Towards dS String Vacua: Warped, Twisted, & Minimal

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Minimal Simple dS Solutions

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Based on arXiv:0810.5328 [hep-th], with
Shajid Haque, Bret Underwood, Thomas Van Riet.

Warped Effective Theory

Closed Strings: arXiv:0803.3068 [hep-th], with Gonzalo Torroba, Bret Underwood, Michael Douglas

Open Strings: arXiv:0812.2247 [hep-th], with Fernando Marchesano & Paul McGuirk.

18 December 1998

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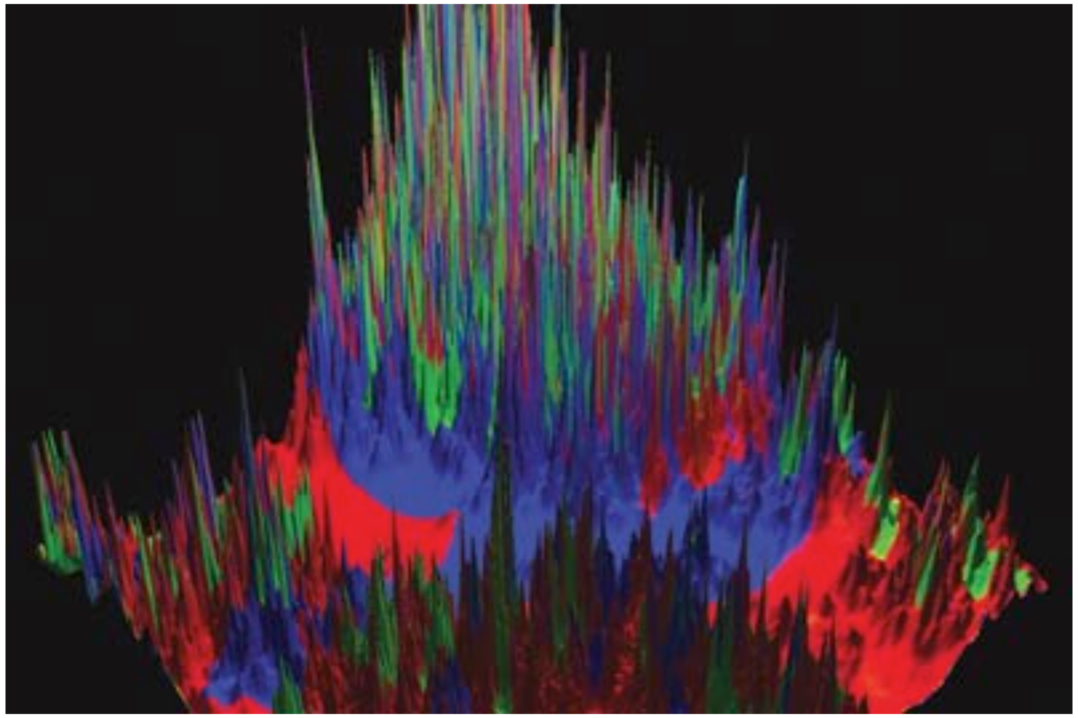
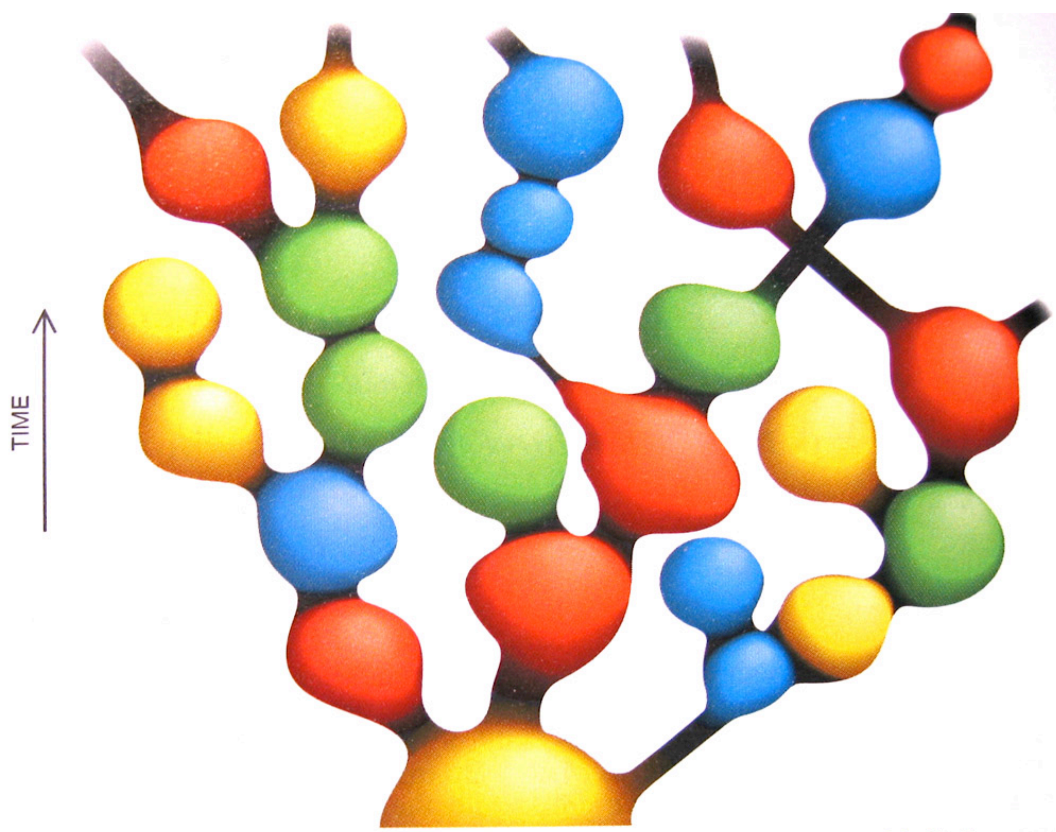
The Accelerating Universe

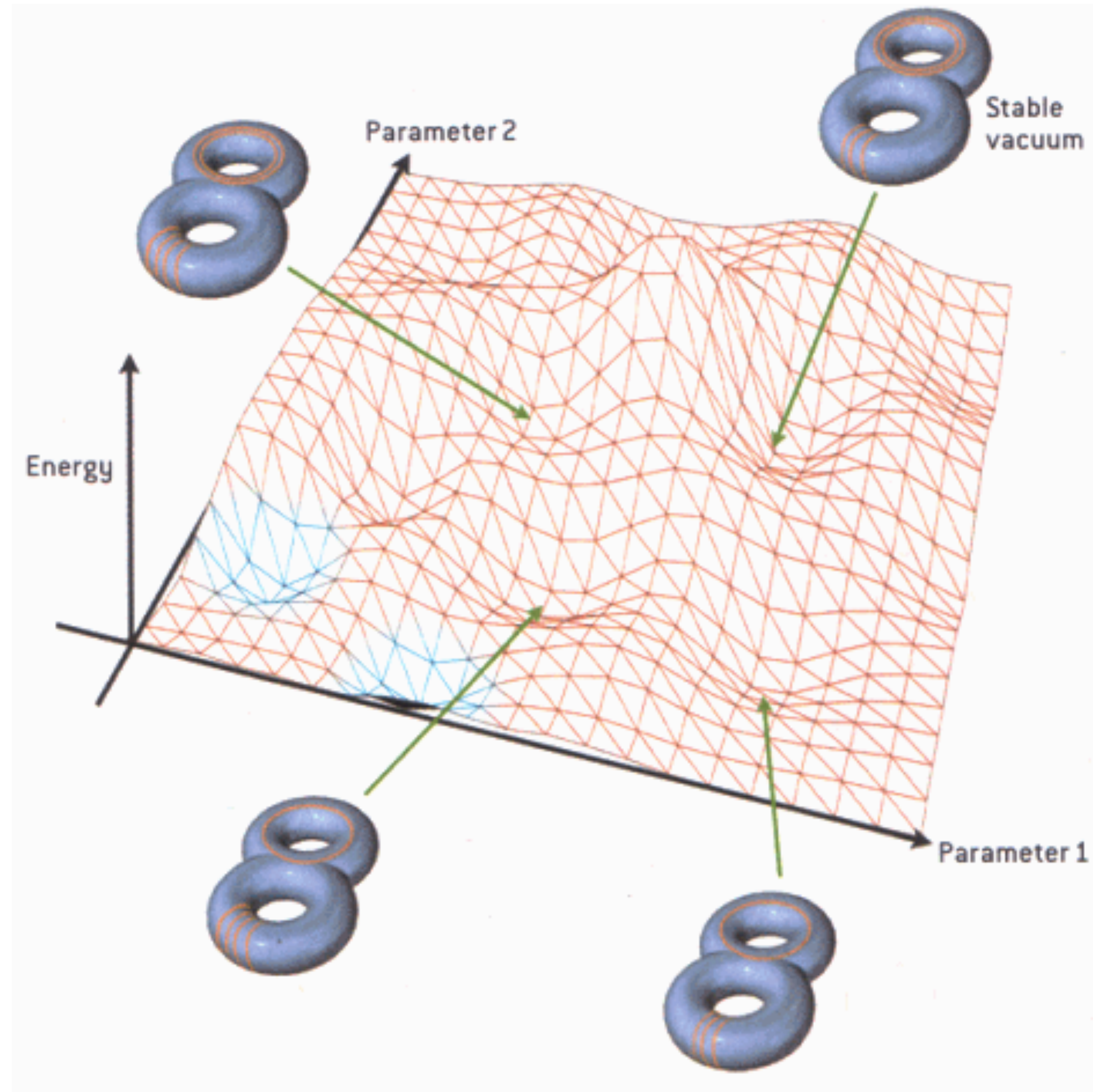
Breakthrough of the Year

American Association for the Advancement of Science

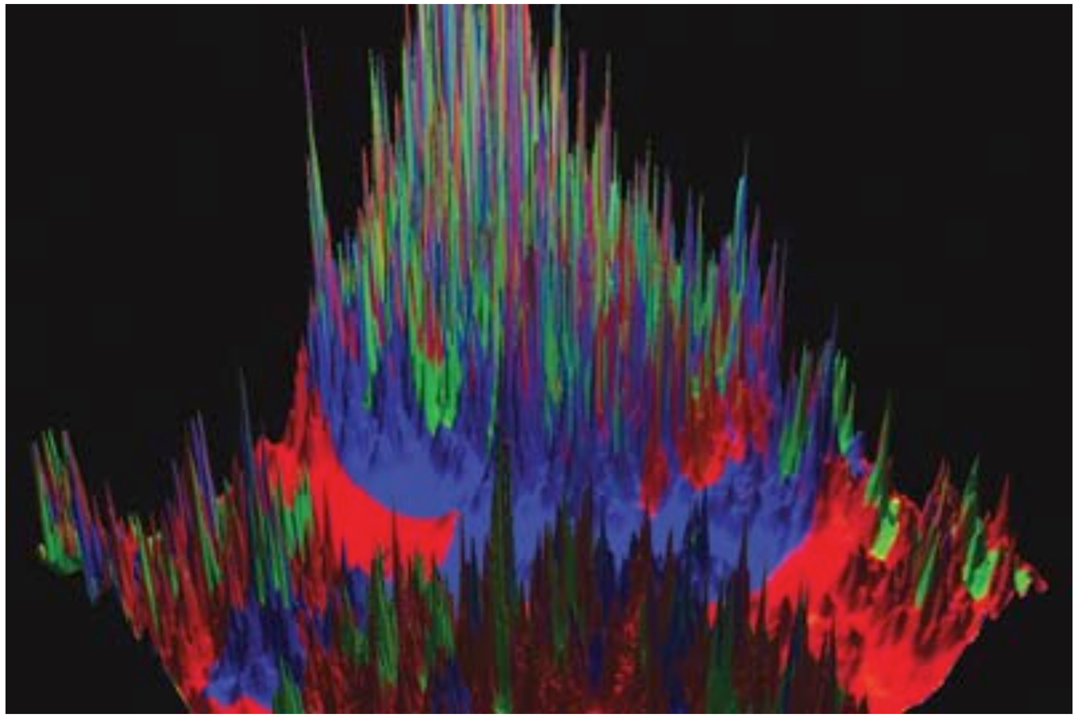


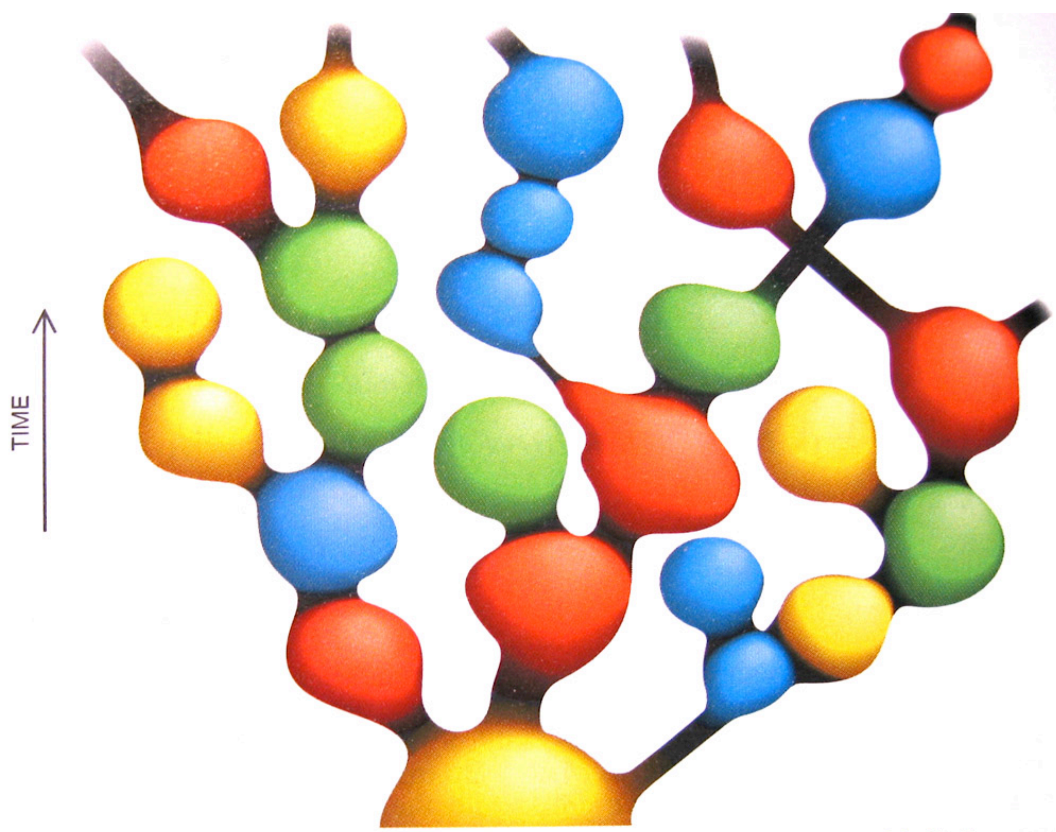


SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.



