The Birds and the Bs

A Case Study of $B_s \to \mu^+\mu^-$ in the MSSM

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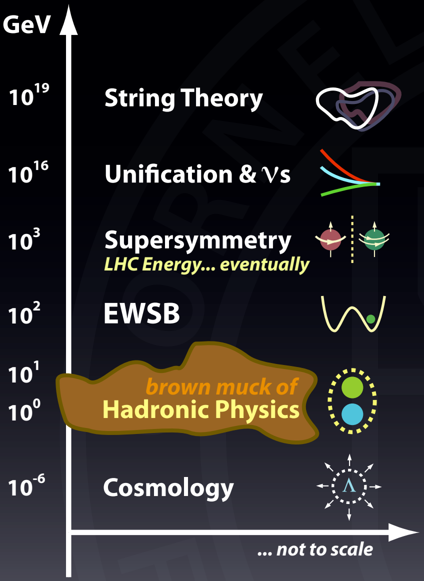
Based on arXiv:0812:4320
In collaboration with A. Dedes, J Rosiek.

Cornell University

Informal CIHEP Pizza Lunch
February 6, 2009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Unification & νs
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Supersymmetry
LHC Energy... eventually
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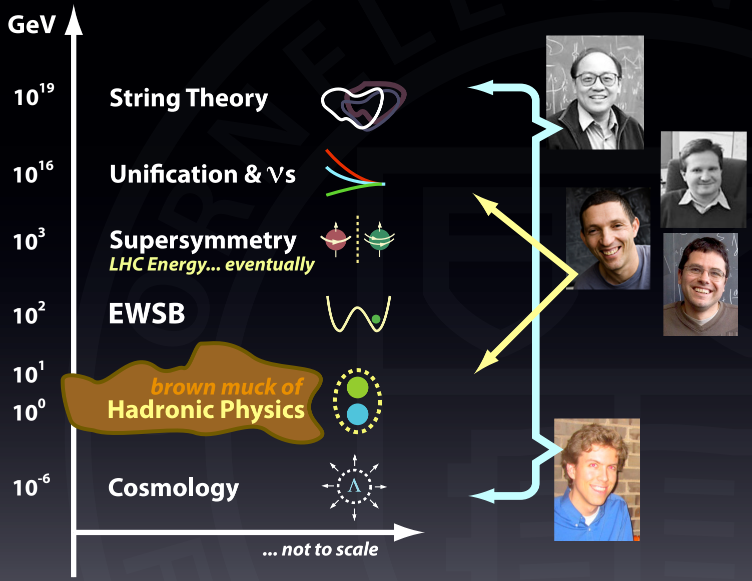
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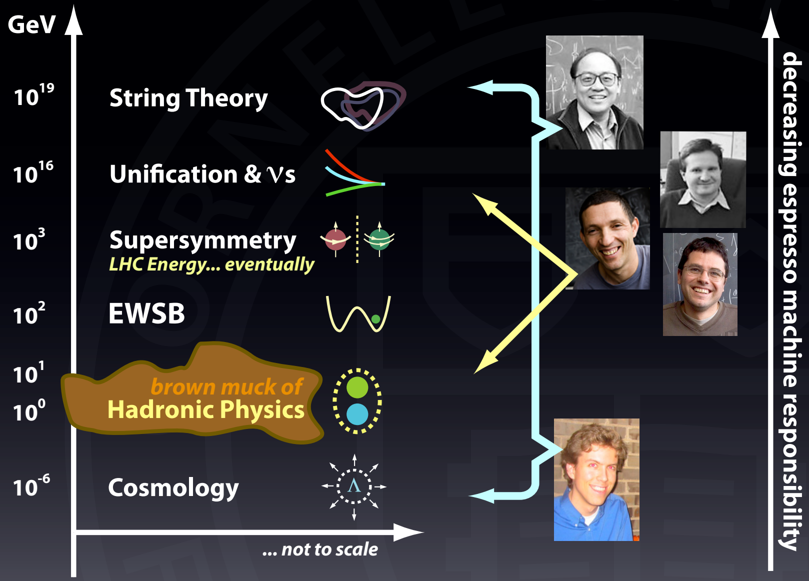
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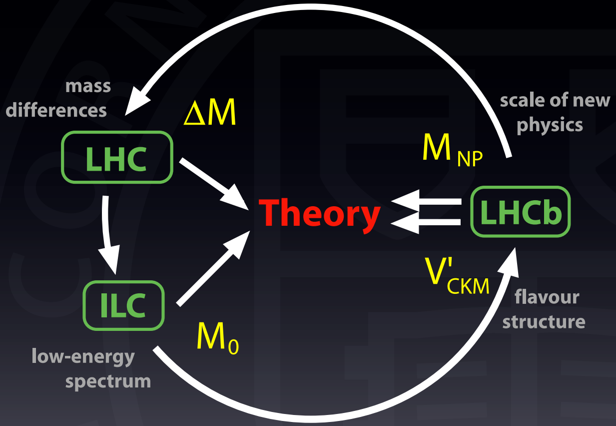
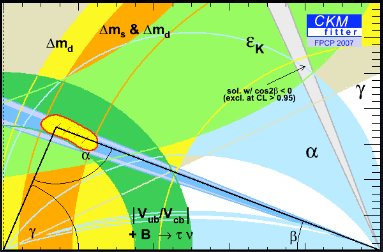
decreasing espresso machine responsibility

