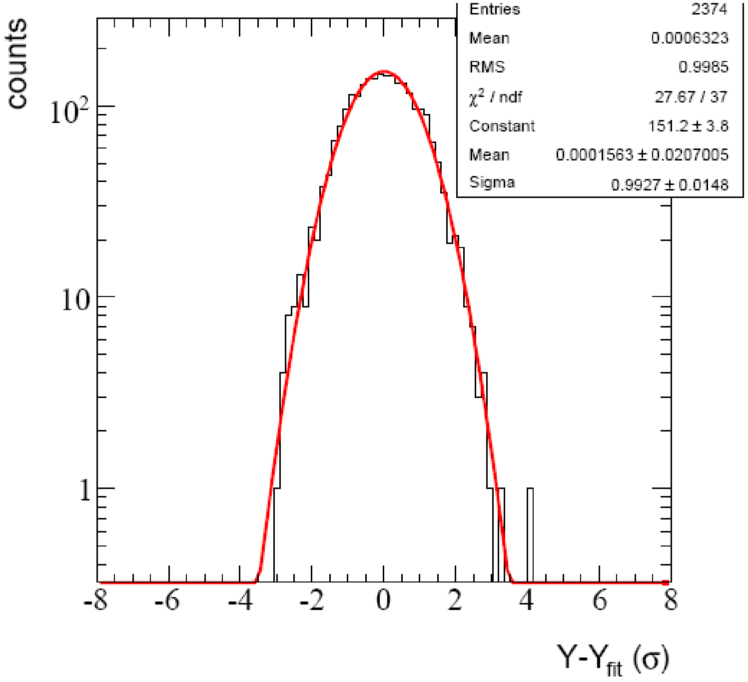
τ_{μ^+}

Quartz data fits well as a simple sum, exploiting the symmetry of the detector. The μSR remnants vanish.









MuLan Systematics and Final Numbers

ppm units

Effect	2006	2007	Comment
Kicker extinction stability	0.20	0.07	Voltage measurements of plates
Upstream muon stops	0.10	0.10	Upper limit from measurements
Overall gain stability:	0.25	0.25	MPV vs time in fill; includes:
Timing stability	0.12	0.12	Laser with external reference ctr.
Pileup correction	0.20	0.20	Extrapolation to zero ADT
Residual polarization	0.10	0.20	Long relax; quartz spin cancelation
Clock stability	0.03	0.03	Calibration and measurement
Total Systematic	**0.42**	**0.42**	**Highly correlated for 2006/2007**
Total Statistical	**1.14**	**1.68**	

$$\tau(\mathrm{R06}) = 2\,196\,979.9 \pm 2.5 \pm 0.9\ \mathrm{ps}$$

AK-3

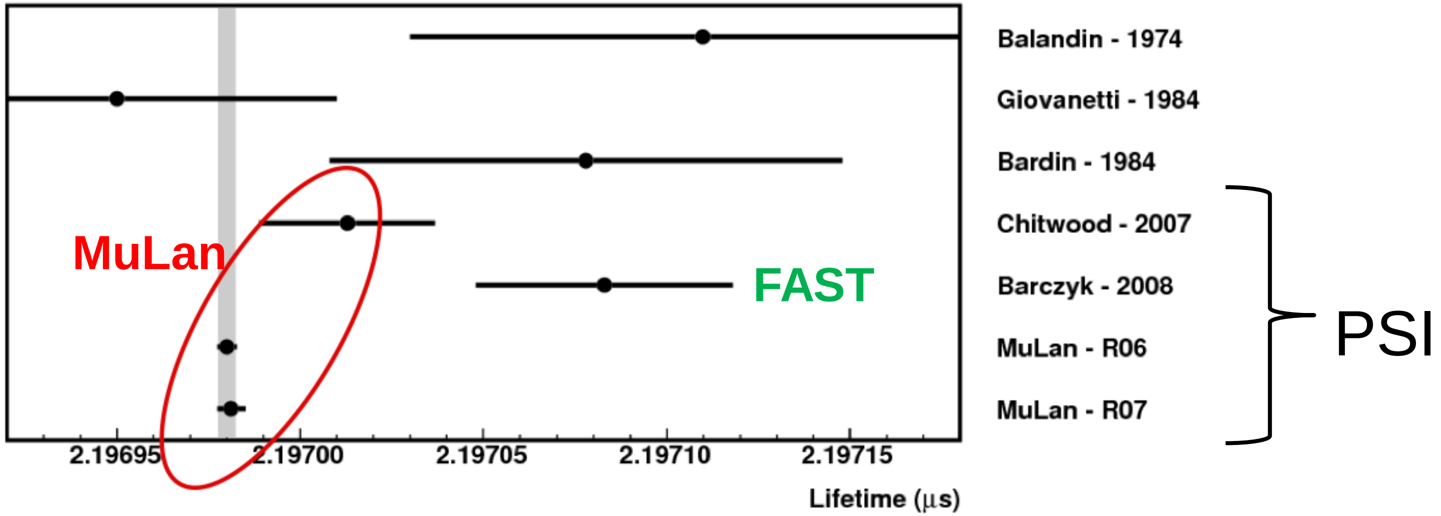
$$\tau(\mathrm{R07}) = 2\,196\,981.2 \pm 3.7 \pm 0.9\ \mathrm{ps}$$

Quartz

$$\Delta\tau(\mathrm{R07} - \mathrm{R06}) = 1.3\ \mathrm{ps}$$

Both measurements were separately blinded

MuLan Final Results on τ_μ:



The most precise particle or nuclear or atomic lifetime ever measured

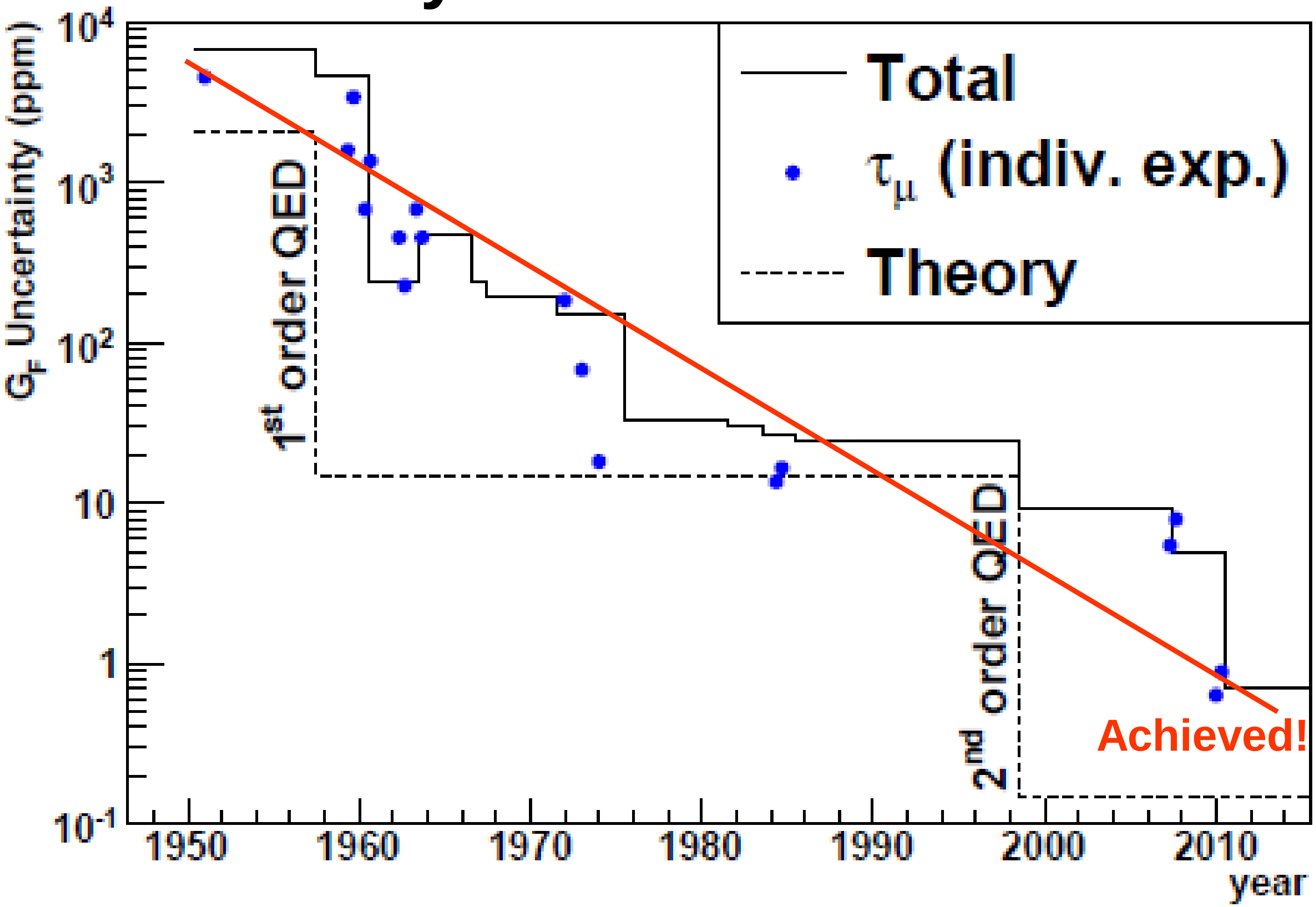
τ(MuLan) = 2 196 980.3 ± 2.2 ps (1.0 ppm)

G_F precision improved by factor of 30 compared to 1999 PDG

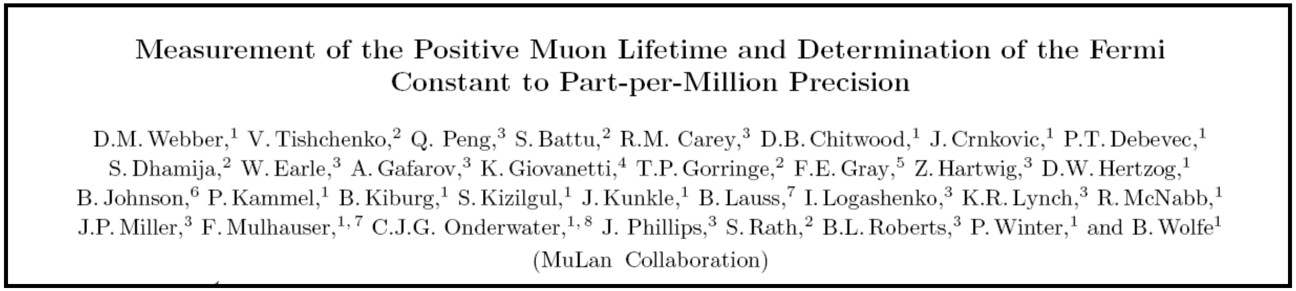
G_F(MuLan) = 1.166 378 7(6) x 10^{-5} GeV^{-2} (0.5 ppm)

PRL 106, 041803 (2011)
Phys. Rev. D 87, 052003 (2013)

G_F & τ_μ precision has improved by ~4 orders of magnitude over 60 years.



Achieved!



Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision

D.M. Webber,[1] V. Tishchenko,[2] Q. Peng,[3] S. Battu,[2] R.M. Carey,[3] D.B. Chitwood,[1] J. Crnkovic,[1] P.T. Debevec,[1] S. Dhamija,[2] W. Earle,[3] A. Gafarov,[3] K. Giovanetti,[4] T.P. Gorringe,[2] F.E. Gray,[5] Z. Hartwig,[3] D.W. Hertzog,[1] B. Johnson,[6] P. Kammel,[1] B. Kiburg,[1] S. Kizilgul,[1] J. Kunkle,[1] B. Lauss,[7] I. Logashenko,[3] K.R. Lynch,[3] R. McNabb,[1] J.P. Miller,[3] F. Mulhauser,[1,7] C.J.G. Onderwater,[1,8] J. Phillips,[3] S. Rath,[2] B.L. Roberts,[3] P. Winter,[1] and B. Wolfe[1]
(MuLan Collaboration)

Some recent Press …

Weak Nuclear Force Is Less Weak

New insights from subatomic particles that fly apart.

Jan 12, 2011

By Phillip F. Schewe
Inside Science News Service

(ISNS) -- The force that governs some of the reactions that keep our sun shining is not quite as weak as scientists had previously thought. As a consequence, our estimation of how energetic the sun actually is just went up by a tiny amount.

View full-size image

Credit: versageek via flickr

Rights Information

The evidence for this weak nuclear force comes from the decay of muons, essentially heavier cousins of the electron, one of the building blocks of atoms.

Just as biologists sometimes study the tiniest and most ephemeral of organisms such as fruit flies, which live for barely a day, to learn things about human disease, so physicists often study the properties of particles that last a fraction of a second to learn about the universe.

The muon lives only about 2 millionths of a second -- 2 microseconds -- far from the realm of human sensation but long enough for scien
digital electronics is so advanced that meas
of a second or less, can easily be made.

Watching muons decay is not like propping
uranium. That's because muons are so sh
medical isotopes. At the Paul Scherrer Insti
create muons amid collisions with a graphi

Researchers then gathered a fine spray of
metal target which was surrounded by a de
over 2 trillion muons provided the best yet v
2.1969803 microseconds.

---

25. January 2011

How strong is the weak force?

New measurement of the muon lifetime – the most precise determination of any lifetime – provides a high-accuracy value for a crucial parameter determining the strength of weak nuclear force. The experiments were performed by an international research team at the accelerator facility of the Paul Scherrer Institute. The results are about to be published in the journal Physical Review Letters.

The weak force is one of the four fundamental forces of Nature. Although we hardly encounter processes governed by the weak force in our everyday life, it is still of crucial importance; e.g., being responsible for the processes that make the Sun shine. An international research team led by scientists from the University of Illinois, Boston University and the University of Kentucky performed experiments at the Paul Scherrer Institute (Villigen, Switzerland) that allowed them to determine a parameter crucial for the strength of the weak force with unprecedented accuracy of 0.6 parts per million. This so called Fermi constant is one of the fundamental natural
ations of processes in the world of elementary particles.

PSI scientist Bernhard Lauss with the detector array used in the determination of the muon lifetime (PSI/F. Reiser)

our understanding of the subatomic world in the 1970s was the
d the electromagnetic interaction – another of the four fundamental
of one and the same interaction. It is called the electroweak
rmined by three parameters, the Fermi constant being one of

Das Wort der Woche: Myon

29.01.2011 | 18:14 | von Thomas Kramar (Die Presse)

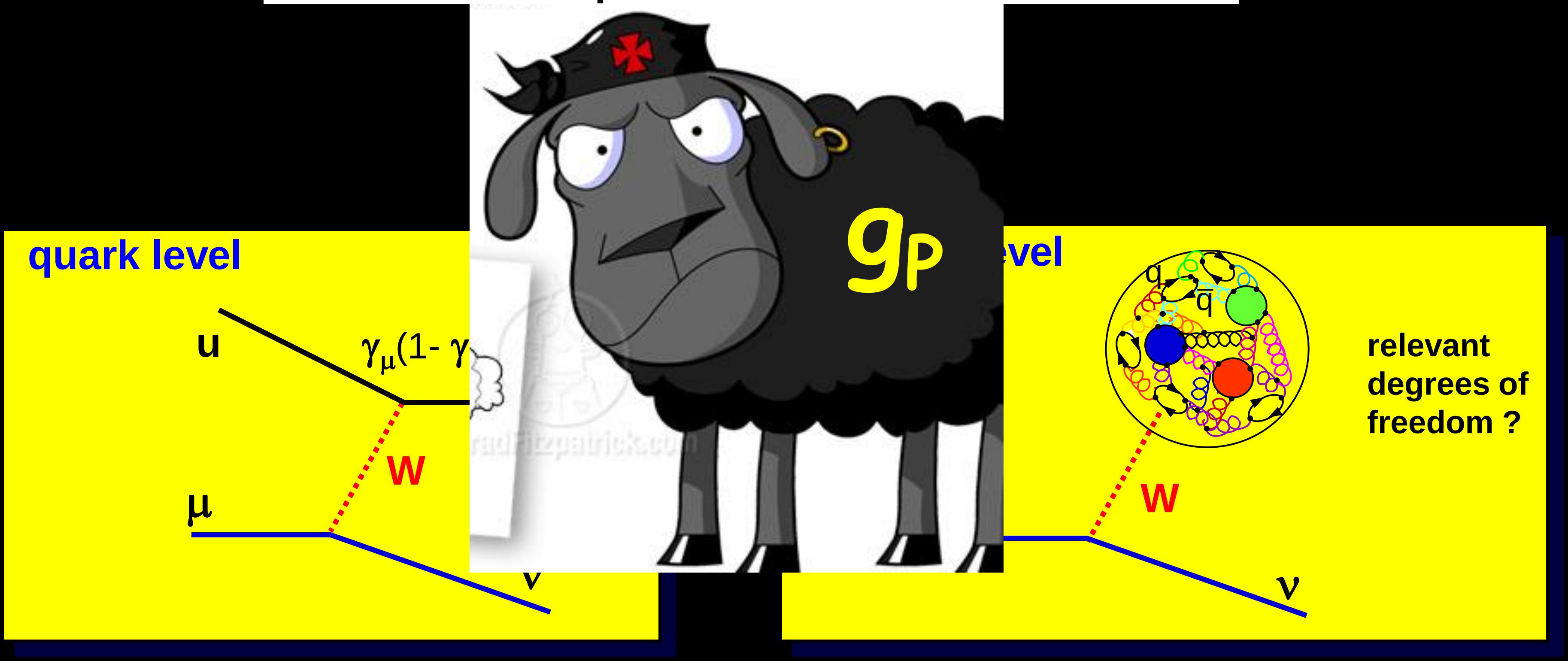
Die mittlere Lebensdauer des Myons – eines schwereren Verwandten des Elektrons – wurde so genau gemessen wie nie zuvor. Das hilft den Physikern, eine der Grundkräfte (noch) besser kennenzulernen.

Wer hat denn das bestellt?" Mit diesem eher unfreundlichen Ausruf begrüßte der Teilchenphysiker (und spätere Nobelpreisträger) Isidor Isaac Rabi das Myon. Tatsächlich: Dieses 1936 bei der Untersuchung kosmischer Strahlung entdeckte Elementarteilchen – daran beteiligt war der österreichische Nobelpreisträger Victor Franz Hess – war für die damalige Teilchenphysik ein Störenfried: ein Teilchen, das dieselben Eigenschaften wie das Elektron hat, aber 207-mal so schwer ist, das hat keine Theorie vorausgesagt!

Inzwischen haben die Physiker damit zu leben gelernt, dass Elementarteilchen schwerere Verwandte haben. Und zwar jeweils zwei: So gibt es zum Elektron nicht nur das Myon, sondern auch das (1975 entdeckte) Tauon, das 3478-mal so schwer ist wie das Elektron. Überhaupt: Jedes der Elementarteilchen, aus denen die Materie aufgebaut ist, kommt in drei „Generationen" mit

Muon Capture on the Proton

The Black Sheep of Form Factors – T. Hemmert



relevant degrees of freedom ?

$$\frac{\Delta\Lambda_S}{\Lambda_S} = 1\% \quad \Rightarrow \quad \frac{\Delta g_P}{g_P} \approx 6.1\%$$

Muon Capture on the proton and Axial Nucleon Structure

$$\mu^- + p \rightarrow n + \nu_\mu$$

Capture rate Λ_S :

$$\boxed{\mathcal{M} = \frac{-iG_F V_{ud}}{\sqrt{2}} \overline{u}(p_\nu)\gamma_\alpha(1-\gamma_5)u(p_\mu)\overline{u}(p_f)\tau_- \left[V^\alpha - A^\alpha\right] u(p_i)}$$

Lorentz, T invariance gives these possibilities

$$\boxed{\begin{aligned} V_\alpha &= g_V(q^2)\,\gamma_\alpha + \frac{i\,g_M(q^2)}{2\,M_N}\,\sigma_{\alpha\beta}\,q^\beta \\ A_\alpha &= g_A(q^2)\,\gamma_\alpha\,\gamma_5 + \frac{g_P(q^2)}{m_\mu}\,q_\alpha\,\gamma_5 \end{aligned}}$$

How does Λ_S depend on precision of the form factors ?

Well known

$$\left\{\boxed{\begin{aligned} \left(\frac{\partial\Lambda_S}{\Lambda_S}\right)_{\Delta g_V} &= 0.024\% \\ \left(\frac{\partial\Lambda_S}{\Lambda_S}\right)_{\Delta g_M} &= 0.01\% \\ \left(\frac{\partial\Lambda_S}{\Lambda_S}\right)_{\Delta g_A} &= 0.38\% \end{aligned}}\right.$$

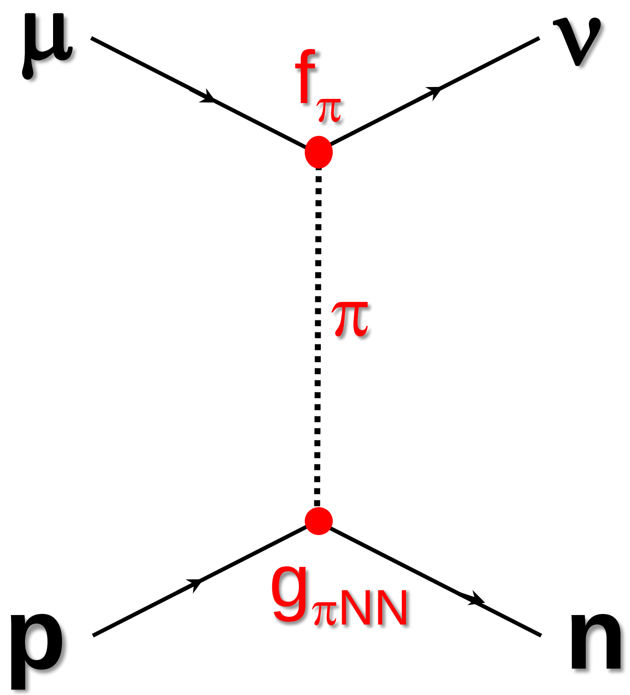
$$\boxed{\frac{\Delta\Lambda_S}{\Lambda_S} = 1\% \quad \Rightarrow \quad \frac{\Delta g_P}{g_P} \approx 6.1\%}$$