"The Landscape" (Picture from *Scientific American*)





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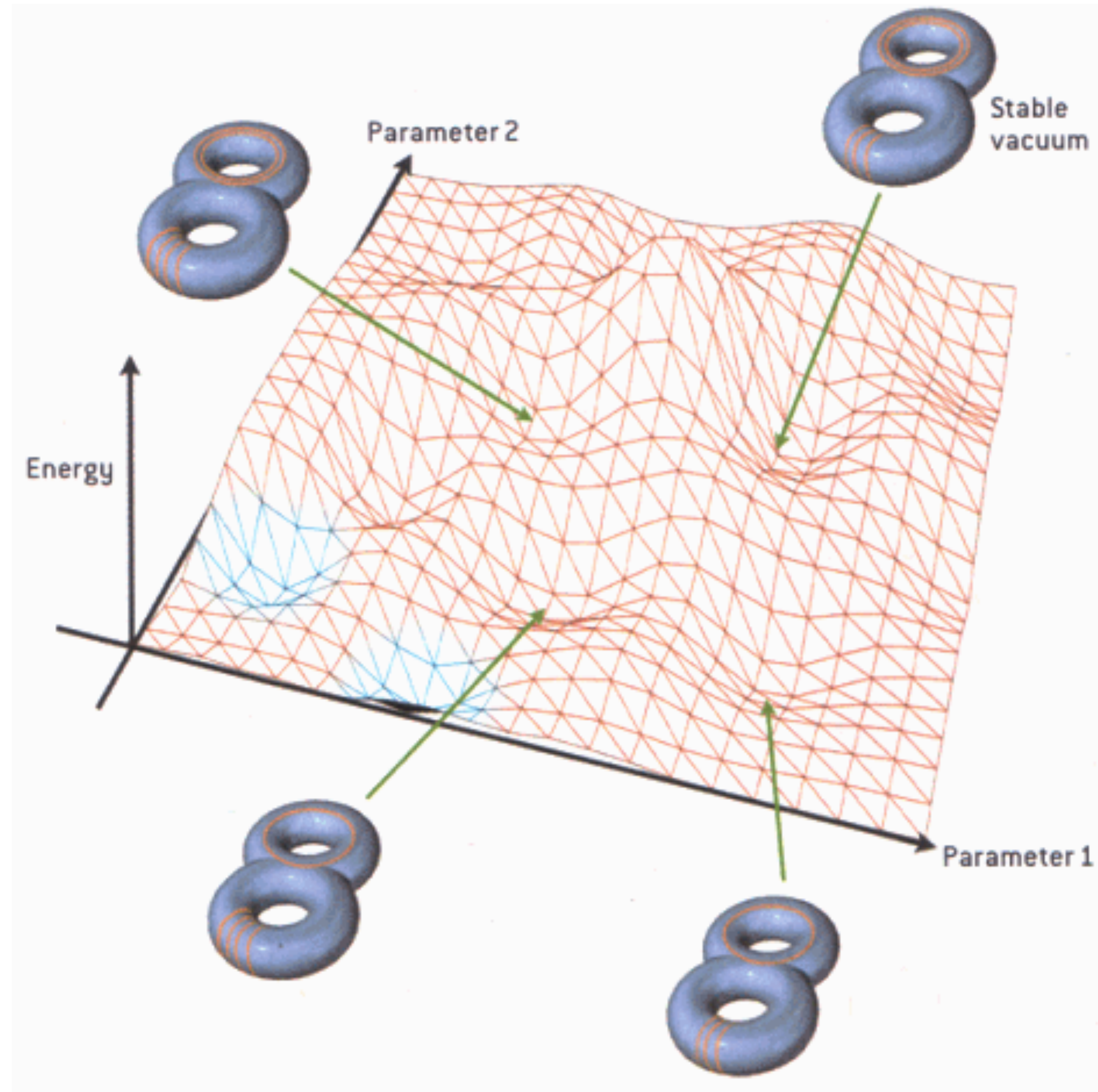


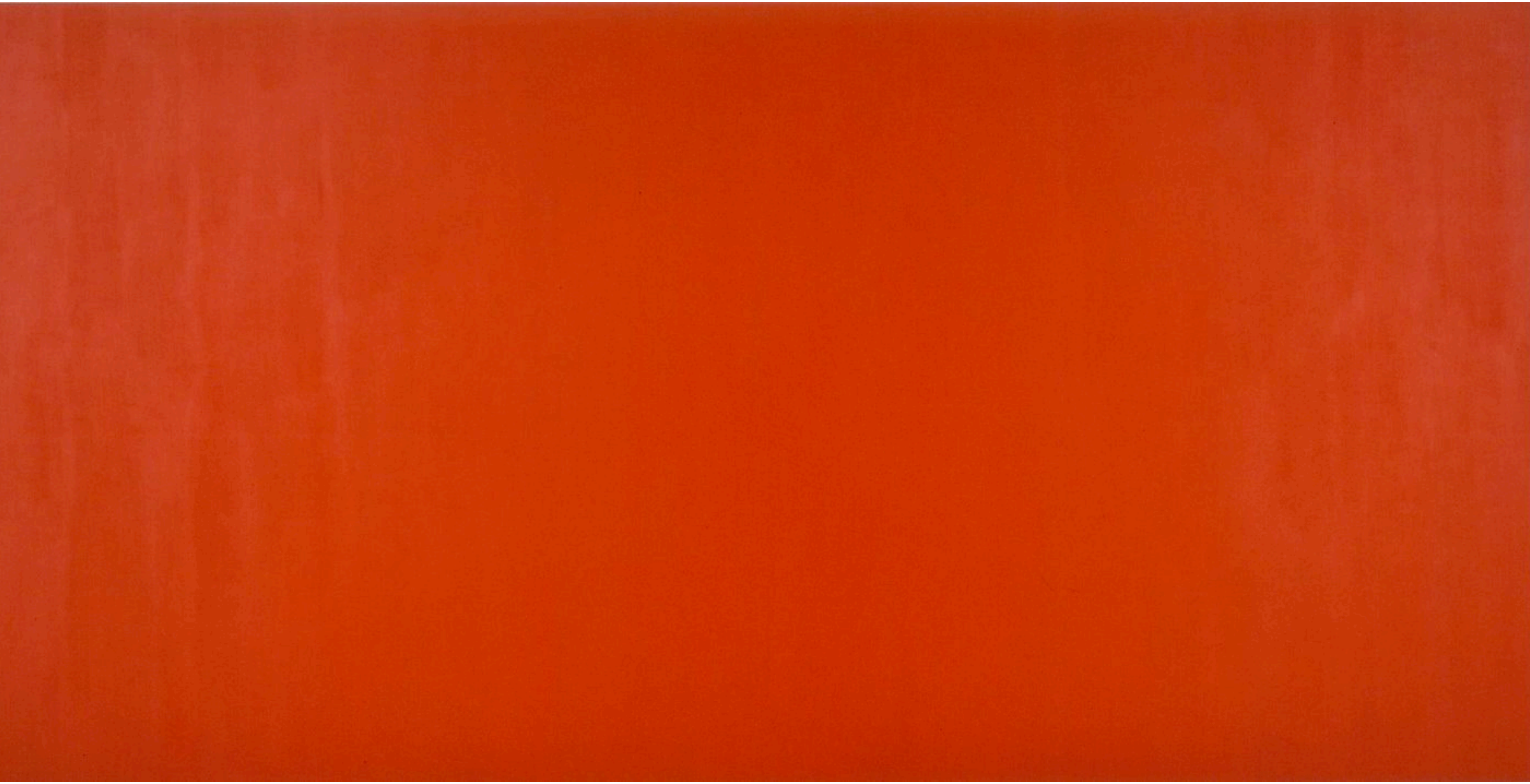
"The Landscape" (Picture from *Scientific American*)

e.g., Kachru, Kallosh, Linde, Trivedi; and many others.

WIKIPEDIA
The Free Encyclopedia

Minimalism describes movements in various forms of art and design, especially visual art and music, where the work is stripped down to its most fundamental features. As a specific movement in the arts it is identified with developments in post-World War II Western Art, most strongly with American visual arts in the late 1960s and early 1970s. Prominent artists associated with this movement include Donald Judd, Agnes Martin and Frank Stella. It is rooted in the reductive aspects of Modernism, and is often interpreted as a reaction against Abstract Expressionism and a bridge to Postmodern art practices.

Richard Pousette-Dart, *Symphony No. 1, The Transcendental*, oil on canvas, 1941-42, Metropolitan Museum of Art



Barnett Newman, *Anna's light,* 1968

Barnett Newman, *Onement 1*, 1948. Museum of Modern Art, New York. The first example of Newman using the so-called "zip" to define the spatial structure of his paintings.

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SU-ITP-07/20

Simple de Sitter Solutions

Eva Silverstein

Department of Physics and SLAC
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We present a framework for de Sitter model building in type IIA string theory, illustrated with specific examples. We find metastable dS minima of the potential for moduli obtained from a compactification on a product of two Nil three-manifolds (which have negative scalar curvature) combined with orientifolds, branes, fractional Chern-Simons forms, and fluxes. As a discrete quantum number is taken large, the curvature, field strengths, inverse volume, and four dimensional string coupling become parametrically small, and the de Sitter Hubble scale can be tuned parametrically smaller than the scales of the moduli, KK, and winding mode masses. A subtle point in the construction is that although the curvature remains consistently weak, the circle fibers of the nilmanifolds become very small in this limit (though this is avoided in illustrative solutions at modest values of the parameters). In the simplest version of the construction, the heaviest moduli masses are parametrically of the same order as the lightest KK and winding masses. However, we provide a method for separating these marginally overlapping scales, and more generally the underlying supersymmetry of the model protects against large corrections to the low-energy moduli potential.

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- NSNS, RR fluxes
- Fractional CS forms
- KK5, NS5-branes

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\Title {\vbox{\baselineskip12pt\hbox{SLAC-PUB-13016} \hbox{SU-ITP-07/20} \hbox{}}} {\vbox{
%\centerline{ Simple Stringy Models of Dynamical Supersymmetry Breaking}
\centerline{Simple de Sitter Solutions}} } \centerline{Eva Silverstein }

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%Complicated de Sitter Solutions
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%\centerline{^{1}Department of Particle Physics} \centerline{Weizmann Institute of Science}
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Revisiting a No-go Theorem

Dimensional reduction of massive IIA SUGRA gives:

$$V = V_{\rm metric} + V_3^{NS} + \sum_p V_p^{RR} + V_{O6} + V_{D6} + V_{NS5} + V_{KK5}$$

Focus on 2D slices of the full moduli space:

$\rho \equiv ({\rm Vol})^{1/3}$ volume modulus

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For a vanilla subset of contributions to V:

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + \sum_p pV_p \geq 9V$$

Hertzberg, Kachru, Taylor, Tegmark

This simple relation has interesting consequences:

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Hertzberg, Kachru, Taylor, Tegmark

For inflation, we need V>0, but

$$\epsilon \geq \frac{\tilde{m}_P^2}{2}\left[\left(\frac{\partial \ln V}{\partial\hat{\rho}}\right)^2 + \left(\frac{\partial \ln V}{\partial\hat{\tau}}\right)^2\right] \qquad \geq \frac{27}{13}$$

For vacua:

$$V = -(\sum_p pV_p)/9 \qquad \leq 0$$

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Restoring some of the omitted ingredients may evade this "no-go" theorem.

Question: What is the minimal set we need?