Complimentarity



The program of flavor physics



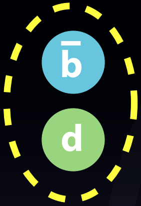
Goniometry

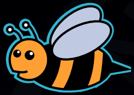
The measurement of angles

- 'Luminosity' frontier
- Many different measurements
- Laboratory: *B* mesons

We will focus on one particular decay ($B_s \to \mu\mu$), but one should always remember that it's just one piece of a larger program.

B-mesons: state-of-the-art flavor laboratories





Meson	**Mass**	**Mean lifetime**
B_d^0	5.280 GeV	1.53×10^{-12}s
B_s^0	5.370 GeV	1.44×10^{-12}s

B-factories ‘traditionally’ run at $\Upsilon(4S)$ resonance, which produce B_d, but not B_s.

B-mesons have just the right mass and width to allow us to measure their oscillation ($\mathcal{CP}$ phase). Asymmetric B-factories allow us to measure the different branching ratios of B and $\overline{B}$ mesons.

Presently we are interested in FCNC B-decays.

The March of the Penguins

Penguin diagram

Allows FCNC sub-diagram to occur on-shell.

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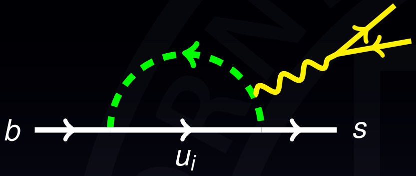

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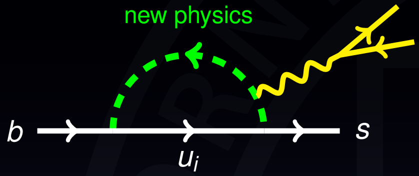

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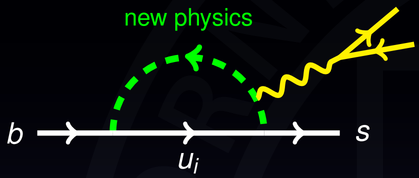

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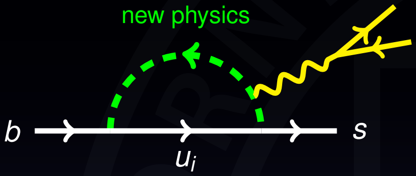

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The March of the Penguins

Where do we look for penguins?

The March of the Penguins

Where do we look for penguins? Antarctica.

The March of the Penguins

Where do we look for penguins? Antarctica.

Very little background, penguin is dominant fauna.