The least well known is g_P

Pseudoscalar form factor g_P

τ_{μ^-}

$$g_P(q^2) = -\frac{2m_N m_\mu g_A(0)}{q^2-m_\pi^2} \cdot$$



PCAC pole term

(Adler, Dothan, Wolfenstein)

$$\boxed{g_P = 8.26 \pm 0.23}$$

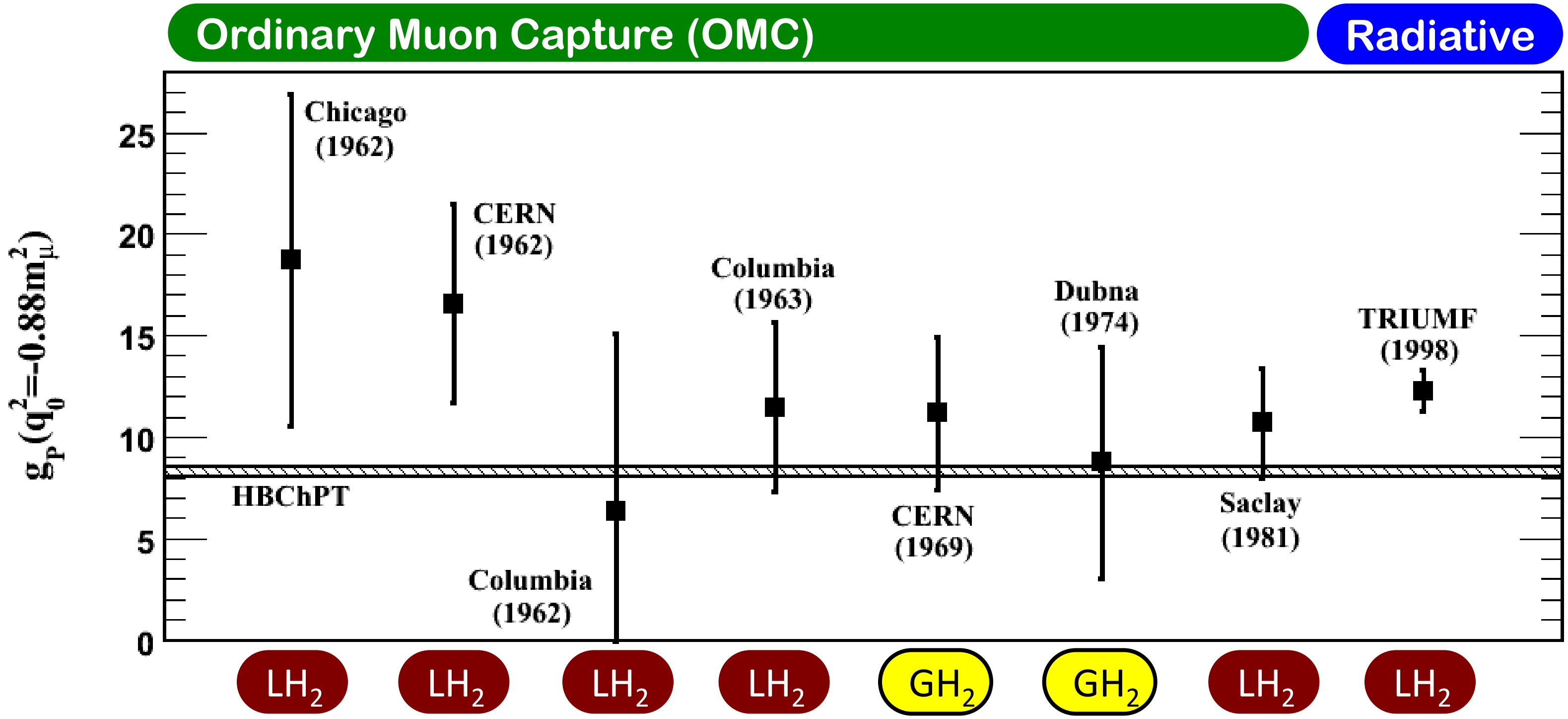
→ **Λ_S = 711.5 ± 4.6 s^{-1}(0.65%)**

- ChPT based on the spontaneous symmetry breaking
- QCD prediction via ChPT @ 2-3% precision level
- Basic test of chiral symmetries and low-energy QCD

Recent review: Kammel, P. and Kubodera, K., Annu. Rev. Nucl. Part. Sci. 60 (2010), 327