Finding dS vacua is as simple as "a,b,c": Silverstein

$$V = a(\rho, M)\tau^{-2} - b(\rho, M)\tau^{-3} + c(\rho, M)\tau^{-4}$$

Geometric flux, NS flux, NS 5-branes, KK 5-branes:

$$a(\rho, M) = \frac{\tilde{C}_f(M)}{\rho} + \frac{\tilde{A}_{KK5}(M)}{\rho} + \frac{\tilde{A}_{NS5}(M)}{\rho^2} + \frac{\tilde{A}_{H3}(M)}{\rho^3}$$

O6-planes and D6-branes:

$$b(\rho, M) = +n_{O6} f(M) - n_{D6} g(M)$$

RR-flux (and by extension, fractional Wilson lines):

$$c(\rho, M) = \rho^3 \tilde{m}^2 + \rho \tilde{A}_2(M) + \frac{\tilde{A}_4^{elec}(M)}{\rho} + \frac{\tilde{A}_6(M)}{\rho^3}$$

A useful crutch of finding dS vacua is to consider:

$$\frac{4ac}{b^2} \approx 1$$

By analyzing the dilaton direction, can see dS vacua exist only if:

$$1 < \frac{4ac}{b^2} < \frac{9}{8}$$

Maloney, Silverstein, Strominger

Search for minima of $\delta(\rho, M) \approx 0$ in the ρ direction:

$$\frac{4ac}{b^2} = 1 + \delta(\rho, M)$$

At the minima:

$$V_{min} \approx \left(\frac{b_0}{2c_0}\right)^4 c_0 \delta_0$$

small & positive

The "no-go" theorem follows because:

$$\frac{4ac}{b^2} = (\text{const}) \sum_p \rho^{-p} \tilde{A}_p(M)$$

with only NSNS and RR fluxes and O6/D6

runaway as $\rho \to \infty$, with $4ac/b^2 \to 0$

Allowing negative internal curvature:

$$\frac{4ac}{b^2} = (\text{const}) \sum_p \tilde{A}_p[\tilde{C}_f \rho^{2-p} + \tilde{A}_{H3}\rho^{-p}]$$

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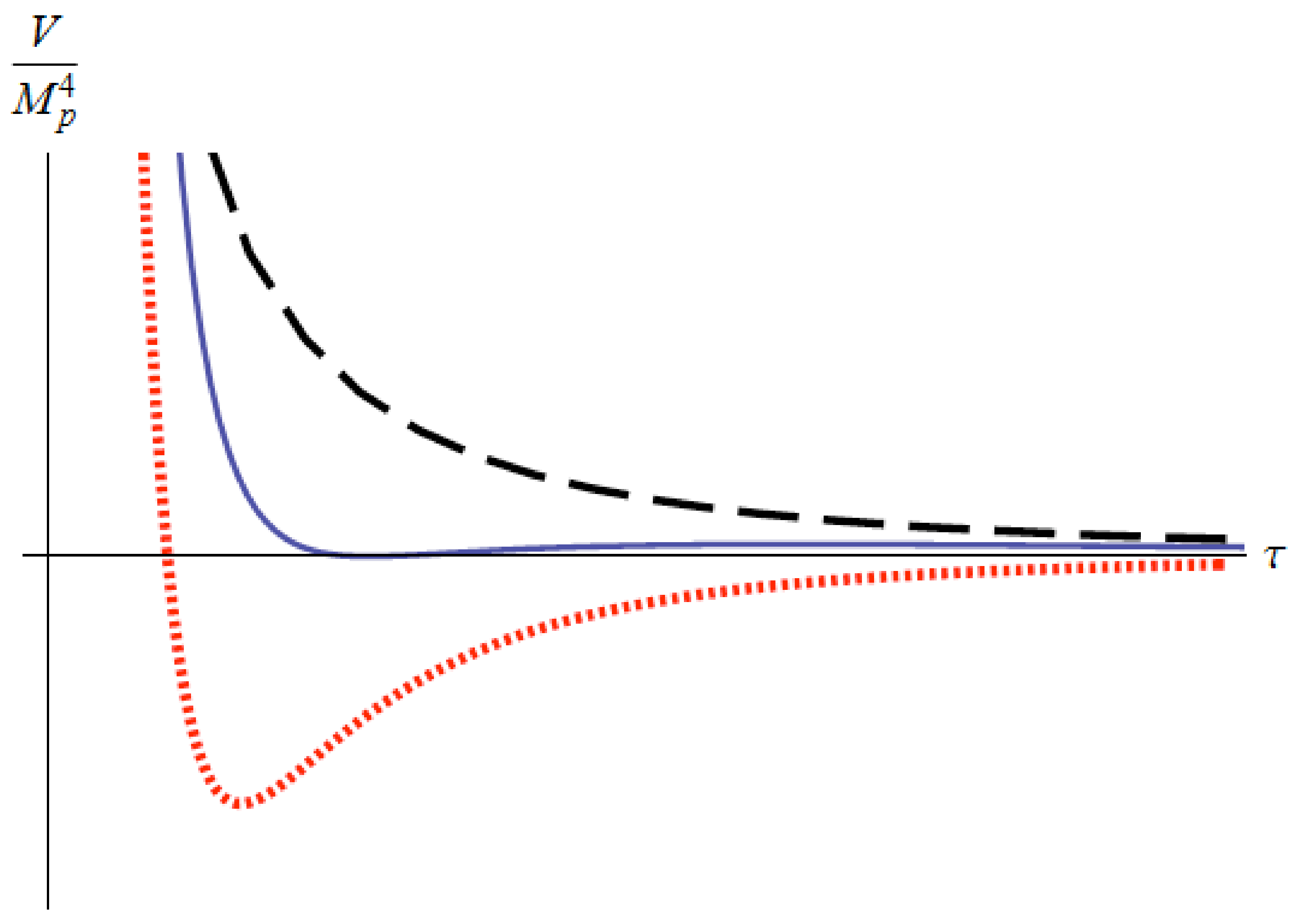
The minimal additional ingredients for IIA dS vacua: negative internal curvature and Romans’ parameter!

Intuitively, we can understand why:



Negative internal scalar curvature acts as an uplifting term.

Model Building

The action for massive IIA SUGRA in string frame:

$$S = \frac{1}{2\kappa_{10}^2}\int^{-2\phi}\Big(\star\mathcal{R} + 4\star d\phi\wedge d\phi - \tfrac{1}{2}\star H_3\wedge H_3\Big)$$
$$-\star F_2\wedge F_2 - \star F_4\wedge F_4 - \star m^2 + CS$$

where $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$

$$\begin{aligned} H_3 &= dB_2 \\ F_2 &= dC_1 + mB_2 \\ F_4 &= dC_3 - C_1\wedge H_3 - \frac{m}{2}B\wedge B \end{aligned}$$

Some flux moduli gain masses
by Stuckelberg couplings
due to m and (later) metric fluxes

The CS term:

$$-dC_3\wedge dC_3\wedge B_2 + \frac{m}{3}B\wedge B\wedge B\wedge dC_3 - \frac{m^2}{20}B\wedge B\wedge B\wedge B\wedge B$$

Start with the string frame metric:

$$\begin{aligned} ds_{10}^2 &= g_{\mu\nu}^{(s)} dx^\mu dx^\nu + g_{mn} dy^m dy^n \\ &= g_{\mu\nu}^{(s)} dx^\mu dx^\nu + \alpha' \rho \, d\tilde{s}_6^2 \end{aligned}$$

Need to introduce O-planes to cancel tadpoles:

$(z_1, z_2, z_3, \tilde{z}_1, \tilde{z}_2, \tilde{z}_3) \longleftrightarrow (\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, z_1, z_2, z_3)$ *defines O6-plane*

As a warmup, consider $\mathcal{M}_6 = \mathcal{M}_3 \times \mathcal{M}_3$

Simplest choice: compact hyperbolic spaces

$$d\mathbb{H}_3^2(\Lambda) = \tfrac{6}{\Lambda} \left(d\varphi^2 + \sinh^2(\varphi) d\Omega_2^2 \right) \;, \qquad \mathcal{R} = -\Lambda$$

Only one modulus: breathing mode

Set $\Lambda = 1$ by rescaling ρ

Compactify by discrete $SO(3,1)$ identifications.

$$\tilde{V}_6 = \int_{(\mathbb{H}_3 \times \mathbb{H}_3)/\mathbb{Z}_2} \epsilon_3 \wedge \tilde{\epsilon}_3 = \frac{e^{2\alpha}}{2} \, ,$$

where dimensionless volume of each hyperboloid:

$$\tilde{V}_3(\mathbb{H}_3) = e^{\alpha} \geq 1$$

α : topological data
discrete, bounded below

Only two moduli:

$$\alpha' \rho = \left(V_6 / \tilde{V}_6 \right)^{1/3} \qquad \tau \equiv e^{-\phi} \rho^{3/2}$$

4D Planck mass depends on their stabilized values:

$$\int d^4 x \sqrt{g_4^{(s)}} \left(\frac{\tau^2 \alpha'^3 \tilde{V}_6}{2\kappa_{10}^2} \right) \mathcal{R}_4^{(s)} + \ldots \qquad M_p^2 = \frac{\tilde{V}_6 \alpha'^3 \tau_0^2}{\kappa_{10}^2} = \frac{V_{6,0}}{\kappa_{10}^2 g_{s,0}^2}$$

Scalar Potential

Dimensionally reduce to 4D Einstein frame:

$$S = \int dx_4 \sqrt{g_4} \Big(\frac{M_p^2}{2} R_4 - \frac{M_p^2}{2} G_{ij} \partial\phi^i \partial\phi^j - V(\phi) \Big) \, ,$$

Simple for CHM: only two moduli and not so many cycles to turn on fluxes

Curvature:
$$V_{CURV} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-2} \rho^{-1} \, .$$

F_0 :
$$V_{F_0} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-4} \rho^3 \, \frac{f_0^2}{16\pi^2} \, .$$

$F_6 = 2\, k_6\, \epsilon_3 \wedge \tilde{\epsilon}_3$:
$$V_{F_6} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-4} \rho^{-3} \, \frac{(2\pi)^{10} \, f_6^2}{e^{4\alpha}} \, .$$

$H_3 = p\epsilon_3^A = \sqrt{2}(\epsilon_3 - \tilde{\epsilon}_3)$:
$$V_{H_3} = \frac{M_p^2 \tau_0^2}{\alpha'} \frac{8\pi^4 h^2}{e^{2\alpha}} \tau^{-2} \rho^{-3} h^2$$

O-Planes and Tadpoles

The O6-plane source term in IIA action (string frame):

$$2(2\pi)^{-6} l_s^{-7} \int_{O6}^{-\Phi} \sqrt{|g|} - 2\sqrt{2}(2\pi)^{-6} l_s^{-7} \int_{O6} C_7 \, ;$$

Since it wraps Σ_3^S:

$$V_{O6} = -\frac{M_p^2 \tau_0^2}{\alpha} e^{-\alpha'} \, 4\sqrt{8}\, \pi \, \tau^{-3}$$

Bianchi identities:

$$\begin{aligned} dF_2 &= m_0 H_3 + 2\pi\sqrt{2}\, l_s \, \Sigma_3^A \, , \\ dF_4 &= -F_2 \wedge H_3 \, , \end{aligned}$$

Tadpole conditions:

$$\int_{\Sigma_3^i} m_0 H_3 = -2\pi\sqrt{2}\, l_s \int_{\Sigma_3^i} \Sigma_3^A \, ,$$

$$\int_{\Sigma_5} F_2 \wedge H_3 = 0 \, .$$

$\Rightarrow$ Only constraint: $f_0 h = 2$

Searching for dS Vacua

Collecting all contributions to the potential:

$$
\begin{aligned}
\frac{\alpha'}{M_p^2\tau_0^2}a(\rho) &= \frac{1}{\rho} + \frac{32\pi^4}{e^{2\alpha}f_0^2}\rho^{-3}\,, \\
\frac{\alpha'}{M_p^2\tau_0^2}b(\rho) &= e^{-\alpha}\,4\sqrt{8}\,\pi\,, \\
\frac{\alpha'}{M_p^2\tau_0^2}c(\rho) &= \left(\frac{f_0^2}{16\pi^2}\,\rho^3 + \frac{(2\pi)^{10}\,f_6^2}{e^{4\alpha}}\,\rho^{-3}\right).
\end{aligned}
$$

The scalar potential is thus explicitly calculable in terms of microphysical flux quanta!

$$\frac{4ac}{b^2}\Big|_{minimum} \approx 1+\delta \qquad\qquad \tau = \frac{b}{2a} + \mathcal{O}(\delta)$$

CHM is too simple:

$$g_s = \frac{e^{\alpha}}{4\sqrt{2}\pi}\sqrt{\rho_0} + \frac{4\sqrt{2}\pi^3}{e^{\alpha} f_0^2}\frac{1}{(\sqrt{\rho_0})^3}$$

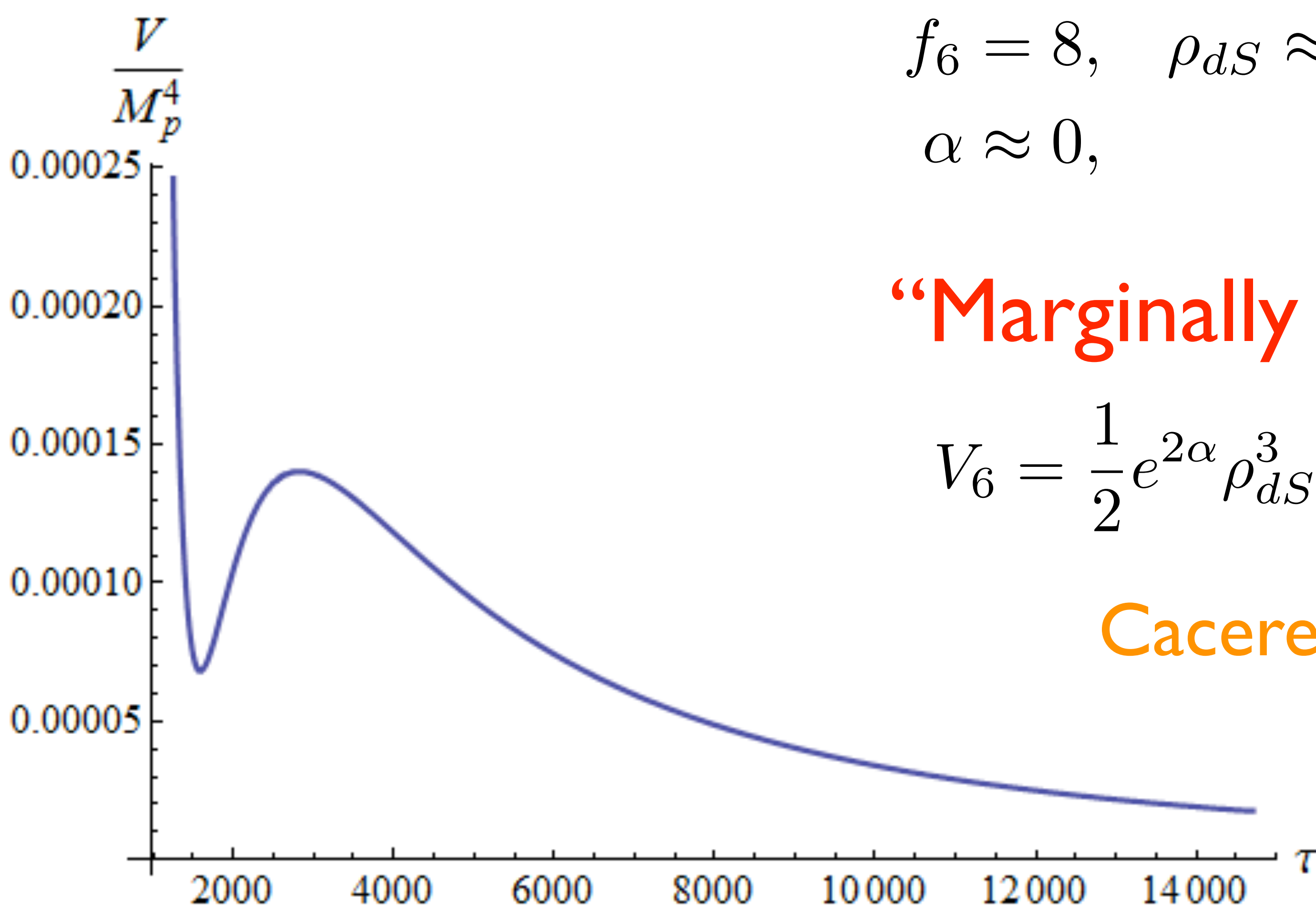
A trade-off between weak coupling and large volume

An example:

$$f_0 = 2, \quad \tfrac{4ac}{b^2} \approx 1.03, \quad \frac{V_{dS}}{M_p^4} \approx 7.9 \times 10^{-5}$$

$$f_6 = 8, \quad \rho_{dS} \approx 90.614, \quad \tau_{dS} \approx 1.47 \times 10^3$$

$$\alpha \approx 0,$$



"Marginally perturbative":

$$V_6 = \frac{1}{2}e^{2\alpha}\rho_{dS}^3 \approx 3.72 \times 10^5, \quad g_s \approx 0.56$$

Caceres, Kaplunovsky, Mandelberg (96)
Hebecker, Trapletti (04)

Separation of Scales

Canonically normalized moduli:

$$\hat{\tau} = \sqrt{2} M_p \ln \tau$$
$$\hat{\rho} = \sqrt{\frac{3}{2}} M_p \ln \rho \,.$$

have masses of the same scale as the KK modes.