The March of the Penguins

Where do we look for SUSY penguins? $B_s \to \mu^+\mu^-$.

Very little background, penguin is dominant process.

$B_s \to \mu^+\mu^-$: Very little background

The Standard Model background is suppressed by...

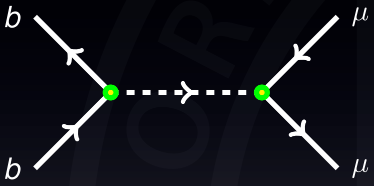
- **Loop**: no tree-level contribution, $(16\pi^2)^{-1}$
- **FCNC**: 'GIM' suppression, $|V^\dagger V|_{bs}$
- **Helicity**: Lepton mass insertion, m_μ / M_{B_s}

Channel	Expt.	Bound (90% CL)	SM Prediction
$B_s^0 \to \mu^+\mu^-$	CDF II	$< 4.7 \times 10^{-8}$	$(4.8 \pm 1.3) \times 10^{-9}$
$B_d^0 \to \mu^+\mu^-$	CDF II	$< 1.5 \times 10^{-8}$	$(1.4 \pm 0.4) \times 10^{-10}$
$B_s^0 \to \mu^+ e^-$	CDF II	$< 2.0 \times 10^{-7}$	≈ 0
$B_d^0 \to \mu^+ e^-$	CDF II	$< 6.4 \times 10^{-8}$	≈ 0

Clean dilepton signal, only hadronic uncertainty is f_B. 'Ideal' for LHC.

$B_s \to \mu^+\mu^-$: Penguin is the dominant process

In the MSSM, the **Higgs-penguin** mediated $B_s \to \mu^+\mu^-$ diagram is sensitive to $\tan\beta$. Recall: $\tan\beta = v_u/v_d$.



$$\begin{pmatrix} \bar{s}_R & \bar{b}_R \end{pmatrix} \begin{pmatrix} m_s & 0 \\ y_b \epsilon v_u & m_b \end{pmatrix} \begin{pmatrix} s_L \\ b_L \end{pmatrix}$$

Amplitude is enhanced by $\tan^3\beta$.

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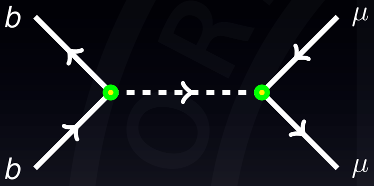

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$$y_{b,\ell} = \frac{m_{b,\ell}}{v_d} \propto \frac{1}{\cos\beta} \xrightarrow{\tan\beta \gg 1} \tan\beta$$

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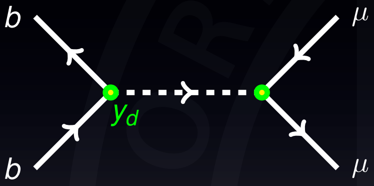

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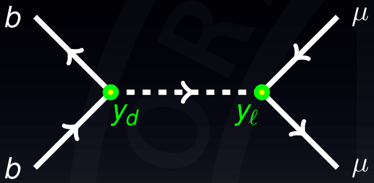

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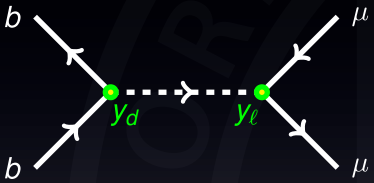

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s-b mixing:

$$\sin\theta \approx y_b \epsilon v_u / m_b \approx \epsilon \tan\beta$$

$$y_{b,\ell} = \frac{m_{b,\ell}}{v_d} \propto \frac{1}{\cos\beta} \xrightarrow{\tan\beta \gg 1} \tan\beta$$

Amplitude is enhanced by $\tan^3\beta$.

The March of the Penguins

The standard model background...