The experimental determinations of g_P prior to MuCap were far less precise

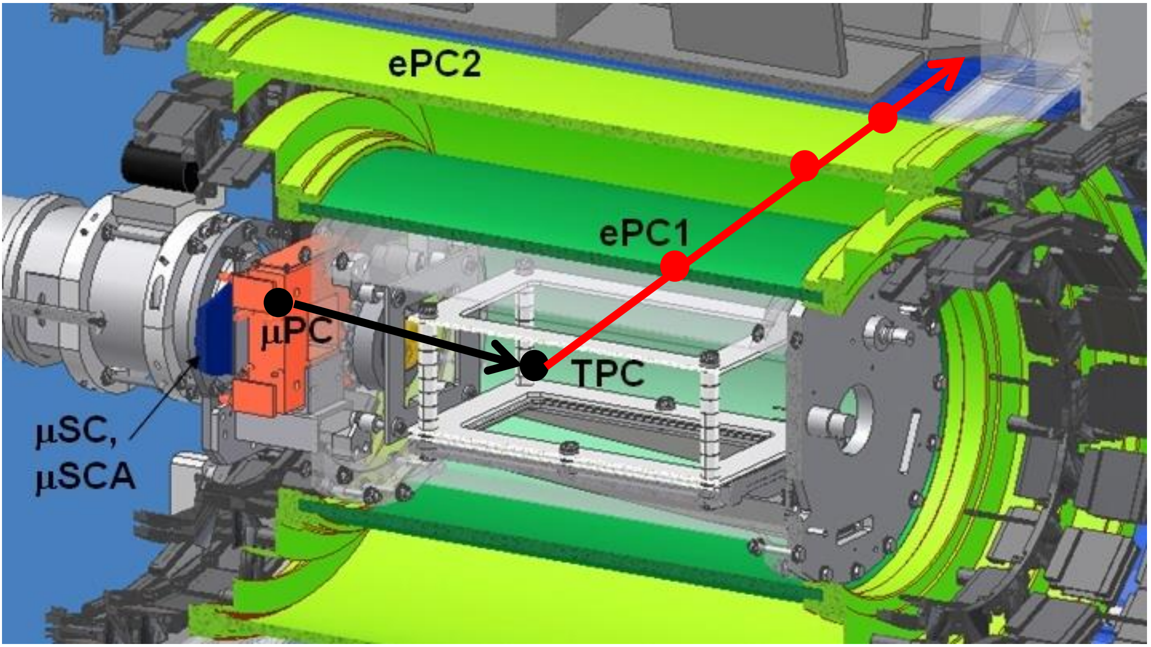
τ_{μ^-}



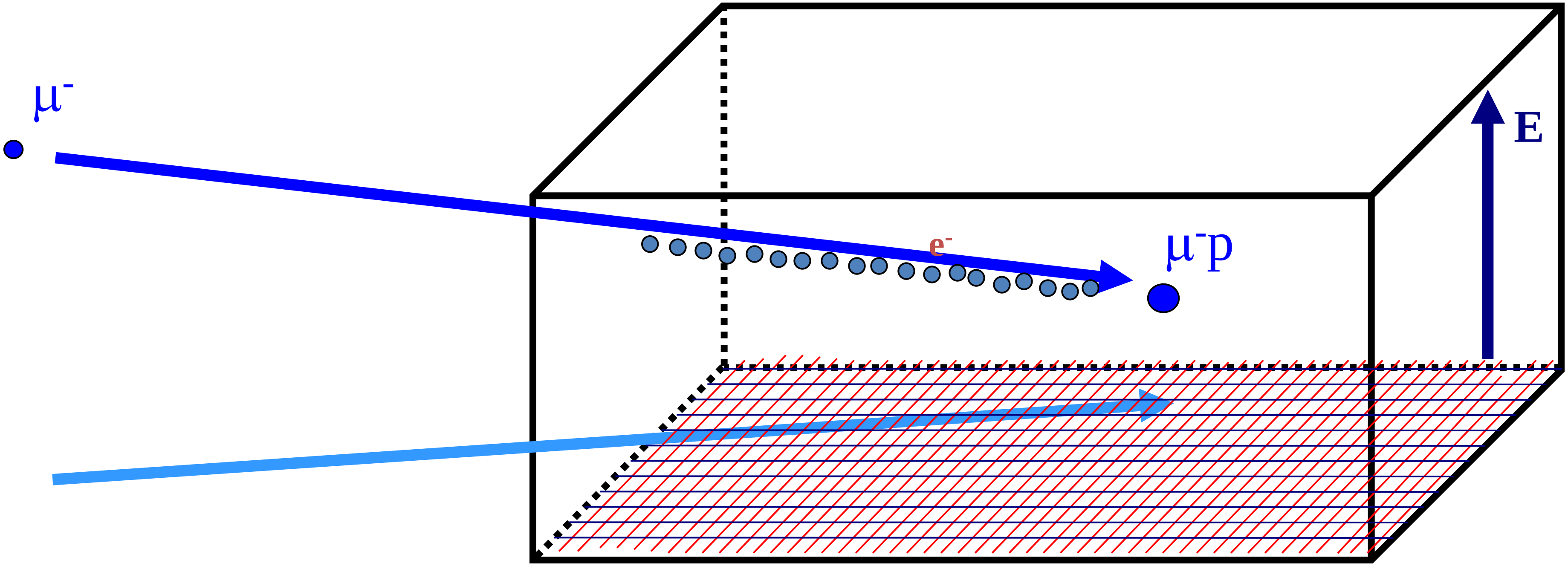
Liquid H_2 target

Gas (low density) H_2 target

The MuCap Experimental Concept

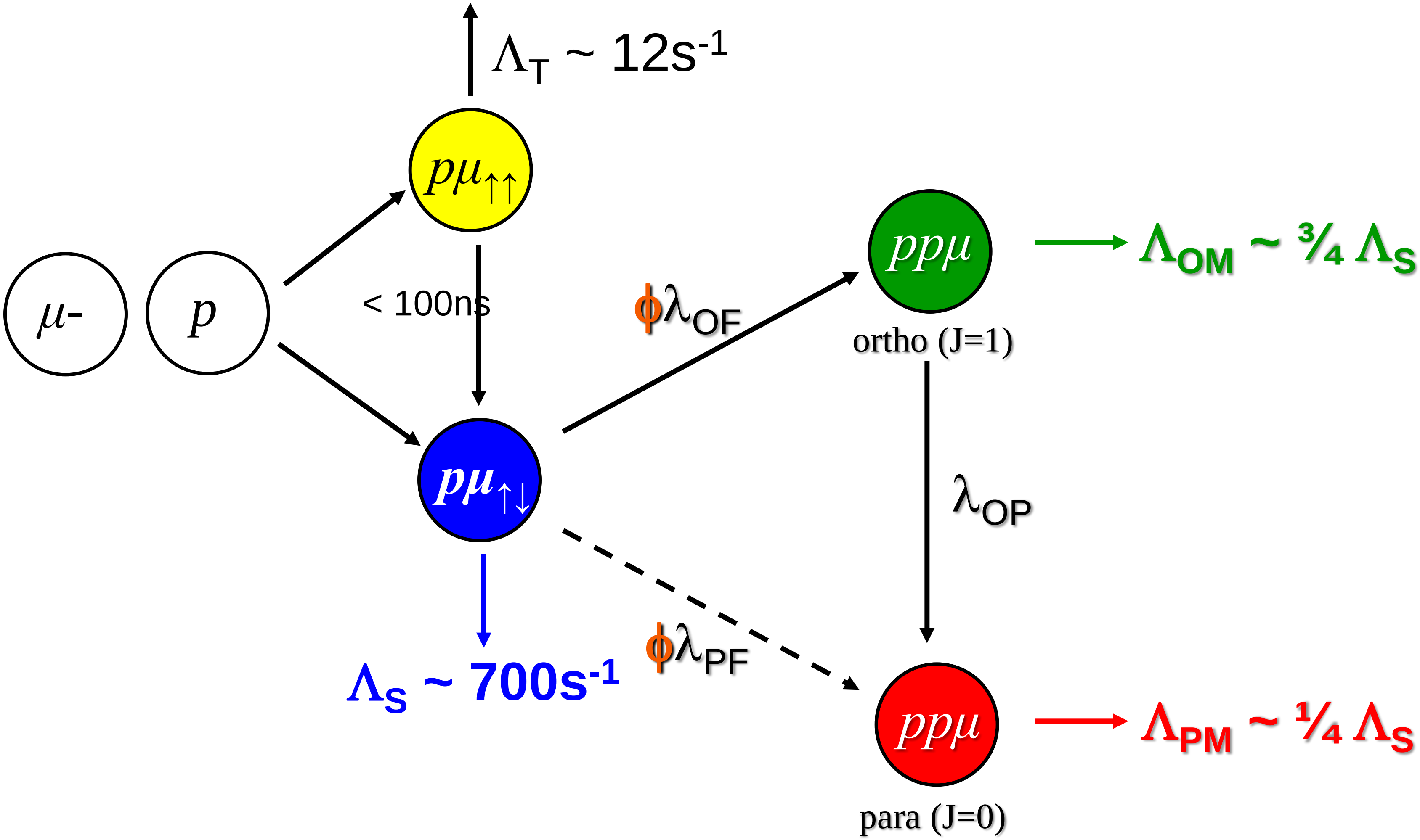


- Stop in pure hydrogen (gas)
- Gas impurities < 10 ppb
- Isotopic impurity < 6 ppb
- Image muon stop with TPC
- Measure the disappearance rate (effective lifetime)



Muon kinetics

ϕ: Hydrogen density, (LH_2: ϕ=1)



$\Lambda_T \sim 12 s^{-1}$

$p\mu_{\uparrow\uparrow}$

μ-

p

< 100ns

$p\mu_{\uparrow\downarrow}$

$\Lambda_S \sim 700 s^{-1}$

$\phi\lambda_{OF}$

$\phi\lambda_{PF}$

$pp\mu$

ortho (J=1)

$\Lambda_{OM} \sim \frac{3}{4} \Lambda_S$

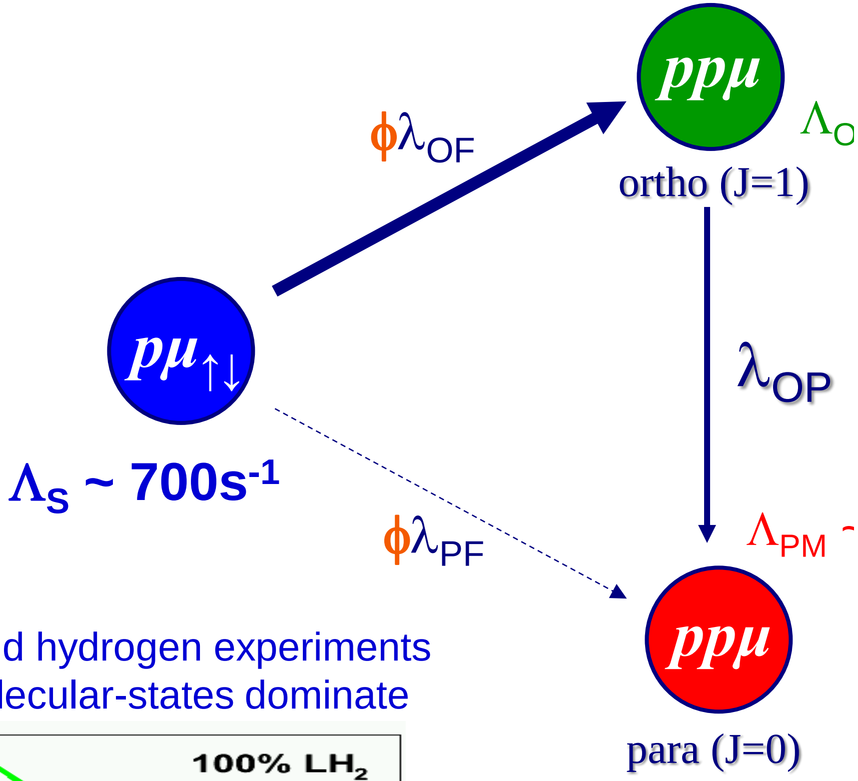
λ_{OP}

$pp\mu$

para (J=0)

$\Lambda_{PM} \sim \frac{1}{4} \Lambda_S$

Capture from **μp singlet** is in competition with capture from ppμ molecular states: depends on density and time



$pp\mu$

ortho (J=1)

Λ_O

$\phi\lambda_{OF}$

$p\mu_{\uparrow\downarrow}$

λ_{OP}

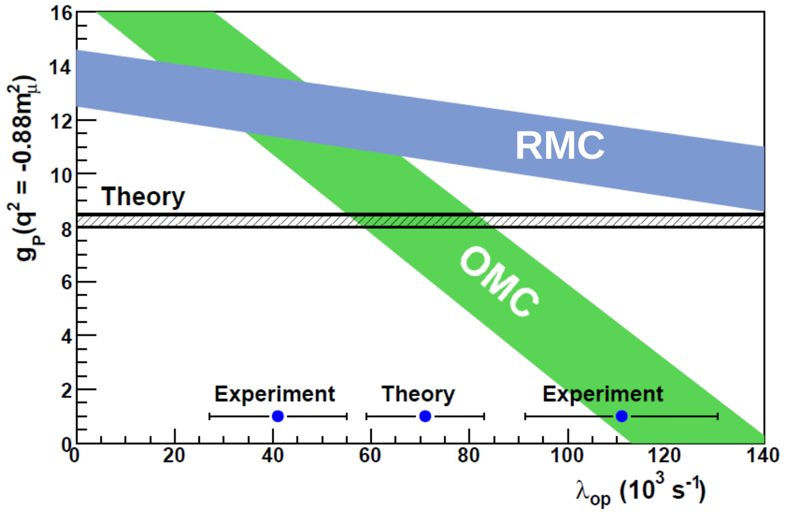
$\Lambda_S \sim 700 s^{-1}$

$\phi\lambda_{PF}$

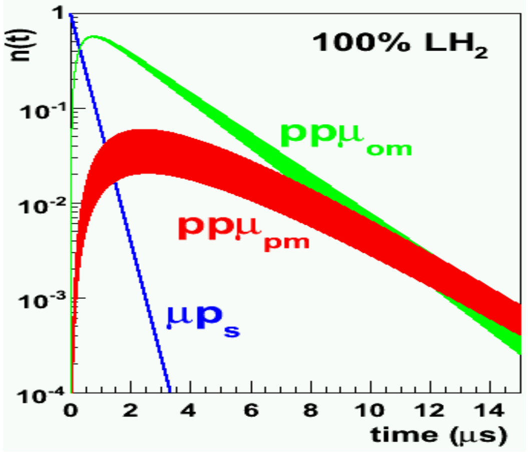
Λ_{PM}

$pp\mu$

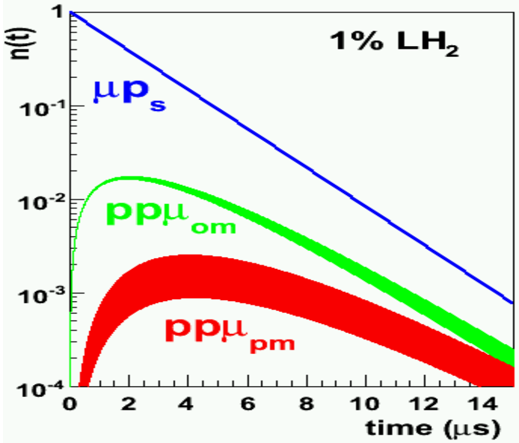
para (J=0)



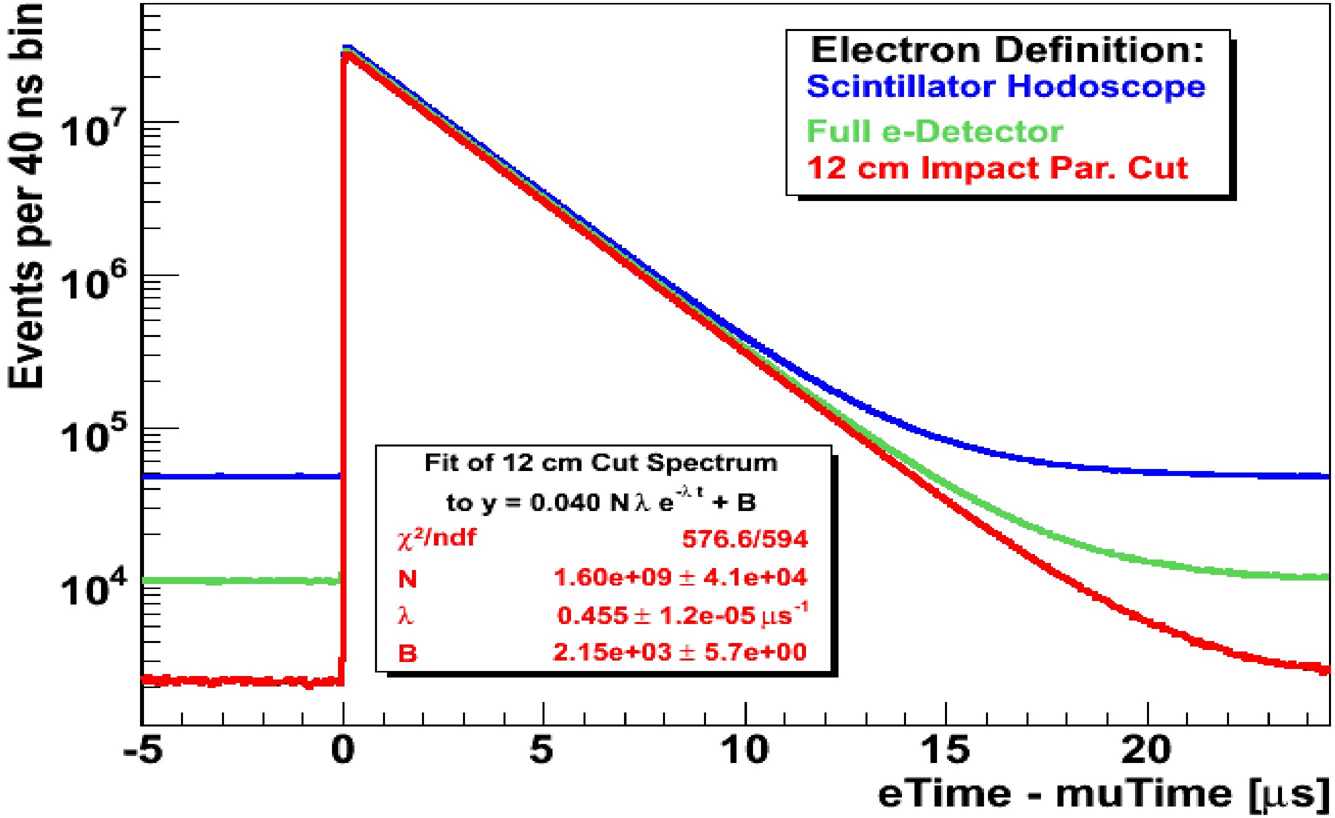
Liquid hydrogen experiments
Molecular-states dominate