Similar to strongly warped flux compactifications

Giddings, Maharana; GS, Torroba, Underwood, Douglas

CHM reduction is a consistent truncation in the SUGRA sense, like the Freund-Rubin vacua:

Minimizing 4D potential $\Longleftrightarrow$ Solving 10D EOM by setting KK modes=0.

Twisted Tori

- Negative curvature, flux backreaction included.
- More tunable parameters, possibly find vacua with parametrically small coupling & large volume.
- Potentially find dS solutions as spontaneously SUSY breaking vacua in gauged SUGRA.
- Monodromy in the CMB. Silverstein, Westphal [c.f. McAllister, Silverstein, Westphal]
- Standard Model Building Camara, Font, Ibanez

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- A lot more moduli

Twisted Tori

For twisted tori of the form: $G_3 \times G_3$

3D Group manifolds classified by Bianchi:

$$g^{-1}dg = \eta^a T_a$$

η^a : Maurer-Cartan forms

T_a : Generators of Lie Algebra

MC equations: $d\eta^a = -f^a_{bc}\eta^b \wedge \eta^c$

Bianchi type	Algebra	(q_1, q_2, q_3)
I	$U(1)^3$	$(0,0,0)$
II	Heis$_3$	$(0,0,Q_1)$
VI_0	ISO(1,1)	$(0,-Q_1,Q_2)$
VII_0	ISO(2)	$(0,Q_1,Q_2)$
$VIII$	SO(2,1)	$(Q_1,-Q_2,Q_3)$
IX	SO(3)	(Q_1,Q_2,Q_3)

Twisted Tori

For twisted tori of the form: $G_3 \times G_3$

3D Group manifolds classified by Bianchi:

$$g^{-1}dg = \eta^a T_a$$

η^a : Maurer-Cartan forms

T_a : Generators of Lie Algebra

MC equations: $d\eta^a = -f^a_{bc}\eta^b \wedge \eta^c$

$$f^a_{bc} = \epsilon_{bcd} Q^{ad}$$

$$Q = \begin{pmatrix} q_1 & & \\ & q_2 & \\ & & q_3 \end{pmatrix}$$

Bianchi type	Algebra	(q_1, q_2, q_3)
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Negative curvature

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Nilmanifold

Negative curvature

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IX	SO(3)	(Q_1,Q_2,Q_3)

Nilmanifold

Solmanifold

Negative curvature

Compactify by discrete identification of G: quantization of structure constants Q.

(Co)homology can be computed: a lot more moduli.

Discrete torsion cycles: fractional Wilson lines Silverstein

but K-theory constraints

Metric flux contribution to 4D potential:

$$V_{\mathrm{metric}} = \left(\frac{\alpha'^2}{2\kappa_{10}^2}\right) Vol_6 \rho^{-1}\tau^{-2}\tilde{C}_f(M) \qquad M_{ab} = \begin{pmatrix} M_3 & M_{a\tilde{a}} \\ M_{a\tilde{a}} & M_3 \end{pmatrix}$$

$$\alpha'^{-3}\,\tilde{C}_f(M) = -(\mathrm{Tr}[QM_3])^2 + 2\mathrm{Tr}[QM_3QM_3]$$

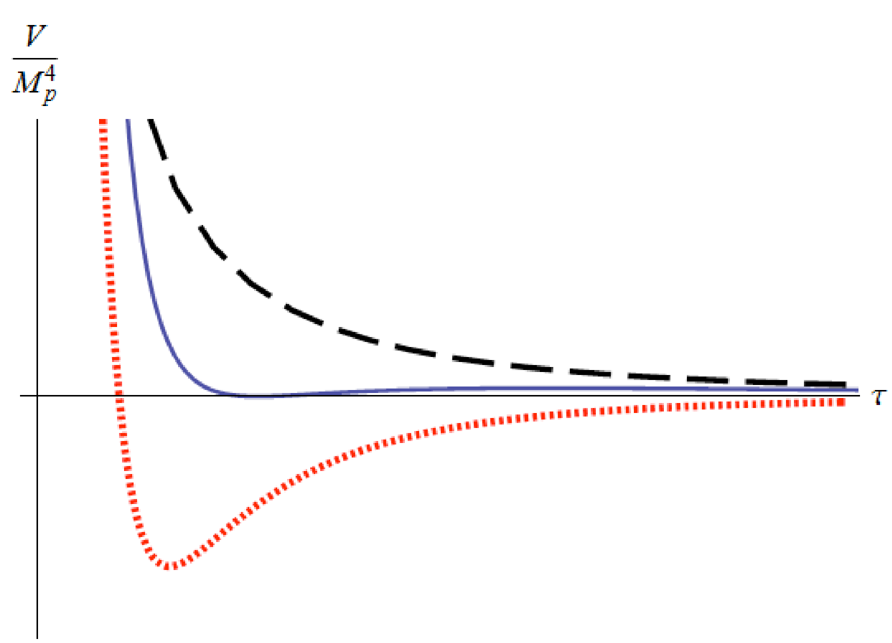
Moduli dependence of a,b,c gives runaway directions in field space: $\phi_i \sim \rho^{-\alpha}$ ⟹ need KK5-branes

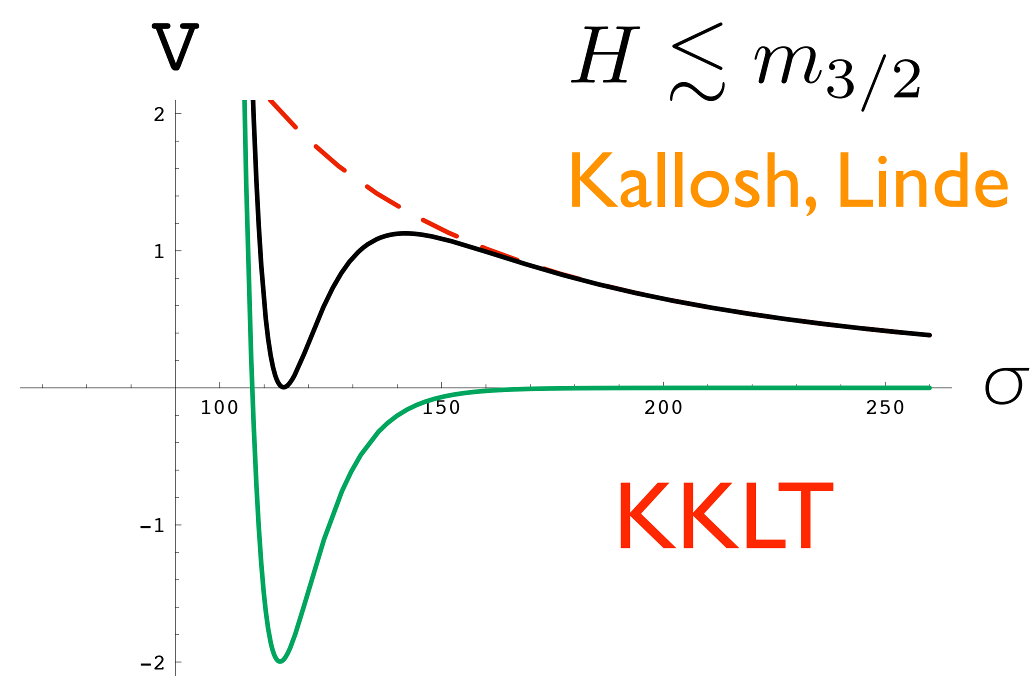
Seems to have no such dangerous runaway for Nil6.

Comments

- Minimal ingredients to construct simple dS vacua. Parametrically weak coupling/large volume solutions require more “knobs”.

- Minimal dS vacua can in principle be constructed as spontaneously ~~SUSY~~ vacua of gauged SUGRA:



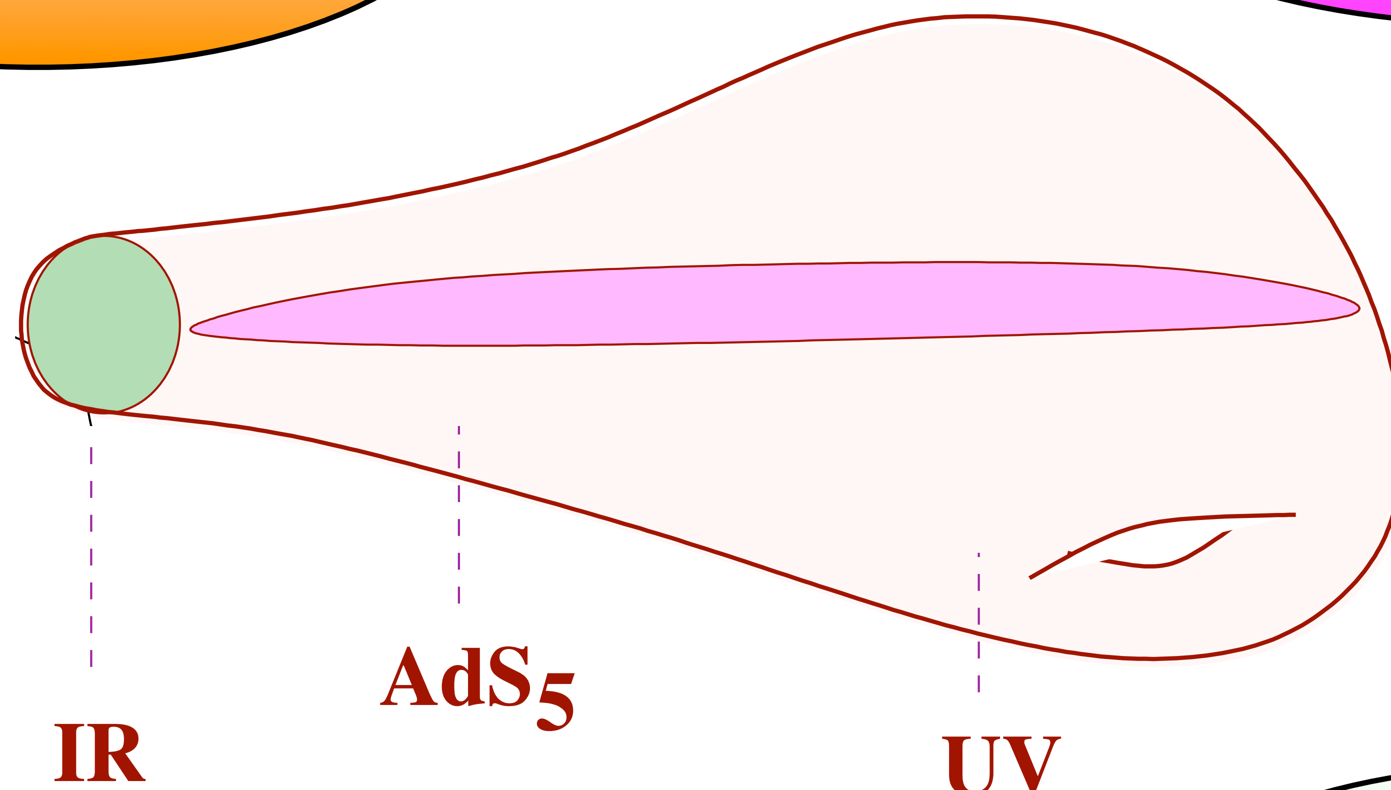


- Dark Energy + Inflation + Standard Model

The Ubiquitous Throat

Gauge/gravity correspondence

Inflation



RS/Technicolor/ SUSY mediation

Moduli stabilization/ KKLT

Warped EFT

- Understanding warped dynamics is essential for drawing precise predictions in such string theory models of particle physics & inflation.
- Closed string sector: Many subtle issues with strong warping such as compensators, gauge redundancies and constraints, backreaction, separation of scales, ...

  GS, Torroba, Underwood, Douglas (STUD)
- Open string sector (Standard Model): wavefunctions in warped backgrounds are prequisites for extracting Kahler potential, Yukawa couplings and flavor, SUSY mediation, technicolor model building, ...