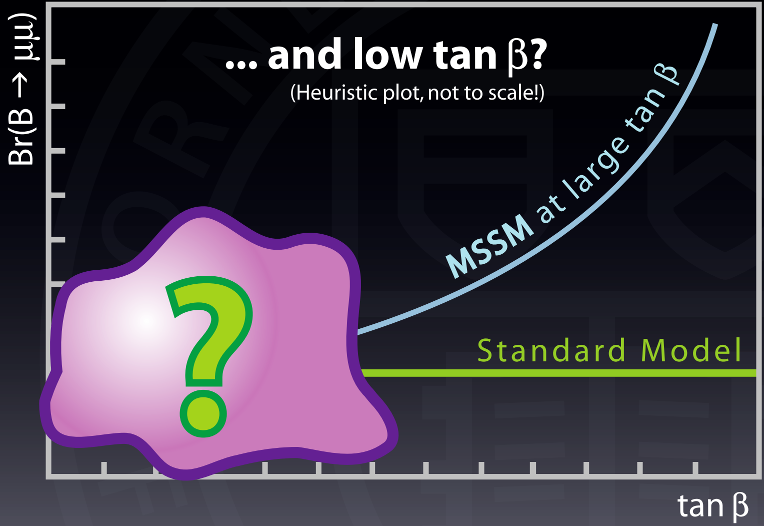
The March of the Penguins

The standard model background... and SUSY at large tan β

$Br(B_s \to \mu\mu) \approx 5 \cdot 10^{-7} (\tan\beta/50)^6 (300\ \text{GeV}/M_{A^0})^4$

Motivation: Grand unification, mSUGRA + $(g-2)_\mu$

But what about **low** tan β?



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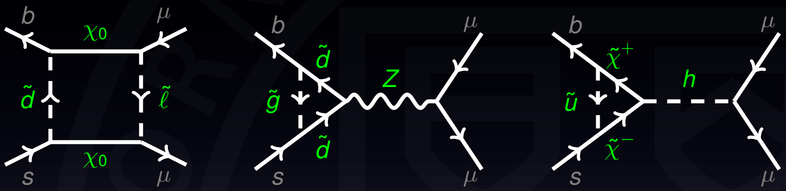
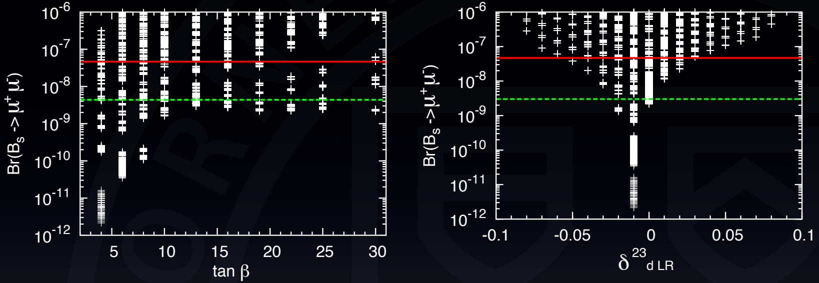


No photon penguin by Ward identity.

- Higgs penguin no longer dominant
- One has to consider interference with other diagrams
- Possibility: cancellation below SM prediction?

Low tan β scan



Scan over MSSM parameter space with respect to SM prediction and experimental limit, taking into account existing experimental bounds.

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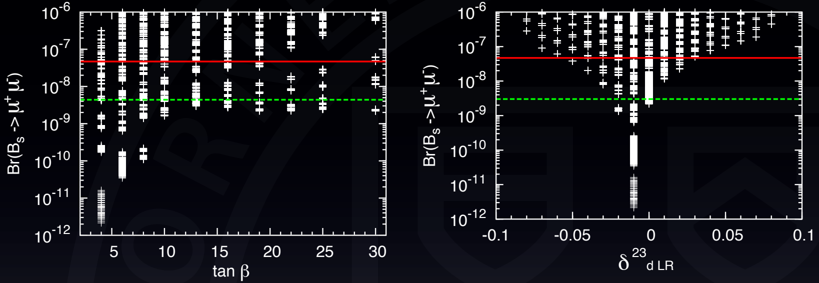