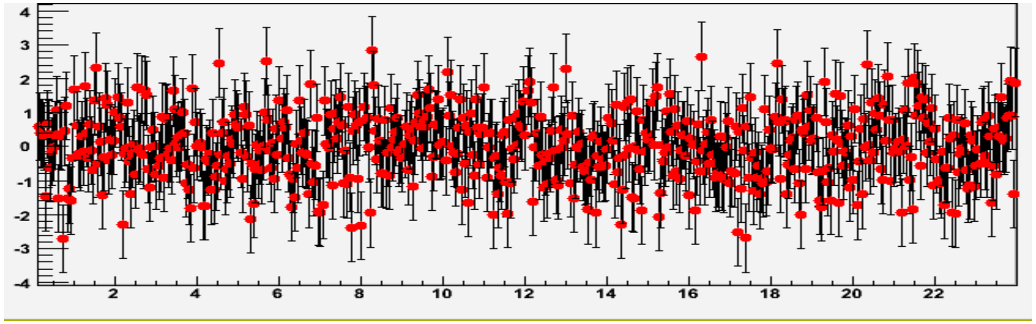
MuCap conditions
Singlet-state dominates



MuCap Negative Muon Lifetime Spectra

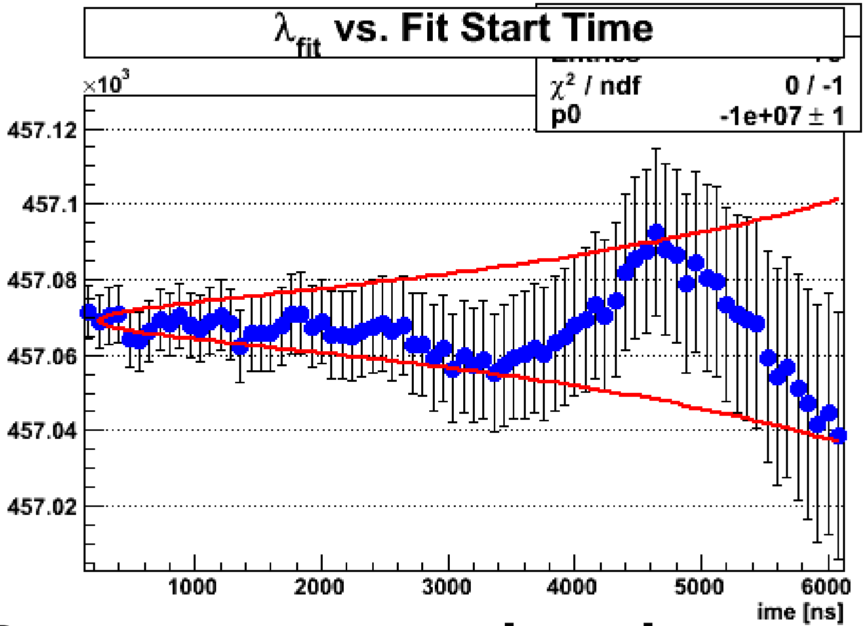


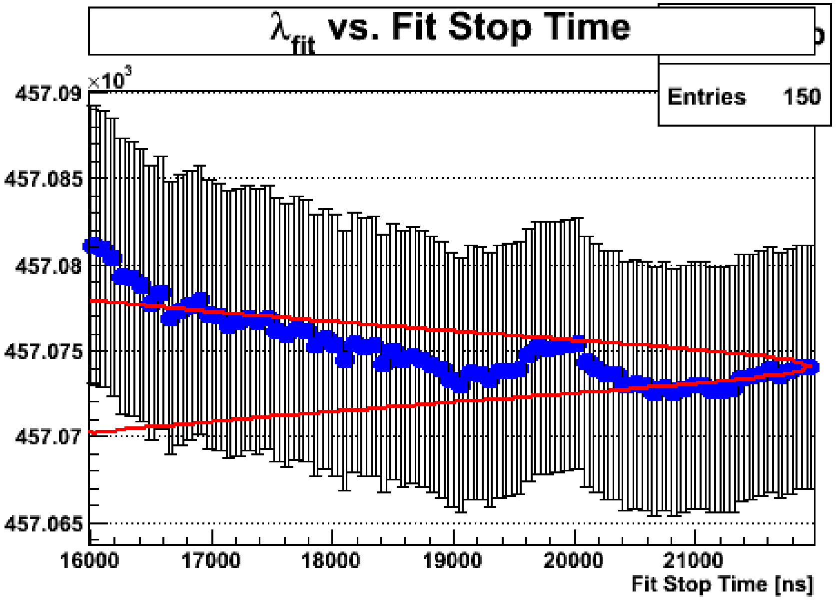
Typical fit normalized residuals



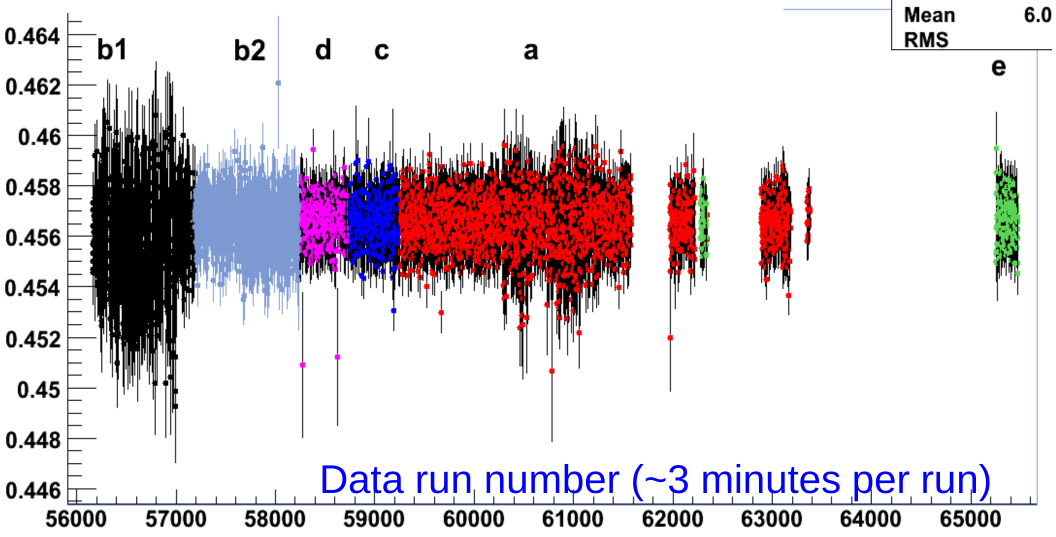
Start- and stop-time-scans

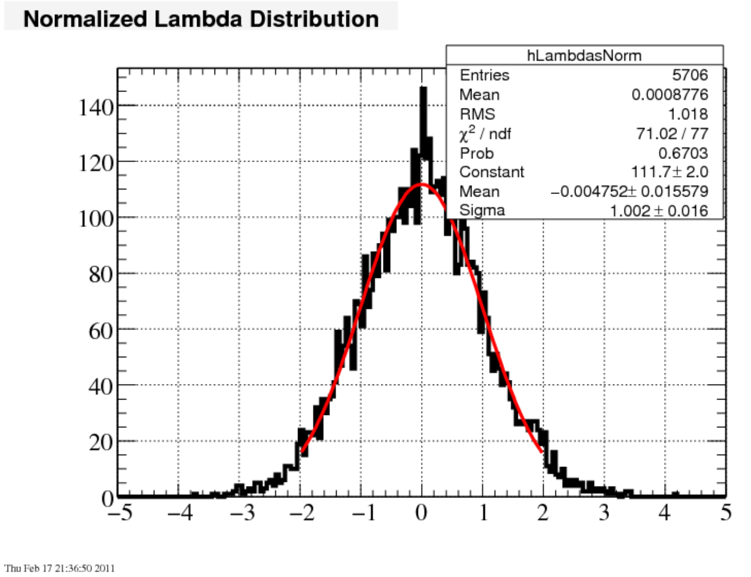
τ_{μ^-}





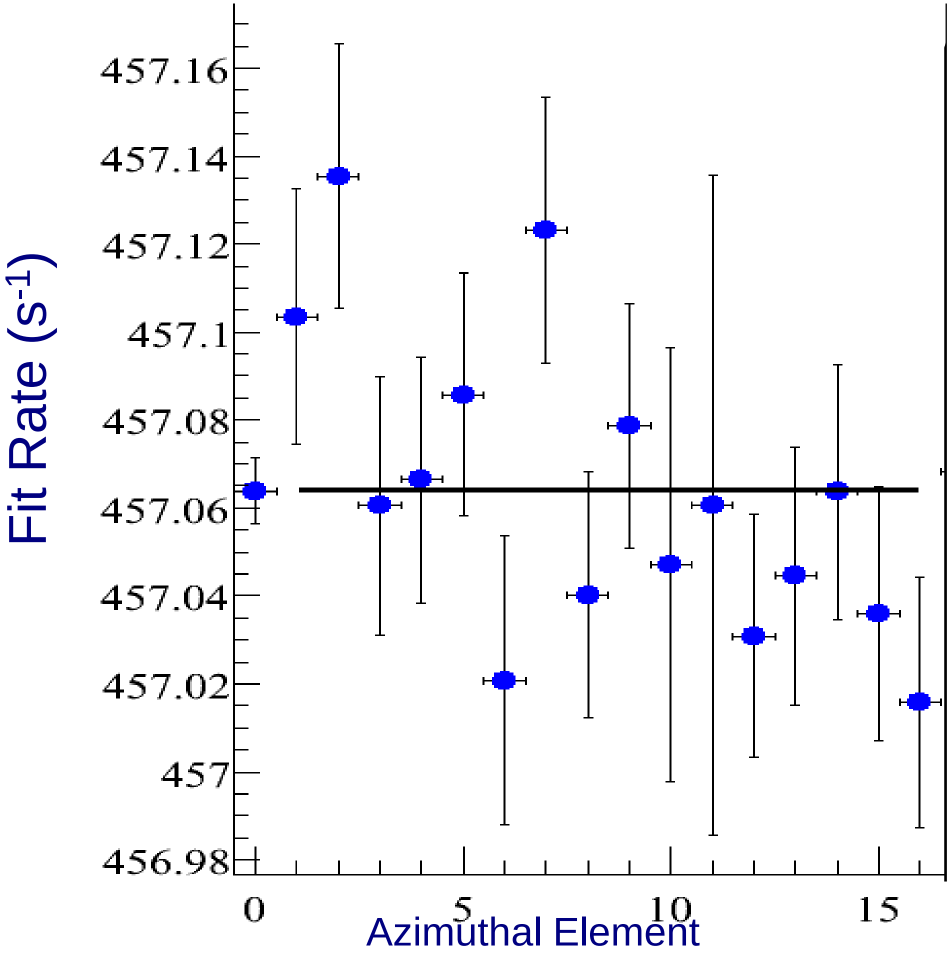
Rate versus run duration



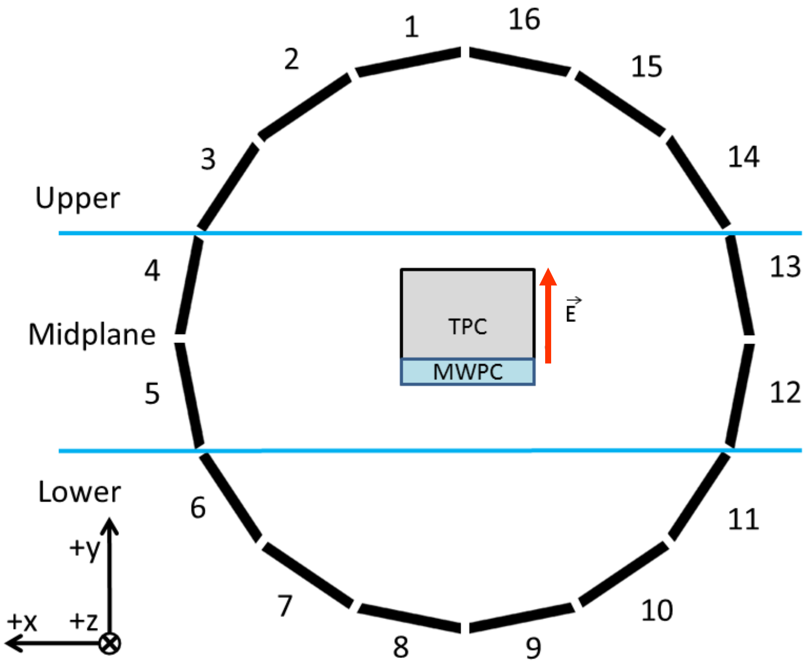


The disappearance rate is independent of azimuth

(non trivial since TPC is asymmetric vertically owing to drift direction)