  Marchesano, McGuirk, GS

Warped EFT:
Closed String Sector

GS, Torroba, Underwood, Douglas

Warped Kahler Potential

- The warping corrected Kahler potential for the complex moduli sector was conjectured to be:

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega \wedge \overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha \wedge \chi_{\overline{\beta}}$$

DeWolfe-Giddings

suggested by the fact that

$$\mathrm{V}_{CY} = \int d^6y\sqrt{g_6} \rightarrow V_W = \int d^6y\sqrt{\tilde{g}_6}e^{-4A(y)}$$

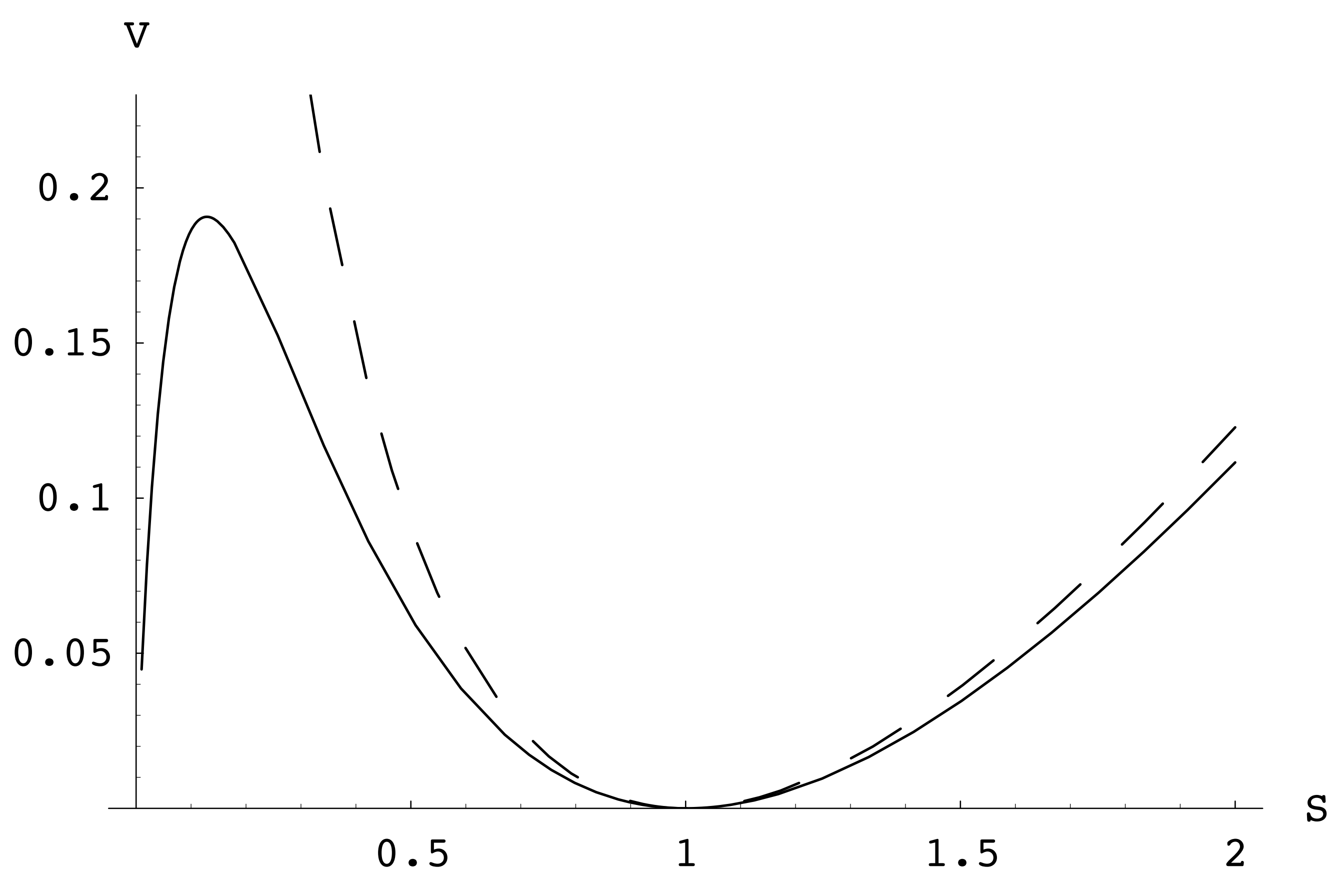
- For the warped deformed conifold:

$$\mathrm{G}_{S\overline{S}} = -\partial_S\partial_{\overline{S}}K = \frac{1}{V_W}\left[c\log\frac{\Lambda_0^3}{|S|} + c'\frac{(g_sN\alpha')^2}{|S|^{4/3}}\right]$$

Douglas, Shelton, Torroba

Warped Kahler Potential

Warping corrections change qualitatively the moduli (and hierarchy) stabilization potential:



c.f. inflaton potential, Yukawa couplings, soft terms, etc.

Issues with Strong Warping

D=10 String Theory

↓ Low Energy

D=10 SUGRA with fluxes

↓ KK Dimensional Reduction

D=4 N=1 SUGRA EFT

↓

String vacua, inflation, de-Sitter, MSSM…

Ex: GKP and KKLT

Type IIB String Theory in D=10

↓ Low Energy

IIB Supergravity in D=10

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left\{R_{10} - \frac{|G_3|^2}{2\mathrm{Im}\tau} - \frac{1}{4}|\tilde{F}_5|^2\right\} + \mathrm{CS} + \mathrm{local}$$

↓ KK Dimensional Reduction

N=1 SUGRA in D=4

$$K = -3\log(\rho+\bar{\rho}) - \log(\tau+\bar{\tau}) - \log\left(\int J^3\right) - \log\left(\int \Omega\wedge\bar{\Omega}\right)$$

$$W = \int G_3\wedge\Omega + W_{np}$$

Issues with Strong Warping

D=10 String Theory

↓ Low Energy

D=10 SUGRA with fluxes

↓ KK Dimensional Reduction

D=4 N=1 SUGRA EFT

↓

String vacua, inflation, de-Sitter, MSSM…

Many subtleties with warped KK reduction:

- General KK ansatz (compensators)
- Mixing/sourcing of KK modes with moduli
- Backreaction of moduli on warp factor
- 10D Gauge redundancies
- 10D Constraint equations

In *warped backgrounds* these issues are all highly coupled to each other!

KK Scale in Warped Background

	Moduli	KK modes
Unwarped	$m_z^2 \sim \dfrac{1}{\alpha'}$	$m_{KK}^2 \sim \dfrac{1}{L^2}$

KK Scale in Warped Background

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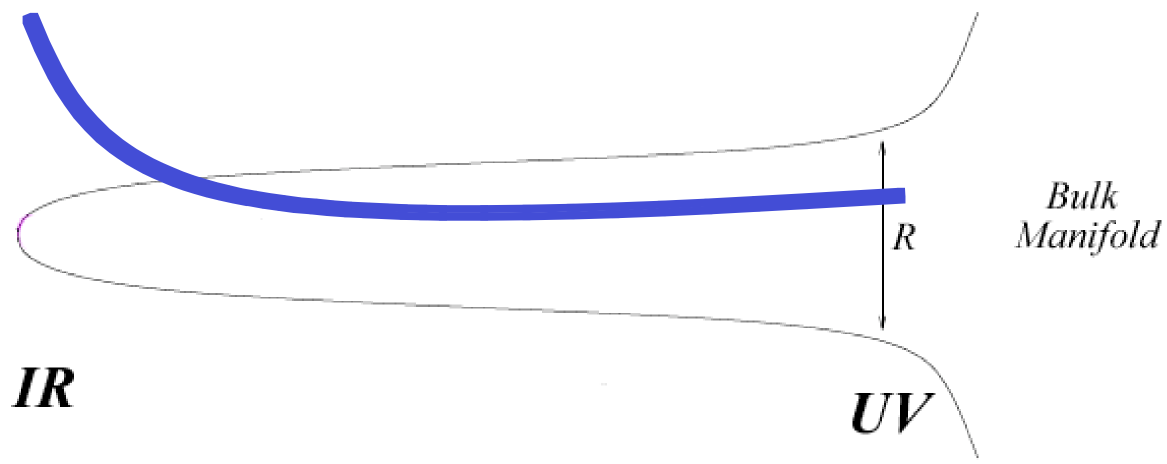
Unwarped	$m_z^2 \sim \frac{1}{\alpha'}$	$m_{KK}^2 \sim \frac{1}{L^2}$

Strong warping



DeWolfe, Giddings;
Giddings, Maharana;
Frey, Maharana;
Burgess, Camara, de Alw
Giddings, Maharana,
Quevedo, Suruliz; ...

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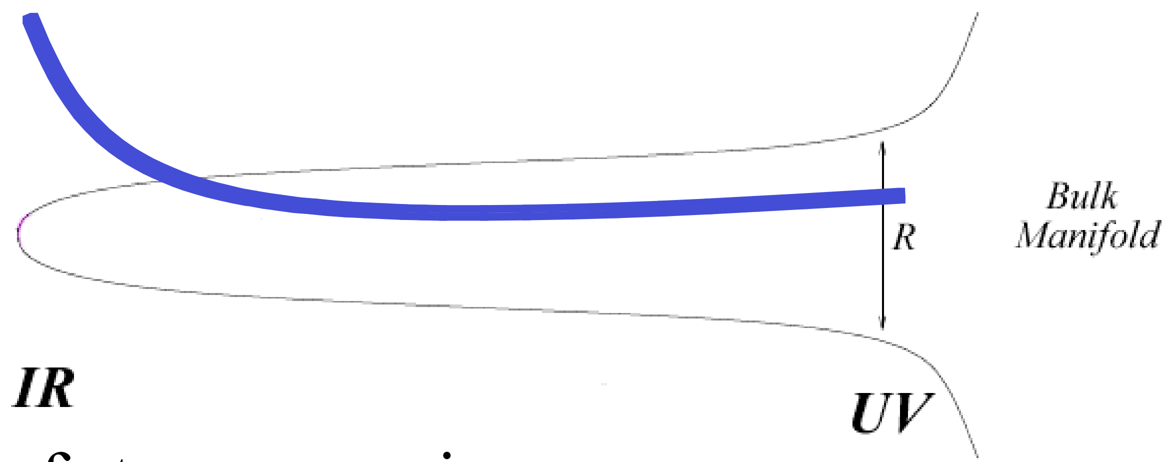

DeWolfe, Giddings;
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Fields localize to region of strong warping.

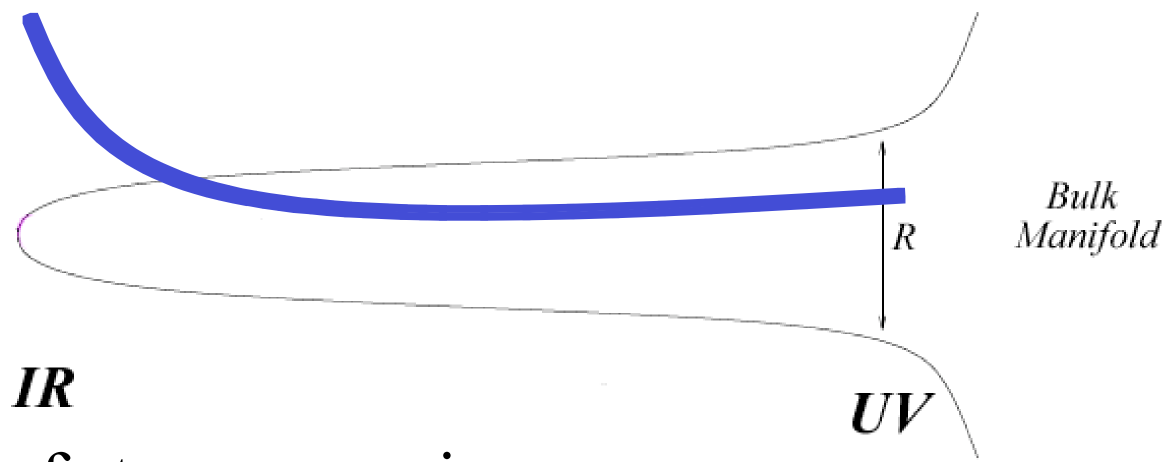
KK Scale in Warped Background

Moduli | KK modes

Unwarped

$$m_z^2 \sim \frac{1}{\alpha'} \quad \Big| \quad m_{KK}^2 \sim \frac{1}{L^2}$$

Strong warping



DeWolfe, Giddings; Giddings, Maharana; Frey, Maharana; Burgess, Camara, de Alw Giddings, Maharana, Quevedo, Suruliz; ...

Fields localize to region of strong warping.

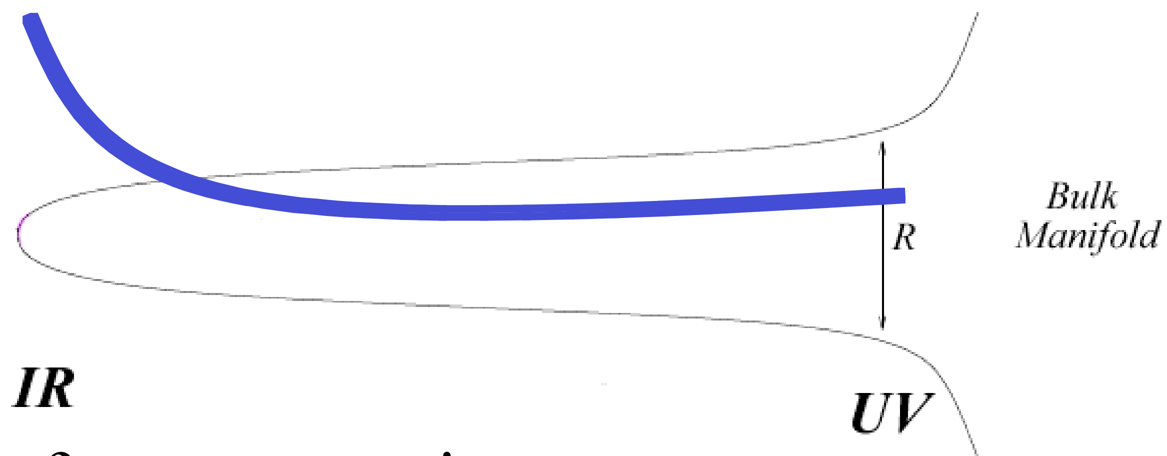
Masses redshifted

$$m_z^2 \sim e^{2A_0}\frac{1}{\alpha'} \quad \Big| \quad m_{KK}^2 \sim e^{2A_0}\frac{1}{\alpha'}$$

KK Scale in Warped Background

	Moduli	KK modes
Unwarped	$m_z^2 \sim \dfrac{1}{\alpha'}$	$m_{KK}^2 \sim \dfrac{1}{L^2}$

Strong warping



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Fields localize to region of strong warping.