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Mass insertion parameterizes flavor violation: $\delta^{IJ}{}_{QXY} = \frac{(M_Q^2)^{IJ}_{XY}}{\sqrt{(M_Q^2)^{IJ}_{XX}(M_Q^2)^{IJ}_{YY}}}$

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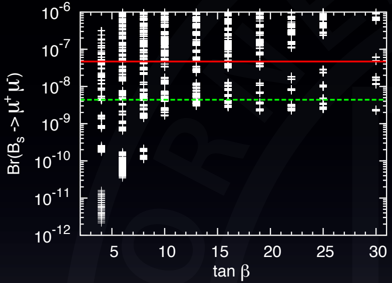

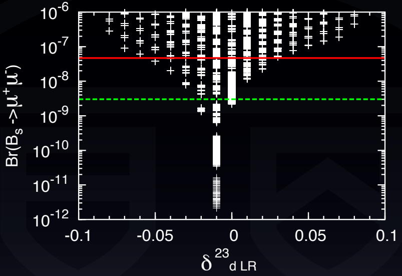

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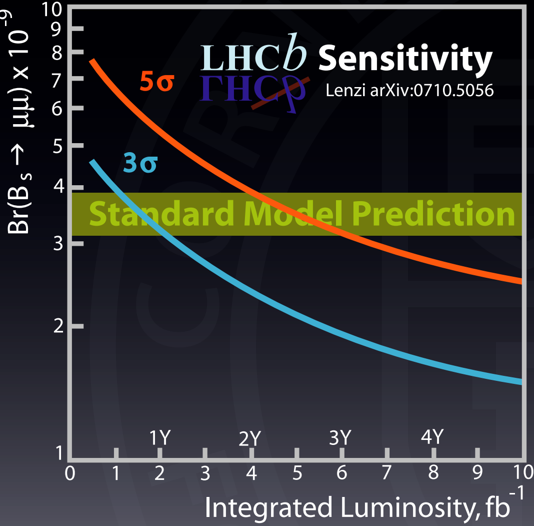
Funnel region: Pseudoscalar and axial contributions cancel, scalar contribution is negligible; e.g. models where MSSM is extended with an additional light *CP*-odd Higgs.

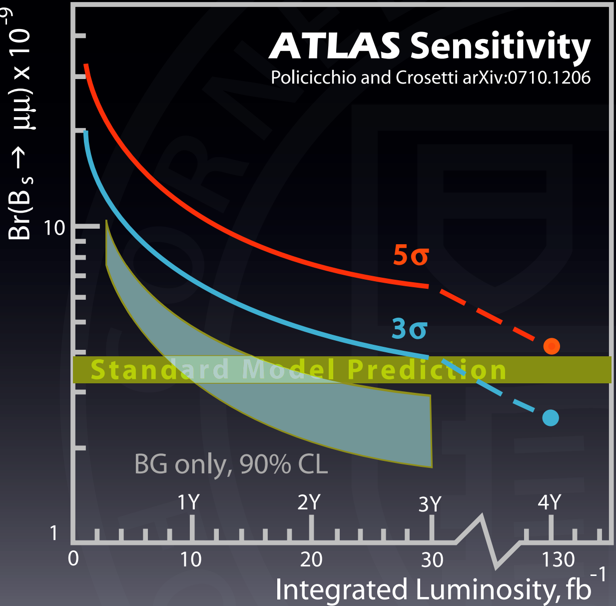
LHC*b* 'benchmark' process



Potential...
'Signal' in 1Y
'Discovery' in 3Y

Implications on
LHCb upgrade?
(B_s or B_d?)

General purpose detectors...