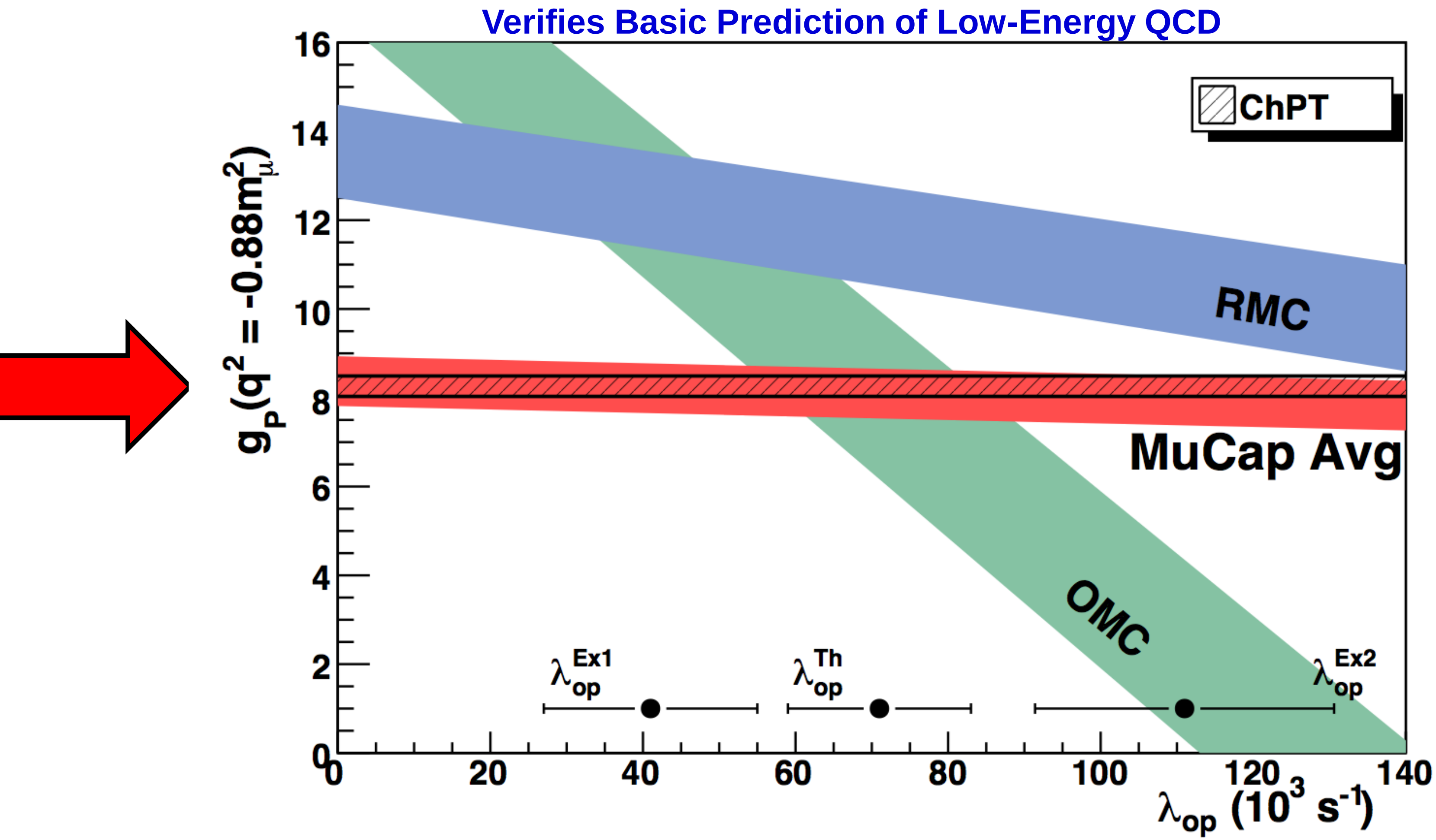


Beam View of MuCap Detector



Precise and unambiguous MuCap result solves longstanding puzzle

τ_{μ^-}



g_P(MuCap) = 8.06 ± 0.55

g_P(theory) = 8.26 ± 0.23

Phys.Rev.Lett. 110 (2013) 012504

τ_{μ^+}

Compare Lifetimes !

τ_{μ^-}

(If you believe χPT more than CPT ☺)

Free μ+ τ_{MuLan} = 2 196 980.3 ± 2.2 ps

Effective* μ- τ_{MuCap} = 2 196 963 +/- 42 ps

Difference:

$$\frac{\tau^+ - \tau^-}{\tau_{\text{avg}}} = (7 \pm 19\ )\ \text{ppm}$$

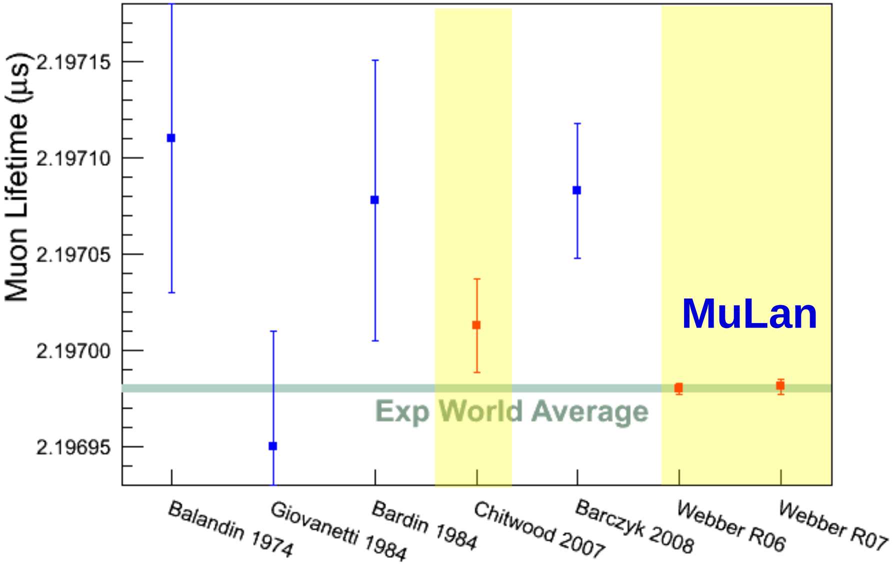
*Important assumptions

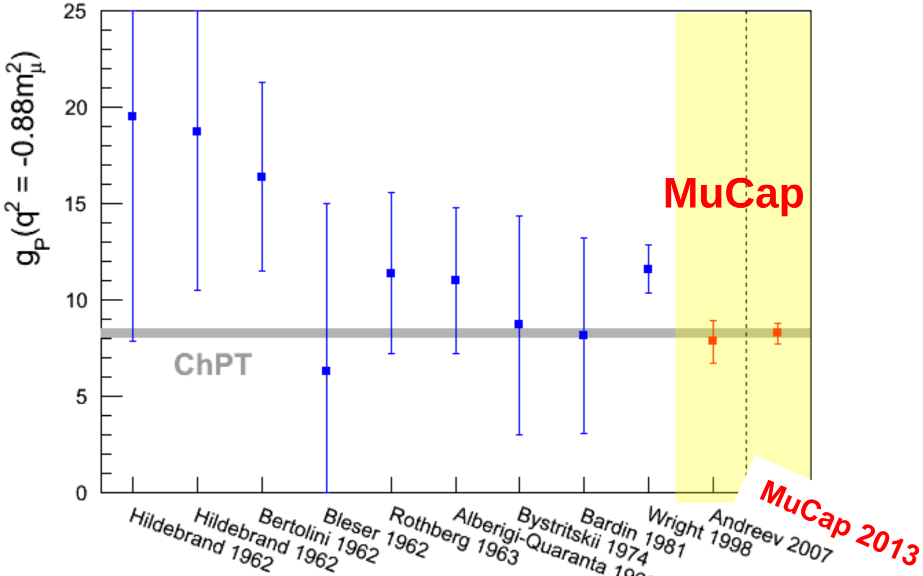
- Assume χ**PT** prediction is exact
- Correct for μp atomic shift (easy)
- Correct for impurity distortion (expt. errors are included)

Summary

- **MuLan** has finished and published
  - 1.0 ppm final error achieved, as proposed
    - The most precise particle or nuclear lifetime ever measured
  - Most precise Fermi constant
  - Modest check of muonium versus free muon

- **MuCap** has finished and published
  - First unambiguous determination of g_P





MuLan at PSI

Backups

MuSun: muon capture on the deuteron

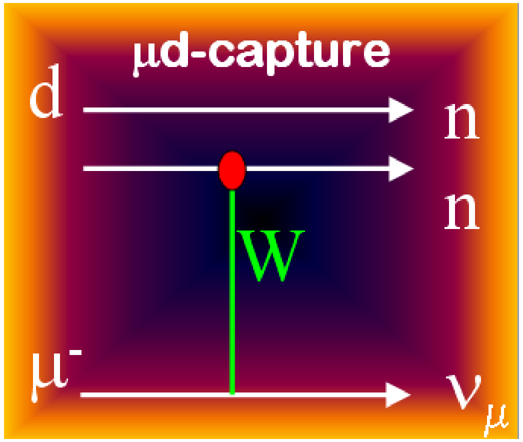
Goal: Measure rate Λ_d from $\mu d(\uparrow\downarrow)$ to < 1.5 %

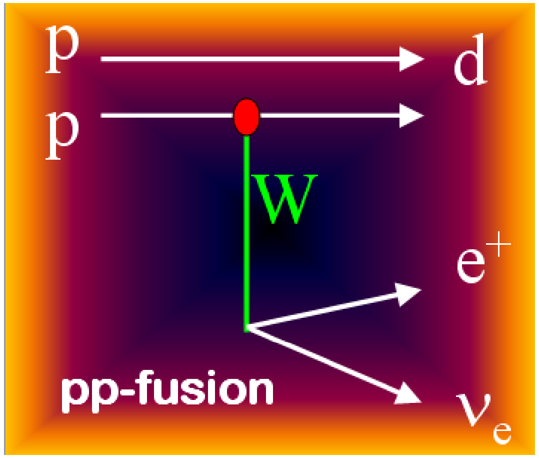
Several fundamental astrophysics processes depend on weak interaction in deuterium