Masses redshifted	$m_z^2 \sim e^{2A_0}\dfrac{1}{\alpha'}$	$m_{KK}^2 \sim e^{2A_0}\dfrac{1}{\alpha'}$

No mass hierarchy between moduli and KK modes for integrating out heavy fields.

Warped Kahler Potential

Previous proposal: (DeWolfe, Giddings)

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega\wedge\overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha\wedge\chi_{\overline{\beta}}$$

did not account for all these subtle issues with warping.

Ansatz for fluctuations: (DeWolfe, Giddings)

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}(\tilde{g}_{mn} + \delta\tilde{g}_{mn})dy^m dy^n$$

... does not solve 10D EOM! Giddings, Maharana; STUD

More general ansatz does, but extremely messy ...

$$ds^2_{10} \to ds^2_{10} + 2\partial_\mu\partial_\nu S^\alpha e^{2A}K_\alpha(y)dx^\mu dx^\nu + 2e^{2A}B_{\alpha m}(y)\partial_\mu S^\alpha dx^\mu dy^m\,.$$

Linearized Einstein Equations

$$
\begin{aligned}
\delta G^{\mu}_{\nu} =&\delta^{\mu}_{\nu} u^I \delta_I \left\{ e^{2A} \left[-2\tilde{\nabla}^2 A + 4(\widetilde{\nabla A})^2 - \frac{1}{2}\tilde{R} \right] \right\} + e^{-2A} \left(\partial^{\mu}\partial_{\nu} u^I - \delta^{\mu}_{\nu} \Box u^I \right) (4\delta_I A - \frac{1}{2}\delta_I \tilde{g}) \\
&+ \left(\partial^{\mu}\partial_{\nu} u^I - \delta^{\mu}_{\nu} \Box u^I \right) e^{2A} \tilde{\nabla}^p (B_{Ip} - \partial_p K_I) \\
&+ e^{-2A} f^K \delta_K G^{(4)\mu}_{\nu} - \frac{1}{2} \left(\delta_K g^{\mu}_{\nu} - \delta^{\mu}_{\nu} \delta_K g^{\lambda}_{\lambda} \right) e^{2A} \tilde{\nabla}^2 f^K \ ,
\end{aligned}
\tag{A.14}
$$

$$
\begin{aligned}
\delta G^{\mu}_{m} = \delta R^{\mu}_{m} =&e^{-2A} \partial^{\mu} u^I \Bigg\{ 2\partial_m \delta_I A - 8\partial_m A \delta_I A - \frac{1}{2}\partial_m \delta_I \tilde{g} + \partial_m A \delta_I \tilde{g} \\
&- 2\partial^{\tilde{p}} A \delta_I \tilde{g}_{mp} + \frac{1}{2} \tilde{\nabla}^p \delta_I \tilde{g}_{mp} \\
&- \frac{1}{2} \tilde{\nabla}^p \left[e^{4A} \left(\tilde{\nabla}_p B_{Im} - \tilde{\nabla}_m B_{Ip} \right) \right] + 2(\partial_m A B_{Ip} - \partial_p A B_{Im}) \tilde{\nabla}^p e^{4A} \\
&+ \frac{1}{2} e^{8A} B_{Im} \tilde{\nabla}^2 e^{-4A} - e^{4A} \tilde{R}^n_m B_{In} \Bigg\} \ ,
\end{aligned}
\tag{A.15}
$$

$$
\begin{aligned}
\delta G^{m}_{n} =&u^I \delta_I \left\{ e^{2A} \left[\tilde{G}^m_n + 4(\widetilde{\nabla A})^2 \delta^m_n - 8\nabla_n A \tilde{\nabla}^m A \right] \right\} - \frac{1}{2} e^{-2A} \Box u^I \tilde{g}^{mk} \delta_I \tilde{g}_{kn} \\
&+ \delta^m_n e^{-2A} \Box u^I (-2\delta_I A + \frac{1}{2}\delta_I \tilde{g}) \\
&\Box u^I \Bigg(\frac{1}{2} e^{-2A} \left\{ \tilde{\nabla}^m \left[e^{4A} \left(B_{In} - \partial_n K_I \right) \right] + \tilde{\nabla}_n \left[e^{4A} \left(B^{\tilde{m}}_I - \partial^{\tilde{m}} K_I \right) \right] \right\} \\
&- \delta^m_n \tilde{\nabla}^p \left[e^{2A} \left(B_{Ip} - \partial_p K_I \right) \right] \Bigg) \\
&+ \frac{1}{2} \delta_K g^{\mu}_{\mu} \left\{ -\frac{1}{2} e^{-2A} \left[\tilde{\nabla}^m \left(e^{4A} \partial_n f^K \right) + \tilde{\nabla}_n \left(e^{4A} \partial^{\tilde{m}} f^K \right) \right] + \delta^m_n \tilde{\nabla}^p \left[e^{2A} \partial_p f^K \right] \right\} \\
&- \frac{1}{2} \delta^m_n f^K e^{-2A} \delta_K R^{(4)} \ .
\end{aligned}
\tag{A.16}
$$

$$
\delta T^{\mu}_{\nu} = -\delta^{\mu}_{\nu} \frac{1}{4\kappa_{10}^2} \left\{ u^I \delta_I \left[e^{-6A} (\widetilde{\nabla\alpha})^2 \right] - 2e^{-6A} \Box u^I S_{Im} \partial^{\tilde{m}} \alpha - 2\Box u^I K_I e^{-6A} (\widetilde{\nabla\alpha})^2 \right\} \ , \tag{A.37}
$$

$$
\delta T^{\mu}_{m} = \frac{1}{2\kappa_{10}^2} \partial^{\mu} u^I e^{-6A} \left[\partial_m S_{Ip} - \partial_p S_{Im} + \partial_m \alpha B_{Ip} - \partial_p \alpha B_{Im} \right] \partial^{\tilde{p}} \alpha \ , \tag{A.38}
$$

$$
\boxed{\delta G^M_N = \kappa_{10}^2 \delta T^M_N}
$$

$$
\begin{aligned}
\delta T^{m}_{n} =& -\frac{1}{2\kappa_{10}^2} u^I \delta_I \left\{ e^{-6A} \left[\partial_n \alpha \partial^{\tilde{m}} \alpha - \frac{1}{2} \delta^m_n (\widetilde{\nabla\alpha})^2 \right] \right\} \\
&+ \frac{e^{-6A}}{2\kappa_{10}^2} \Box u^I \left\{ S_{In} \partial^{\tilde{m}} \alpha + \partial_n \alpha S^{\tilde{m}}_I - \delta^m_n S_{Ip} \partial^{\tilde{p}} \alpha + 2K_I \left[\partial_n \alpha \partial^{\tilde{m}} \alpha - \frac{1}{2} \delta^m_n (\widetilde{\nabla\alpha})^2 \right] \right\} \ .
\end{aligned}
\tag{A.39}
$$

Giddings, Maharana

Gauge Invariance & Compensators

Previous proposal: (DeWolfe, Giddings)

$$\mathrm{K} = -\log\left(\int e^{-4A}\Omega\wedge\overline{\Omega}\right) \Rightarrow G_{\alpha\overline{\beta}} = -\frac{1}{V_W}\int e^{-4A}\chi_\alpha\wedge\chi_{\overline{\beta}}$$

is not diffeomorphism invariant:

$$\chi \rightarrow \chi + d\alpha$$

This turns out to be equivalent to the failure of the metric ansatz in solving the EOM.

Need extra terms proportional to $\partial_\mu S^\alpha$

$$ds_{10}^2 \rightarrow ds_{10}^2 + 2\partial_\mu\partial_\nu S^\alpha e^{2A}K_\alpha(y)dx^\mu dx^\nu + 2e^{2A}B_{\alpha m}(y)\partial_\mu S^\alpha dx^\mu dy^m\,.$$

metric compensators

(Analogously, also flux compensators)

Compensators in E&M

Consider a U(1) gauge field:

$$S = -\frac{1}{4}\int d^{10}x\sqrt{g_{10}}F^{MN}F_{MN}$$

and a family of solutions to $\quad D^M F_{MN} = 0$

parametrized by moduli u^I: $\quad A_M = (A_\mu = 0, A_i(y;u))$

Promoting $u^I \to u^I(x)$, the kinetic terms give:

$$G_{IJ} = \int d^6y\sqrt{g_6}g^{ij}\frac{\partial A_i}{\partial u^i}\frac{\partial A_j}{\partial u^J}$$

not gauge invariant under $\quad \delta A_i = \partial_i \epsilon$

Compensators in E&M

The error is in assuming that: $A_\mu = 0$

still holds for time-dependent moduli.

This is incorrect because the 10D EOM:

$$D^M F_{M\mu} = 0 \Rightarrow \partial_\mu \partial^i A_i = \partial^i \partial_i A_\mu$$

Constraint equations:
no second order
time derivatives

cannot be solved by: $\partial_\mu A_i \neq 0 \ , \quad A_\mu = 0$

Instead, the time-dependence forces a non-zero:

$$A_\mu = \Omega_I \partial_\mu u^I \ , \quad \partial^i \partial_i \Omega_I = \partial^i \frac{\partial A_i}{\partial u^I}$$

Ω_I : compensator field

Compensators in E&M

Effect of compensator on dimensionally reduced action:

$$\frac{\partial A_i}{\partial u^I} \to \delta_I A_i \equiv \frac{\partial A_i}{\partial u^I} - \partial_i \Omega_I \text{ so that } \partial^i(\delta_I A_i) = 0$$

Compensator puts $\delta_I A_i$ back into harmonic gauge.

The field space metric is simply:

$$G_{IJ} = \int d^6 y \sqrt{g_6}\; g^{ij} \delta_I A_i \delta_J A_j$$

Natural mathematical definition (Singer): fluctuation $\delta_I A_i$ orthogonal to gauge transformation, w.r.t G_{IJ}

Warped Compactifications

Time-dependence of moduli sources off-diagonal metric:

$$ds_{10}^2 = e^{2A(y;u)} g_{\mu\nu}(x) dx^\mu dx^\nu + B_j^I(y) \partial_\mu u^I dx^\mu dy^I + g_{ij}(y;u) dy^i dy^j$$

Compensators put metric back into harmonic gauge.

Hard to generalize YM approach. Two strategies:

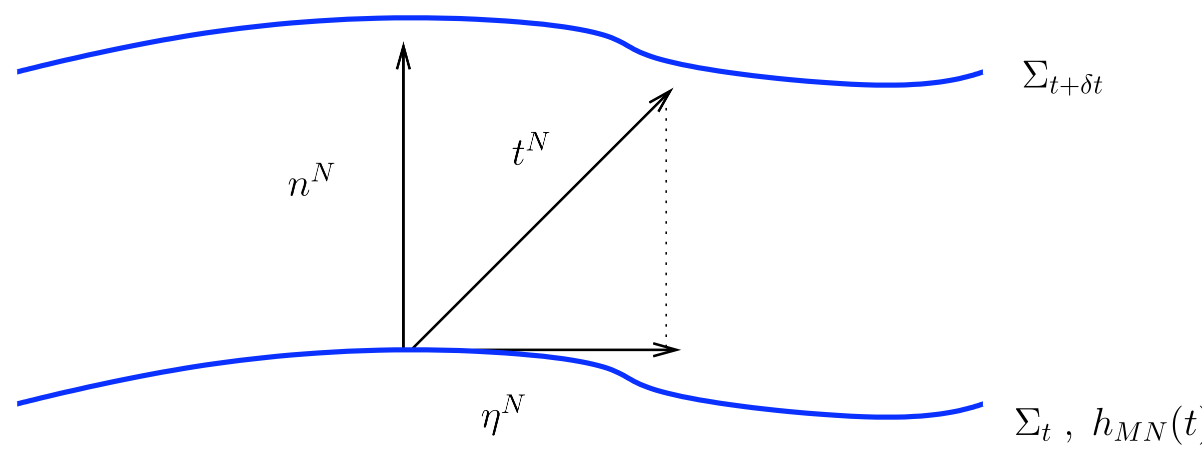
- Lagrangian: gauge-fixed metric ($B_j^I = 0$, compensator gauge), dimensional reduction with 10D constraints.
- Hamiltonian: gauge invariant metric, compensators as Lagrange multipliers enforcing 10D constraints.

Hamiltonian of GR

Split metric into:

h_{MN} space-like piece

η_N tangential shift



Extrinsic curvature: $$K_{MN} = \frac{1}{2}(g^{tt})^{1/2}\left(\dot{h}_{MN} - \nabla_M \eta_N - \nabla_N \eta_M\right)$$

Canonical momentum: $$\pi_{MN} = \frac{\partial \mathcal{L}_{EH}}{\partial \dot{h}_{MN}} = h^{1/2}\left(K_{MN} - h_{MN} K\right)$$

Hamiltonian: $$\mathcal{H}_G = \sqrt{-g_D}\left(-R^{(D-1)} + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2\right) - 2\eta_N \nabla_M\left(\pi^{MN}\right)$$

$\eta_N =$ Lagrange multipliers enforcing the constraints:

$$\nabla_M\left(\pi^{MN}\right) = 0$$

Kinetic Terms

Here, time-dependence of h_{MN} only implicit through $u^I(x)$

Computing the shift vectors: $\eta^i = B^i_I \dot{u}^I$

Therefore, compensators = Lagrange multipliers of $\mathcal{H}_G$!