Conclusion: Lessons

Theory

- There **is** life outside of Minimal Flavor Violation (MFV)
- ... though perhaps only minimal life?
- We can model-build beyond MFV; e.g. 0712.0674, 0712,2074
- Our numerical code is available

Experiment

- Keep an eye out for a measurement of $B_s \to \mu\mu$
- Non-discovery at SM limit could hit at low tan β, beyond-MFV
- Need to think about LHCb upgrade scenarios

Range of input parameters for numerical scan

Parameter	Symbol	Min	Max	Step
Ratio of Higgs vevs	$\tan\beta$	2	30	varied
CKM phase	γ	0	π	$\pi/25$
CP-odd Higgs mass	M_A	100	500	200
SUSY Higgs mixing	μ	-450	450	300
$SU(2)$ gaugino mass	M_2	100	500	200
Gluino mass	M_3	$3M_2$	$3M_2$	0
SUSY scale	M_{SUSY}	500	1000	500
Slepton Masses	$M_{\tilde{\ell}}$	$M_{\text{SUSY}}/3$	$M_{\text{SUSY}}/3$	0
Left top squark mass	$M_{\tilde{Q}_L}$	200	500	300
Right bottom squark mass	$M_{\tilde{b}_R}$	200	500	300
Right top squark mass	$M_{\tilde{t}_R}$	150	300	150
Mass insertion	$\delta^{13}_{dLL}, \delta^{23}_{dLL}$	-1	1	1/10
Mass insertion	$\delta^{13}_{dLR}, \delta^{23}_{dLR}$	-0.1	0.1	1/100

Constraints used in numerical scan

Quantity	Current Measurement	Experimental Error
$m_{\chi_1^0}$	> 46 GeV	
$m_{\chi_1^\pm}$	> 94 GeV	
$m_{\tilde{b}}$	> 89 GeV	
$m_{\tilde{t}}$	> 95.7 GeV	
m_h	> 92.8 GeV	
$\lvert\epsilon_K\rvert$	$2.232 \cdot 10^{-3}$	$0.007 \cdot 10^{-3}$
$\lvert\Delta M_K\rvert$	$3.483 \cdot 10^{-15}$	$0.006 \cdot 10^{-15}$
$\lvert\Delta M_D\rvert$	$< 0.46 \cdot 10^{-13}$	
ΔM_{B_d}	$3.337 \cdot 10^{-13}$ GeV	$0.033 \cdot 10^{-13}$ GeV
ΔM_{B_s}	$116.96 \cdot 10^{-13}$ GeV	$0.79 \cdot 10^{-13}$ GeV
$\mathrm{Br}(B \to X_s\gamma)$	$3.34 \cdot 10^{-4}$	$0.38 \cdot 10^{-4}$
$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$	$< 1.5 \cdot 10^{-10}$	
$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$	$1.5 \cdot 10^{-10}$	$1.3 \cdot 10^{-10}$
Electron EDM	$< 0.07 \cdot 10^{-26}$	
Neutron EDM	$< 0.63 \cdot 10^{-25}$	

Calculation: Effective Operators

The effective Hamiltonian can be written as

$$\mathcal{H} = \frac{1}{(4\pi)^2} \sum_{X,Y=L,R} \left(C_{VXY}\mathcal{O}_{VXY} + C_{SXY}\mathcal{O}_{SXY} + C_{TX}\mathcal{O}_{TX} \right)$$

Writing flavor indices I, J, K, L, the operators are

$$\mathcal{O}^{IJKL}_{VXY} = (\overline{q^J}\gamma^\mu P_X q^I)(\ell^L \gamma_\mu P_Y \ell^K)$$

$$\mathcal{O}^{IJKL}_{SXY} = (\overline{q^J} P_X q^I)(\ell^L P_Y \ell^K)$$

$$\mathcal{O}^{IJKL}_{TX} = (\overline{q^J}\sigma^{\mu\nu} P_X q^I)(\ell^L \sigma_{\mu\nu} \ell^K)$$

Calculation: Factorization

The hadronic and leptonic parts of the matrix element factorize:

$$\langle \ell, \ell' | \mathcal{H}_{eff} | B(p) \rangle = \sum_{i=ops} \langle \ell, \ell' | \mathcal{O}^i_L | 0 \rangle \langle 0 | \mathcal{O}^i_Q | B(p) \rangle$$