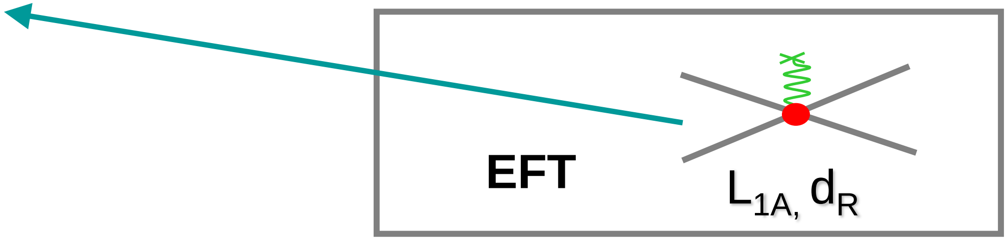
Basic solar fusion: $p + p \rightarrow d + e^+ + \nu$

$\nu_e + d \rightarrow p + p + e^-$ (CC)

$\nu_x + d \rightarrow p + n + \nu_x$ (NC)

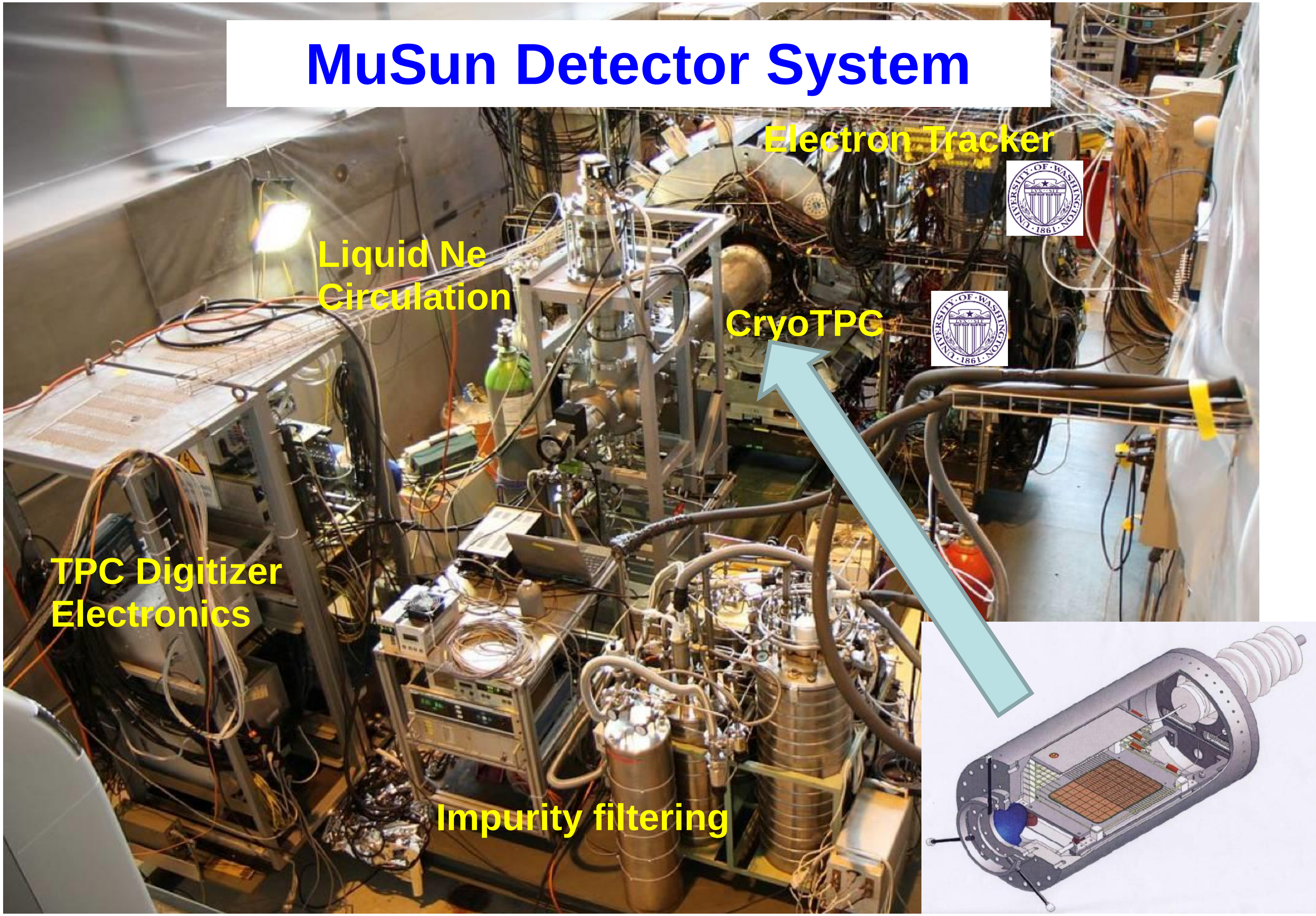






Experiment In Progress at PSI

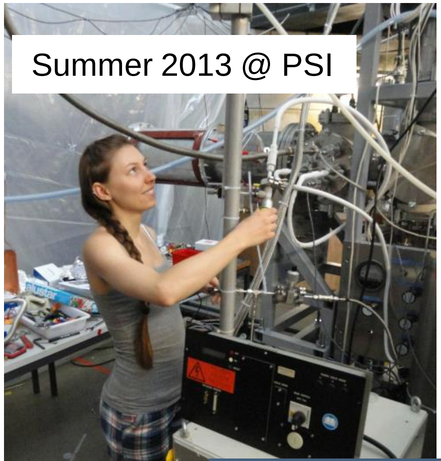
MuSun Detector System



CENPA and MuSun

Cryo-PreAmps, Local TPC optimization, MWPC support

Pictures from PSI and from our UW MuSun Lab setup.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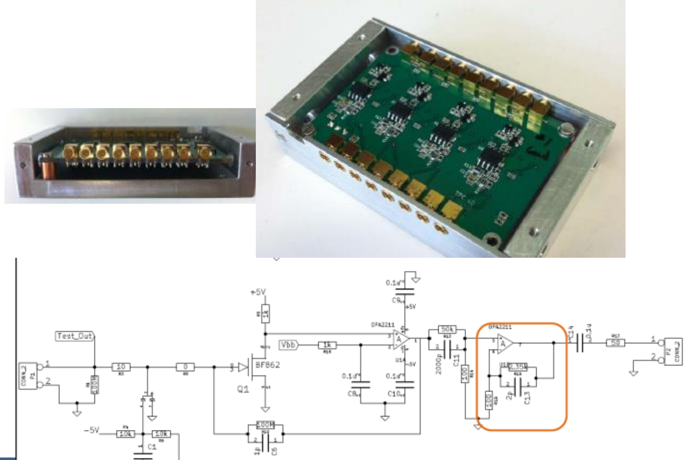
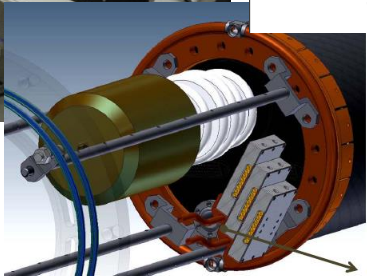
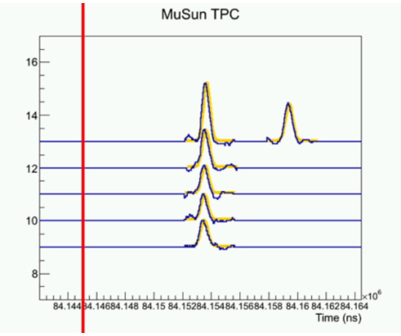
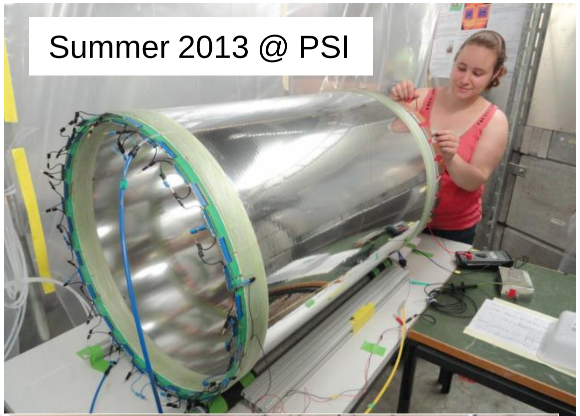
Cryo PreAmp design and construction




3D local design work

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UW MuSun Prep Lab

The clock was provided by an Agilent E4400B Signal Generator, which was stable during the run and found to be accurate to 0.025 ppm.

Agilent E4400 Function Generator

f = 451.0 +/- 0.2 MHz

- **Checked for consistency throughout the run.**
- **Compared to Quartzlock A10-R rubidium frequency standard.**
- **Compared to calibrated frequency counter**

Comparison	10 MHz	60 MHz
Frequency counter	1×10^{-8}	2×10^{-8}
Rubidium atomic clock	4×10^{-8}	3×10^{-8}

Average difference = 0.025 ppm

EW Phenomenology

In the gauge and scalar sectors, the SM Lagrangian contains only four parameters: g, g', μ^2 and h. One could trade them by α, θ_W, M_W and M_H.

Alternatively, we can choose as free parameters:

- **$G_F = (1.166\,378\,8 \pm 0.000\,000\,7) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$**
- **$\alpha^{-1} = 137.035\,999\,084 \pm 0.000\,000\,051$**
- **$M_Z = (91.1875 \pm 0.0021)\ \mathrm{GeV}$**

and the Higgs mass M_H. Uses the three most precise experimental determinations to fix the interaction.

MuCap systematic corrections, uncertainties and final capture rate

Systematic errors	Run 2006		Run 2007		Comment
	Λ (s^{-1})	$\delta\Lambda$ (s^{-1})	Λ (s^{-1})	$\delta\Lambda$(s^{-1})	
High-Z impurities	-7.8	1.87	-4.54	0.93	
μp scatter	-12.4	3.22*	-7.20	1.25*	* = prelim.
μp diffusion	-3.1	0.1	-3.0	0.1	
Fiducial volume cut		3.0		3.0	
Entrance counter inefficiencies		0.5		0.5	
Choice of electron detector def.		1.8*		1.8*	* =prelim.
Total	**-23.3**	**5.14**§	**-14.74**	**3.88**§	§ **= correlated**

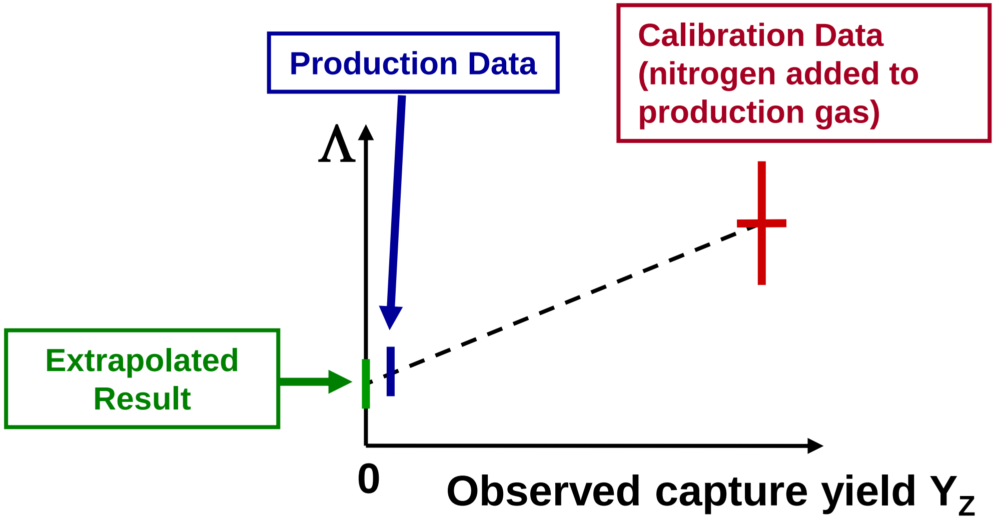
- **$\Lambda_0(06) = 455{,}857.3 \pm 7.7 \pm 5.2$ s^{-1}**
  $\Lambda_0(07) = 455{,}853.1 \pm 8.3 \pm 3.9$ s^{-1}
  - Apply μp atomic correction
  - Subtract μ+ decay rate: $\Lambda_{\mu+} = 455170.05 \pm 0.46$ s^{-1}
  - 3.2% increase in the uncertainty because of ppμ correction