The dynamical variables of H define the metric fluctuations:

$$K_{MN} \sim \dot{u}^I \delta_I h_{MN} \equiv \dot{u}^I \frac{\partial h_{MN}}{\partial u^I} - \nabla_M \eta_N - \nabla_N \eta_M$$

$$\pi_{MN} \sim \dot{u}^I \delta_I \overline{h}_{MN} \equiv \dot{u}^I \left(\delta_I h_{MN} - h_{MN} \delta_I h \right)$$

Only effect of compensators is to shift $\partial_I h_{MN} \to \delta_I h_{MN}$

(“physical” variation) & enforce constraints: $\nabla^M \left(\delta_I \overline{h}_{MN} \right) = 0$

Applications: Warped Compactifications

Conformal Calabi-Yau background:

$$ds_{10}^2 = e^{2A(y;u)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y;u)}\tilde{g}_{mn}(y;u)dy^m dy^n$$

Constraint equations:

$$(1)\ \delta A = \frac{1}{8}\delta\tilde{g} \leftrightarrow \text{Invariance of } V_W = \int d^6y\sqrt{\tilde{g}_6}e^{-4A}$$

$$(2)\ \tilde{\nabla}^{\tilde{m}}(\delta\tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\delta\tilde{g}) = 4\partial^{\tilde{m}}A\delta\tilde{g}_{mn}$$

$$\leftrightarrow \text{“Warped” Harmonic Gauge Condition}$$

Warped moduli space metric:

$$G_{IJ}(u) = \frac{1}{4V_W}\int d^6y\sqrt{\tilde{g}_6}\ e^{-4A}\tilde{g}^{ik}\tilde{g}^{jl}\delta_I\tilde{g}_{ij}\delta_J\tilde{g}_{kl}$$

GS, Torroba, Underwood, Douglas

Warped Deformed Conifold

- Klebanov-Strassler solution:

$$ds_{10}^2 = \frac{|S|^{2/3}}{(g_s N\alpha')} \; I(\tau)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (g_s N\alpha') \; I(\tau)^{1/2}\left[\frac{1}{3K(\tau}\right)^2\left(d\tau^2 + (g^5)^2\right)$$
$$+ K(\tau)\cosh^2\left(\frac{\tau}{2}\right)\left((g^3)^2 + (g^4)^2\right) + K(\tau)\sinh^2\left(\frac{\tau}{2}\right)\left((g^1)^2 + (g^2)^2\right)\Big]$$

where $\quad e^{-4A(\tau)} = \frac{(g_s N\alpha')^2}{|S|^{4/3}} \; I(\tau)$

- S only enters 4D redshift factor, not 6D metric:

$$\delta_S g_{ij} = -\nabla_i \eta_j - \nabla_j \eta_i$$

- Same qualitative feature as DG, differ by order 1 coefficient:

$$G_{S\overline{S}} = \frac{k}{V_W}\frac{(g_s N\alpha')^2}{|S|^{4/3}}$$

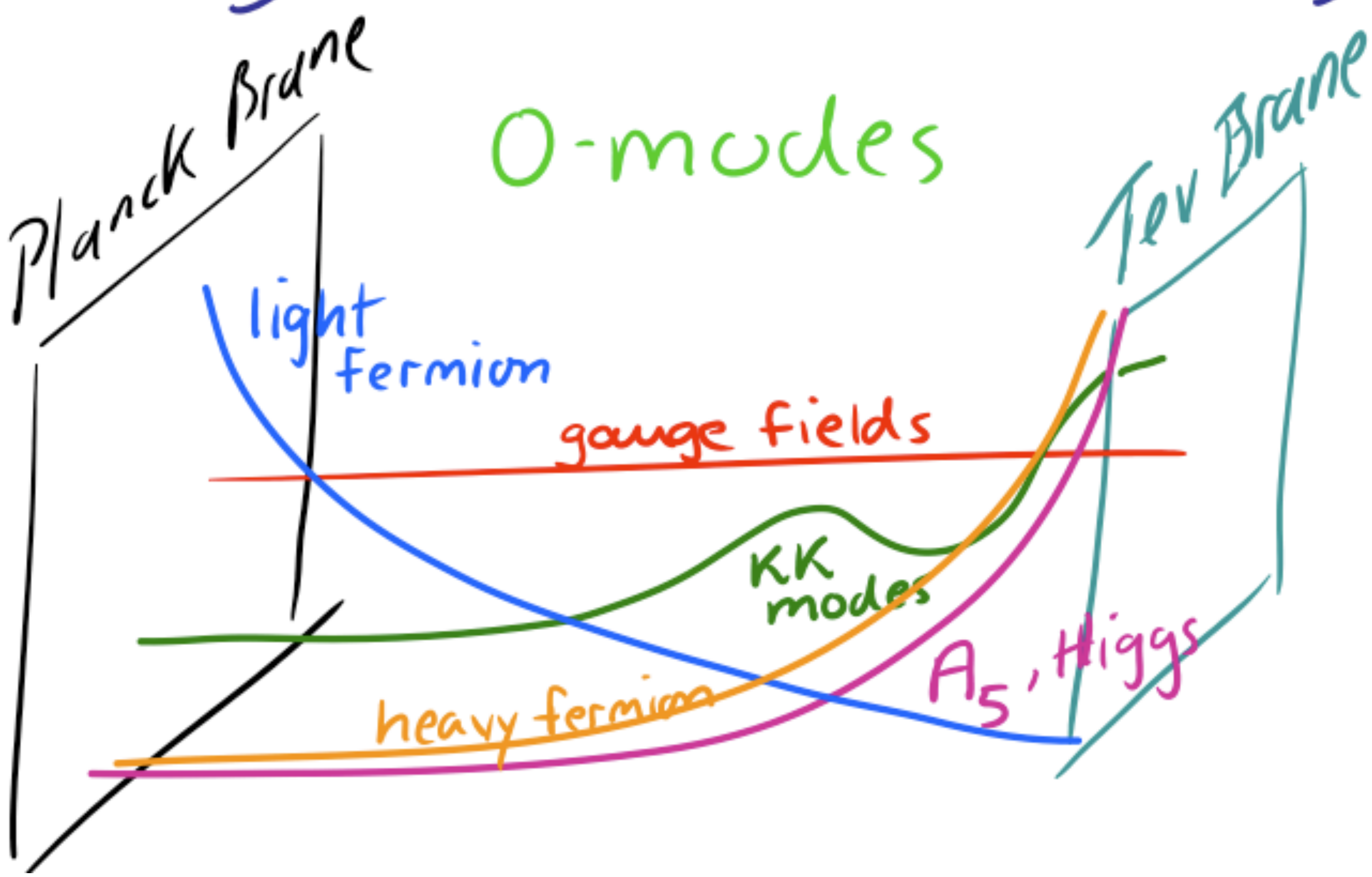
Douglas, Torroba

Warped EFT: Open String Sector

Marchesano, McGuirk, GS

Warped Extra Dimensions

5D Chaos → 4D Cosmos

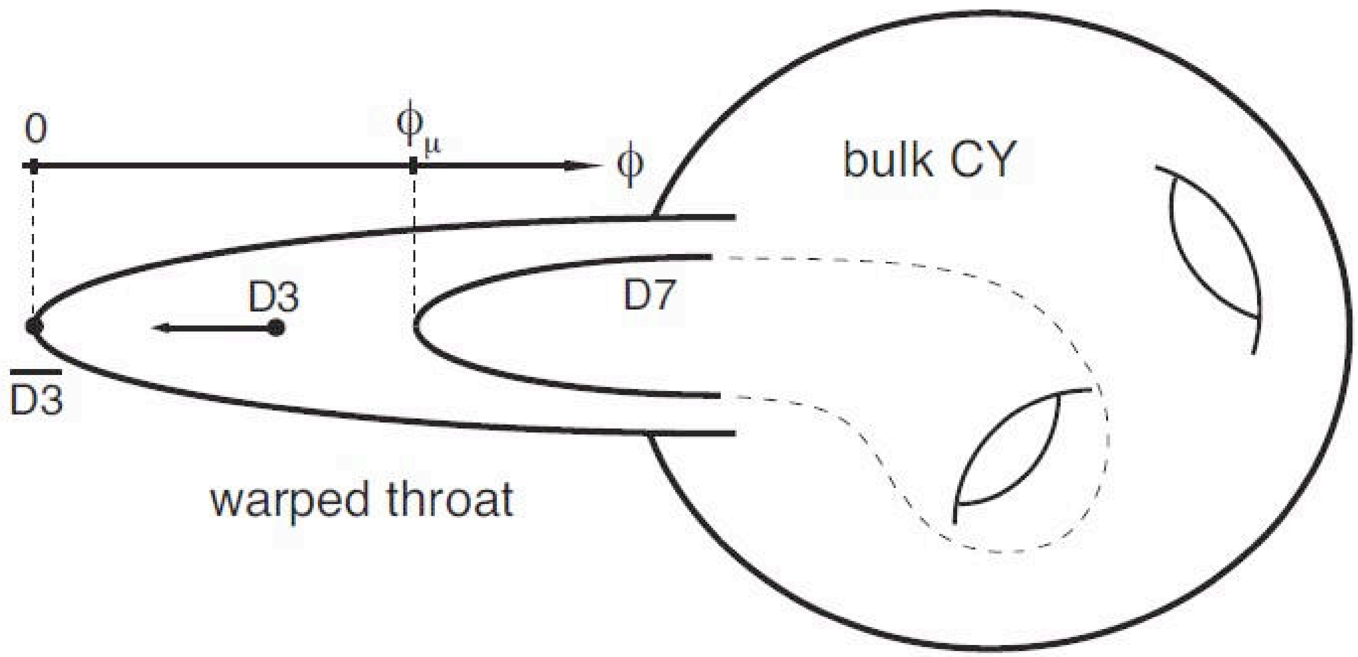


1st approx.
– works (surprisingly) well

[Slide from Sundrum]

Are there new features in string theory embedding?

Warped Extra Dimensions



D7-branes wrap

$$S_4 \subset X_6$$

Type IIB warped background (as in GKP, KKLT):

$$ds_{10}^2 = \Delta^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + \Delta^{1/2}e^\Phi \hat{g}_{mn}dy^m dy^n$$

Consistency requires:

$$F_5 = (1 + *_{10})F_5^{\rm int} \qquad F_5^{\rm int} = \hat{*}_6 d\left(\Delta e^\Phi\right)$$

D-brane Action in Flux Background

The bosonic part: $S^{\rm bos}_{\rm D7} = S^{\rm DBI}_{\rm D7} + S^{\rm CS}_{\rm D7}$

The fermionic part: Martucci, Rosseel, Van den Bleeken, Van Proeyen

$$S^{\rm fer}_{\rm D7} = \tau_{\rm D7} \int d^8\xi \, e^{-\Phi} \sqrt{\left|\det \, P[G]\right|} \, \bar{\Theta} P^{D7}_{-} \left(\Gamma^{\alpha} \mathcal{D}_{\alpha} - \frac{1}{2} \mathcal{O} \right) \Theta$$

obtained from M2-brane action and T-dualities

$\mathcal{D}_{\alpha}$: gravitino variation $\qquad$ $\mathcal{O}$: dilatino variation

10D MW *bispinors* $\quad \Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \; ; \quad P^{D7}_{\pm} = \frac{1}{2} \left(\mathbb{I} \pm \Gamma_{(8)} \otimes \sigma_2 \right)$

κ -symmetry: $\quad \Theta \to \Theta + P^{D7}_{-} \kappa$

In Einstein frame: $G^E_{MN} \equiv e^{-\Phi/2} G^{st}_{MN}$

$$ds^2_{10} = Z^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}\hat{g}_{mn}dy^m dy^n \qquad Z \equiv \Delta e^\Phi$$

$$S^{\rm fer}_{\rm D7} = \tau_{\rm D7}\int d^8\xi\, e^\Phi \sqrt{\left|\det\, P[G^E]\right|}\,\bar{\Theta}P^{\rm D7}_{-}\left(\Gamma^\alpha \mathcal{D}^E_\alpha + \frac{1}{2}\mathcal{O}^E\right)\Theta$$

κ-fixing: removes half of the fermionic d.o.f.

Convenient choices: $\Theta = \begin{pmatrix}\theta \\ 0\end{pmatrix}$ or $P^{\rm D7}_{-}\Theta = 0$

Consider first the simple case: $X_6 = T^6$

$S_4 = T^4 \subset T^6$

Then generalize to (a) Calabi-Yau, (b) Varying dilaton, (c) Other background fluxes, (d) Worldvolume fluxes

Warped Flat Space

With only 5-form fluxes and define: $P_{\pm}^{O3} = \frac{1}{2}\left(\mathbb{I} \pm \Gamma_{(6)} \otimes \sigma_2\right)$

$$\begin{aligned} \mathcal{O} &= 0 \\ \mathcal{D}_\mu &= \nabla_\mu + \frac{1}{8}\not{F}_5^{\text{int}}\Gamma_\mu i\sigma_2 = \partial_\mu - \frac{1}{4}\Gamma_\mu \not{\partial} \ln Z P_+^{O3} \\ \mathcal{D}_m &= \nabla_m + \frac{1}{8}\not{F}_5^{\text{int}}\Gamma_m i\sigma_2 = \partial_m + \frac{1}{8}\partial_m \ln Z - \frac{1}{4}\not{\partial} \ln Z \Gamma_m P_+^{O3} \end{aligned}$$

Pullback to the worldvolume:

$$\Gamma^\mu \mathcal{D}_\mu + \Gamma^a \mathcal{D}_a + \frac{1}{2}\mathcal{O} = \not{\partial}_4^{\text{ext}} + \not{\partial}_4^{\text{int}} + \left(\not{\partial}_4^{\text{int}} \ln Z\right)\left(\frac{1}{8} - \frac{1}{2}P_+^{O3}\right)$$

κ -fixing $\Theta = \begin{pmatrix} \theta \\ 0 \end{pmatrix}$ gives the Dirac action:

$$\begin{aligned} S_{\text{D7}}^{\text{fer}} &= \tau_{\text{D7}} e^{\Phi_0} \int_{\mathbb{R}^{1,3}} d^4x \int_{\mathbf{T}^4} \text{dvol}_{\mathbf{T}^4}\, \bar{\theta} \not{D}^w\, \theta \\ \not{D}^w &= \not{\partial}_4^{\text{ext}} + \not{\partial}_4^{\text{int}} - \frac{1}{8}\left(\not{\partial}_4^{\text{int}} \ln Z\right)\left(1 + 2\Gamma_{\text{Extra}}\right) \end{aligned}$$

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\end{aligned}
$$

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$$
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\not{D}^w &= \not{\partial}_4^{\text{ext}} + \not{\partial}_4^{\text{int}} - \frac{1}{8}\left(\not{\partial}_4^{\text{int}} \ln Z\right)(1 + 2\Gamma_{\text{Extra}})
\end{aligned}
$$