Definition of the **decay constant**, f_B

$$\begin{aligned} \langle 0 | \bar{b} \gamma_\mu P_{L,R} s | B(p) \rangle &= \mp \frac{i}{2} p_\mu f_B \\ \to \langle 0 | \bar{b} P_{L,R} s | B(p) \rangle &= \pm \frac{i}{2} \frac{M_B f_B}{m_b + m_s} \end{aligned}$$

Note that there are no tensor ($\bar{b} \sigma^{\mu\nu} s$) operators by antisymmetry.
f_B contains all the hadronic muck; look it up from non-perturbative methods (i.e. lattice).

Leptonic decay: don't have to worry about jets, inclusive decays, etc.

Calculation: Amplitude

We can now write the amplitude in terms of form factors

$$\mathcal{M} = F_S \bar{\ell}\ell + F_P \bar{\ell}\gamma_5 \ell + F_V p^\mu \bar{\ell}\gamma_\mu \ell + F_A p^\mu \bar{\ell}\gamma_\mu\gamma_5 \ell$$

In terms of the Wilson coefficients, these are

$$F_S = \frac{i}{4}\frac{M_{B_s}^2 f_{B_s}}{m_b + m_s}(C_{SLL} + C_{SLR} - C_{SRR} - C_{SRL})$$

$$F_P = \frac{i}{4}\frac{M_{B_s}^2 f_{B_s}}{m_b + m_s}(-C_{SLL} + C_{SLR} - C_{SRR} + C_{SRL})$$

$$F_V = -\frac{i}{4} f_{B_s}(C_{VLL} + C_{VLR} - C_{VRR} - C_{VRL})$$

$$F_A = -\frac{i}{4} f_{B_s}(-C_{VLL} + C_{VLR} - C_{VRR} + C_{VRL})$$

Calculation: Branching Ratio

$$\mathcal{B}(B_s^0 \to \ell_L^- \, \ell_K^+) = \frac{\tau_{B_s}}{16\pi} \frac{|\mathcal{M}|^2}{M_{B_s}} \sqrt{1 - \left(\frac{m_{\ell_K} + m_{\ell_L}}{M_{B_s}}\right)^2} \sqrt{1 - \left(\frac{m_{\ell_K} - m_{\ell_L}}{M_{B_s}}\right)^2}$$

$$\begin{aligned}
|\mathcal{M}|^2 &= 2|F_S|^2 \left[M_{B_s}^2 - (m_{\ell_L} + m_{\ell_K})^2\right] + 2|F_P|^2 \left[M_{B_s}^2 - (m_{\ell_L} - m_{\ell_K})^2\right] \\
&+ 2|F_V|^2 \left[M_{B_s}^2 (m_{\ell_K} - m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2\right] \\
&+ 2|F_A|^2 \left[M_{B_s}^2 (m_{\ell_K} + m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2\right] \\
&+ 4\,\mathrm{Re}(F_S F_V^*)(m_{\ell_L} - m_{\ell_K}) \left[M_{B_s}^2 + (m_{\ell_K} + m_{\ell_L})^2\right] \\
&+ 4\,\mathrm{Re}(F_P F_A^*)(m_{\ell_L} + m_{\ell_K}) \left[M_{B_s}^2 - (m_{\ell_L} - m_{\ell_K})^2\right] .
\end{aligned}$$

Calculation: $B_s \to \mu^+\mu-$ at low $\tan\beta$

For the case $\ell_K = \ell_L = \mu$, the amplitude-squared is

$$|\mathcal{M}|^2 \approx 2M_{B_q}^2 \left(|F_S|^2 + |F_P + 2\, m_\mu\, F_A|^2\right),$$

where we have also taken $m_\mu / M_B \to 0$.

The minima of this comes from two cases,

$$\begin{aligned} &(1) \quad F_P + 2m_\ell F_A \approx 0, F_P \gg F_S \\ &(2) \quad |F_S| \approx |F_P| \approx |F_A| \approx 0. \end{aligned}$$