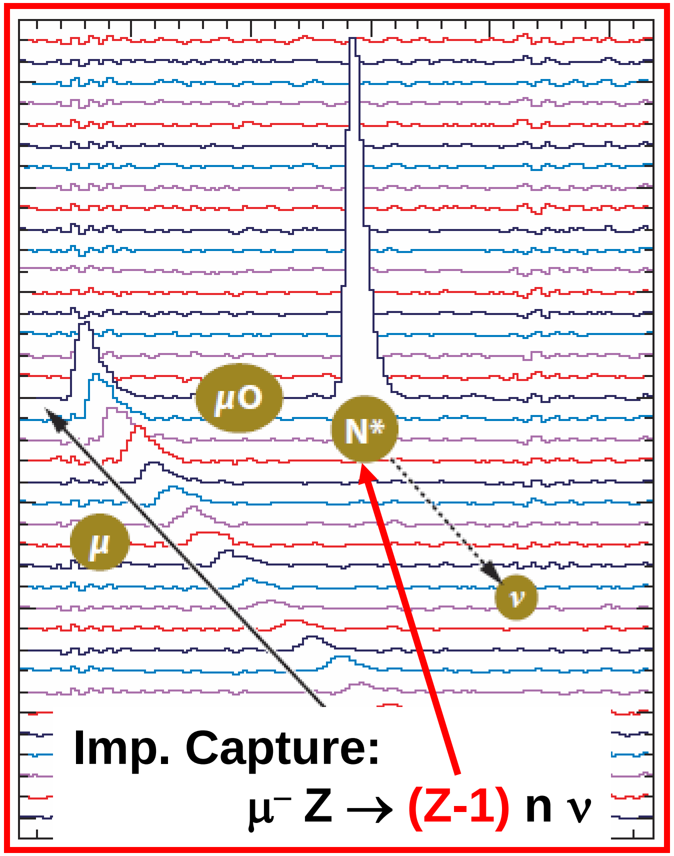
Measured disappearance rates

- **$\Lambda_S(06) = 717.5 \pm 8.0 \pm 5.7$ s^{-1}**
- **$\Lambda_S(07) = 713.1 \pm 8.6 \pm 4.5$ s^{-1}**

$\Delta\Lambda_S(06 – 07) = 4.4$ s^{-1}

Gas impurities are monitored directly. Correction is based on measurement. Calibration done at high concentation





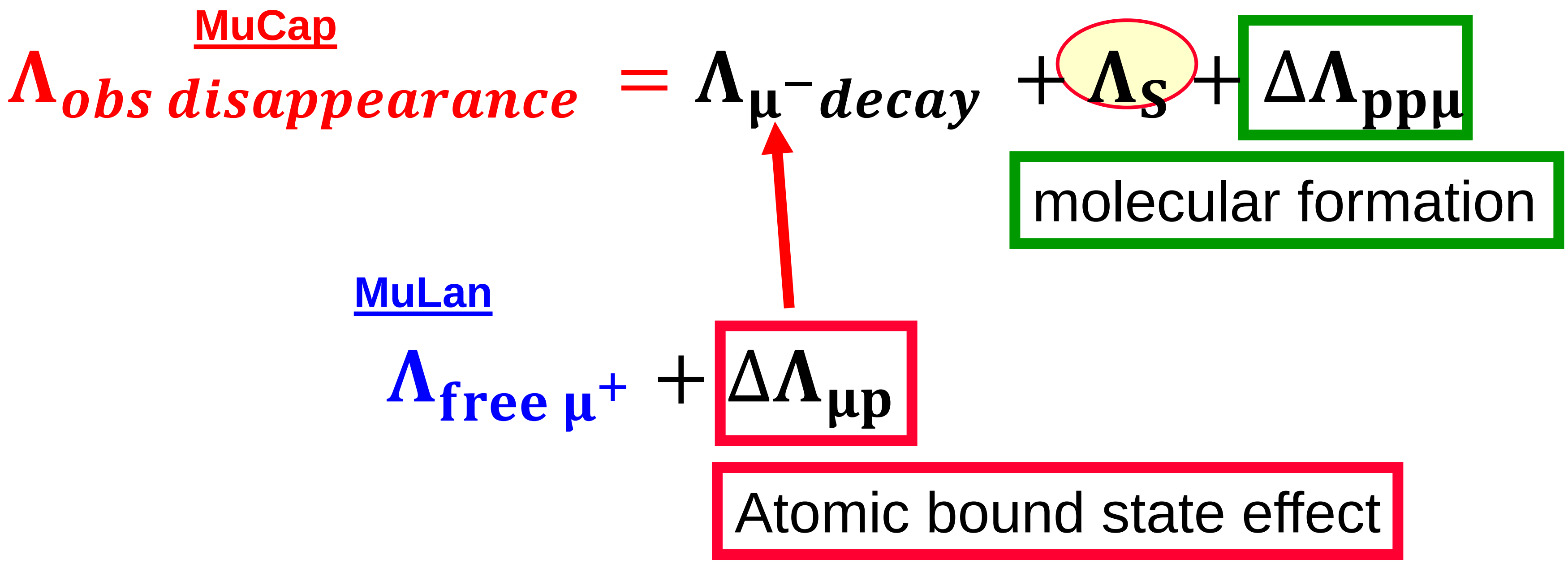
Imp. Capture:

$\mu^- Z \rightarrow (Z\text{-}1)\ n\ \nu$

2004 run: $c_N < 7$ ppb, $c_{H2O} \sim 30$ ppb

2006 / 2007 runs: $c_N < 7$ ppb, $c_{H2O} \sim 10$ ppb

External corrections to Λ



MuCap

$$\Lambda_{obs\ disappearance} = \Lambda_{\mu^- decay} + \Lambda_S + \boxed{\Delta\Lambda_{pp\mu}}$$

molecular formation

MuLan

$$\Lambda_{free\ \mu^+} + \boxed{\Delta\Lambda_{\mu p}}$$

Atomic bound state effect

Λ_S (MuCap) = 715.1 ± 5.4_{stat} ± 5.0_{syst} s^{-1}

* Small revision of molecular correction might affect $\Lambda_S < 0.5\ s^{-1}$ and syst. error

Λ_S (theory) = 711.5 ± 3.5 ± 3 s^{-1}

τ_{μ^+}

Quartz visible μSR. Fit each detector for an "effective lifetime." Would be correct, except for remnant longitudinal polarization relaxation.

$$F(t) = N\left[1 + \frac{1}{3}P_2 \sin(\omega t + \varphi_0)e^{-t/T_2}\right]e^{-t/\tau_\mu} + B,$$

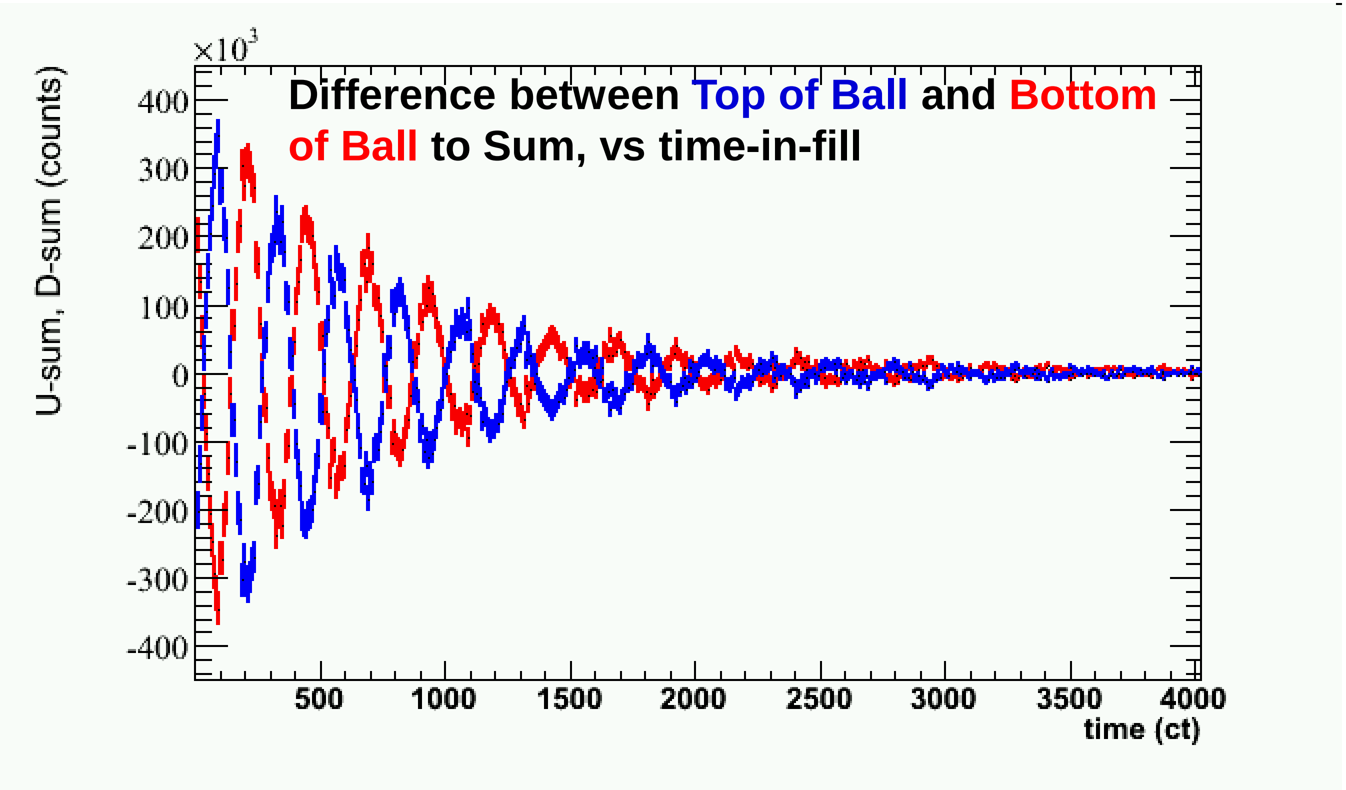


Illustration of free muon precession in top/bottom detector differences

τ_{μ^+}

Longitudinal polarization distorts result in predictable manner depending on location. The ensemble of lifetimes is fit to obtain the actual lifetime. (Method robust in MC studies)

$$R_{\rm eff} \approx R - \frac{1}{3}\frac{R_0}{T_1}\vec{P}_1\hat{r^*}_{\rm D} \times 10^6,$$

Relative effective lifetime (ppm)

(+ blind offset)