F_5

Decompose the 10D MW spinor:

$$\theta \;=\; \chi + B^* \chi^* \qquad \chi \;=\; \theta_{4D} \otimes \theta_{6D}$$

Fermion mass eigenstates:

$$\Gamma_{(4)} \left[\partial\!\!\!/_{\mathbf{T}^4} - \frac{1}{8} \left(\partial\!\!\!/_{\mathbf{T}^4} \ln Z \right) \left(1 + 2\Gamma_{\mathrm{Extra}} \right) \right] \theta^{\zeta}_{6D} \;=\; Z^{1/2} m_{\zeta} (B_6 \theta^{\zeta}_{6D})^*$$

4D zero modes:

$$\theta^0_{6D} \;=\; Z^{-1/8} \eta_- \qquad \text{for} \quad \Gamma_{\mathrm{Extra}}\, \eta_- \;=\; -\eta_-$$ Wilsonlino

$$\theta^0_{6D} \;=\; Z^{3/8} \eta_+ \qquad \text{for} \quad \Gamma_{\mathrm{Extra}}\, \eta_+ \;=\; \eta_+$$ Gaugino, Modulino

c.f. $\theta^0_{6D} = Z^{1/8} \eta$ Acharya, Benini, Valandro

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c.f. $\theta_{6D}^0 = Z^{1/8} \eta$ Acharya, Benini, Valandro

Kinetic terms:

$$S_{\mathrm{D7}}^{\mathrm{fer}} \;=\; \tau_{\mathrm{D7}}\, e^{\Phi_0} \int_{\mathbb{R}^{1,3}} d^4x\, \bar{\theta}_{4D} \not{\partial}_{\mathbb{R}^{1,3}} \theta_{4D} \int_{\mathbf{T}^4} d\mathrm{vol}_{\mathbf{T}^4}\, \eta_-^\dagger \eta_-$$

$$S_{\mathrm{D7}}^{\mathrm{fer}} \;=\; \tau_{\mathrm{D7}}\, e^{\Phi_0} \int_{\mathbb{R}^{1,3}} d^4x\, \bar{\theta}_{4D} \not{\partial}_{\mathbb{R}^{1,3}} \theta_{4D} \int_{\mathbf{T}^4} d\mathrm{vol}_{\mathbf{T}^4}\, Z \eta_+^\dagger \eta_+$$

Open String Bosons

Expand the DBI+CS action to quadratic order:
gauge bosons, wilson lines, moduli wavefunctions are flat.

4D Kinetic terms for bosons and fermions match:

e.g.

$$f_{D7} = \left(8\pi^3 k^2\right)^{-1} \int_{\mathbf{T}^4} \frac{\mathrm{dvol}_{\mathbf{T}^4}}{\sqrt{\hat{g}_{\mathbf{T}^4}}} \left(Z\sqrt{\hat{g}_{\mathbf{T}^4}} + iC_4^{\mathrm{int}}\right) (\alpha^0)^2$$

$$S^{\mathrm{fer}}_{\mathrm{D7}} = \tau_{\mathrm{D7}}\, e^{\Phi_0} \int_{\mathbb{R}^{1,3}} d^4x\, \bar{\theta}_{4D} \not{\partial}_{\mathbb{R}^{1,3}} \theta_{4D} \int_{\mathbf{T}^4} \mathrm{dvol}_{\mathbf{T}^4}\, Z \eta_+^\dagger \eta_+$$

More generally, fields descend from the same multiplet:

$$\int_{\mathbb{R}^{1,3}} \mathrm{d}^4x \bar{\phi} D\phi \int_{\mathrm{int}} \mathrm{dvol}_{\mathrm{int}}\, Z^q \bar{\eta}\eta$$

wavefunction $\sim Z^p$

Comparison to RS

RS			D7		
4D Field	p	q	4D Field	p	q
gauge boson	0	$1/4$	gauge boson/modulus	0	1
gaugino	$3/8$		gaugino/modulino	$3/8$	
matter scalar	$(3-2c)/8$	$(1-c)/2$	Wilson line	0	0
matter fermion	$(2-c)/4$		Wilsonino	$-1/8$	

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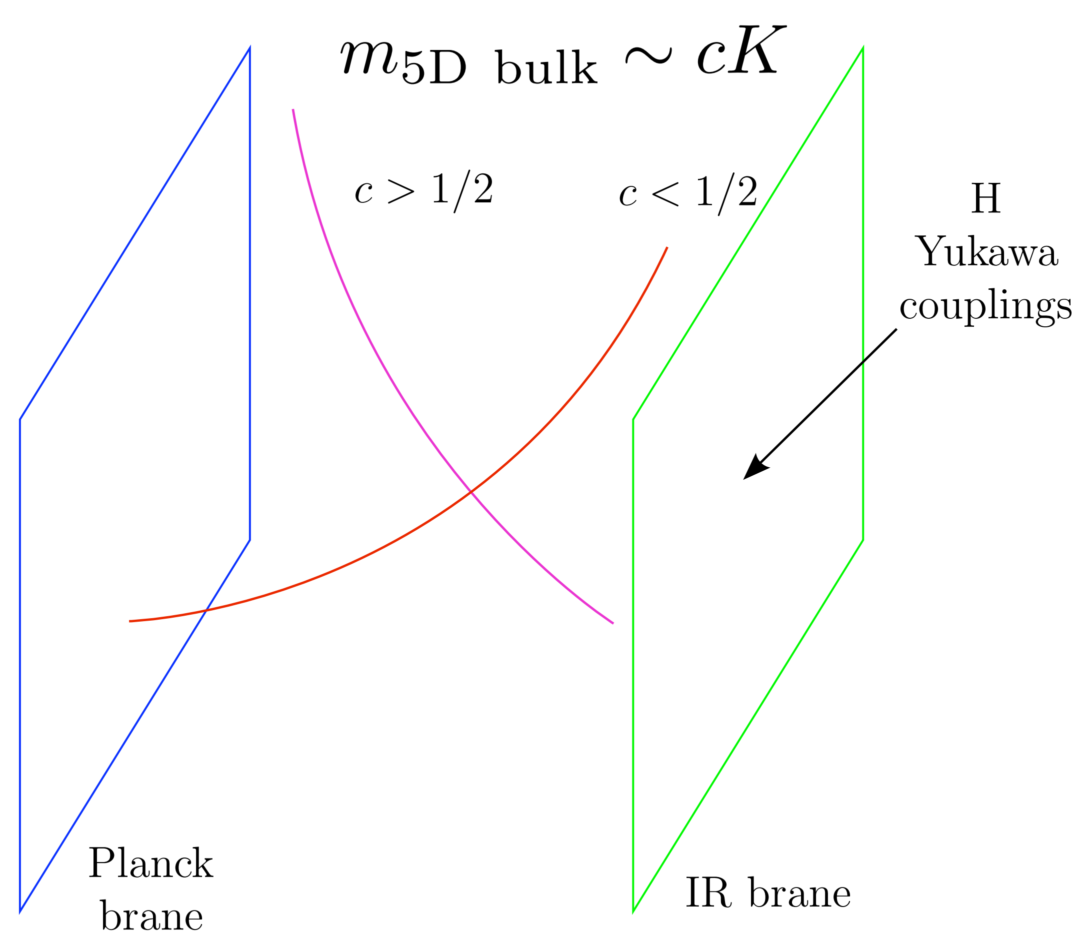

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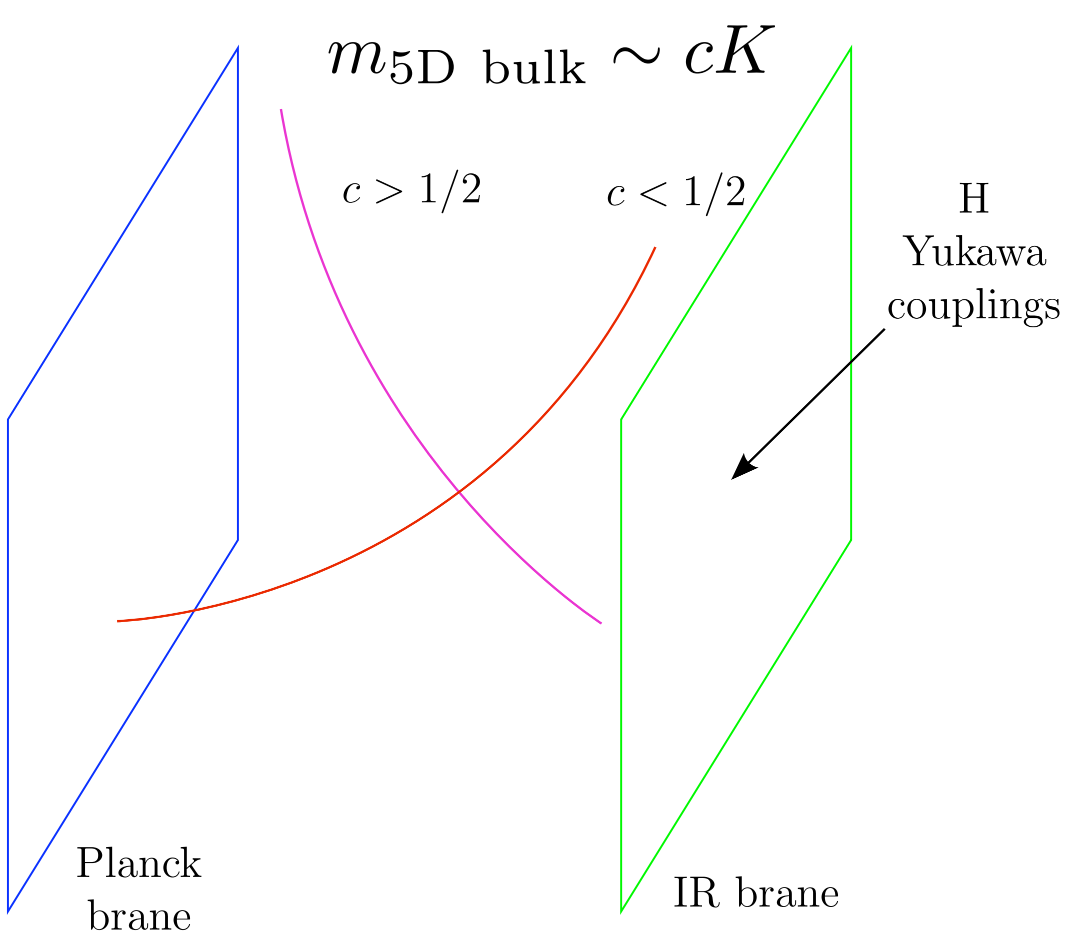

* 5-form fluxes

Comparison to RS

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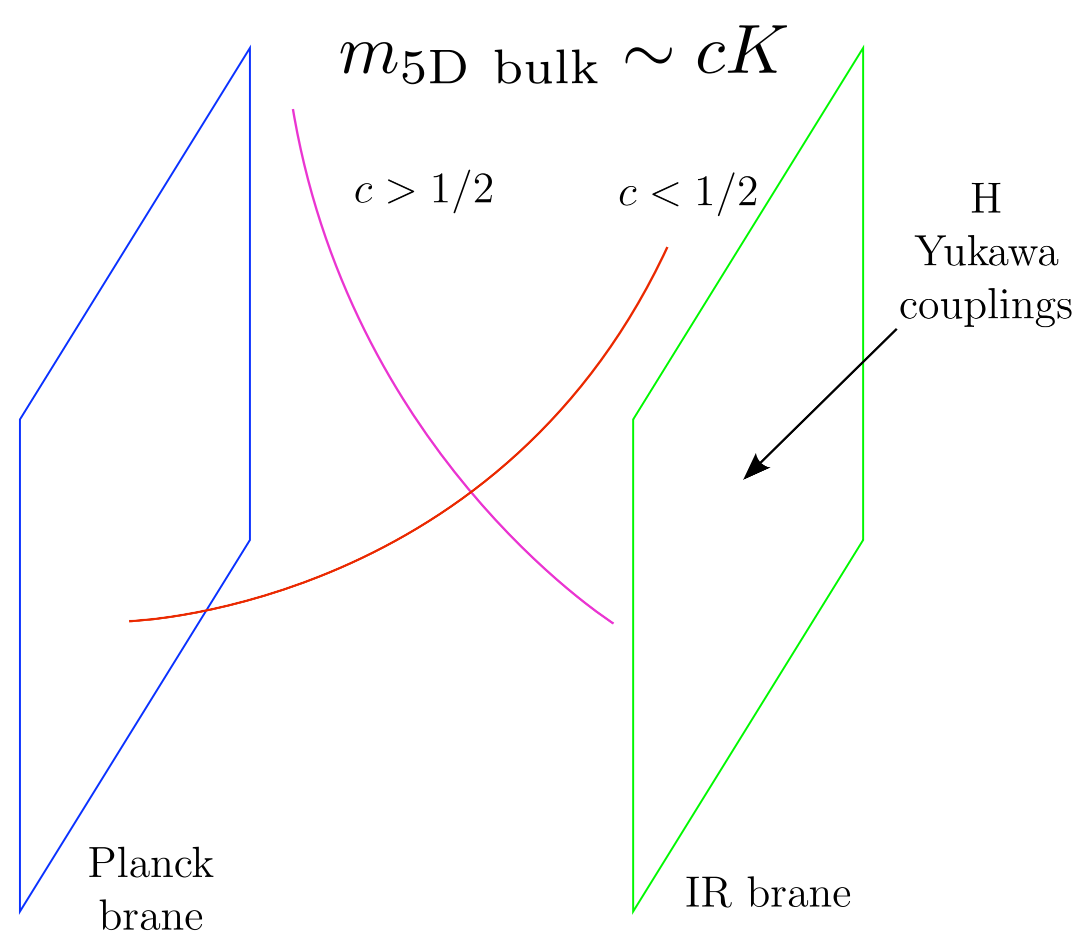

- 5-form fluxes
- 3-form fluxes

Comparison to RS

RS			D7		
4D Field	p	q	4D Field	p	q
gauge boson	0	$1/4$	gauge boson/modulus	0	1
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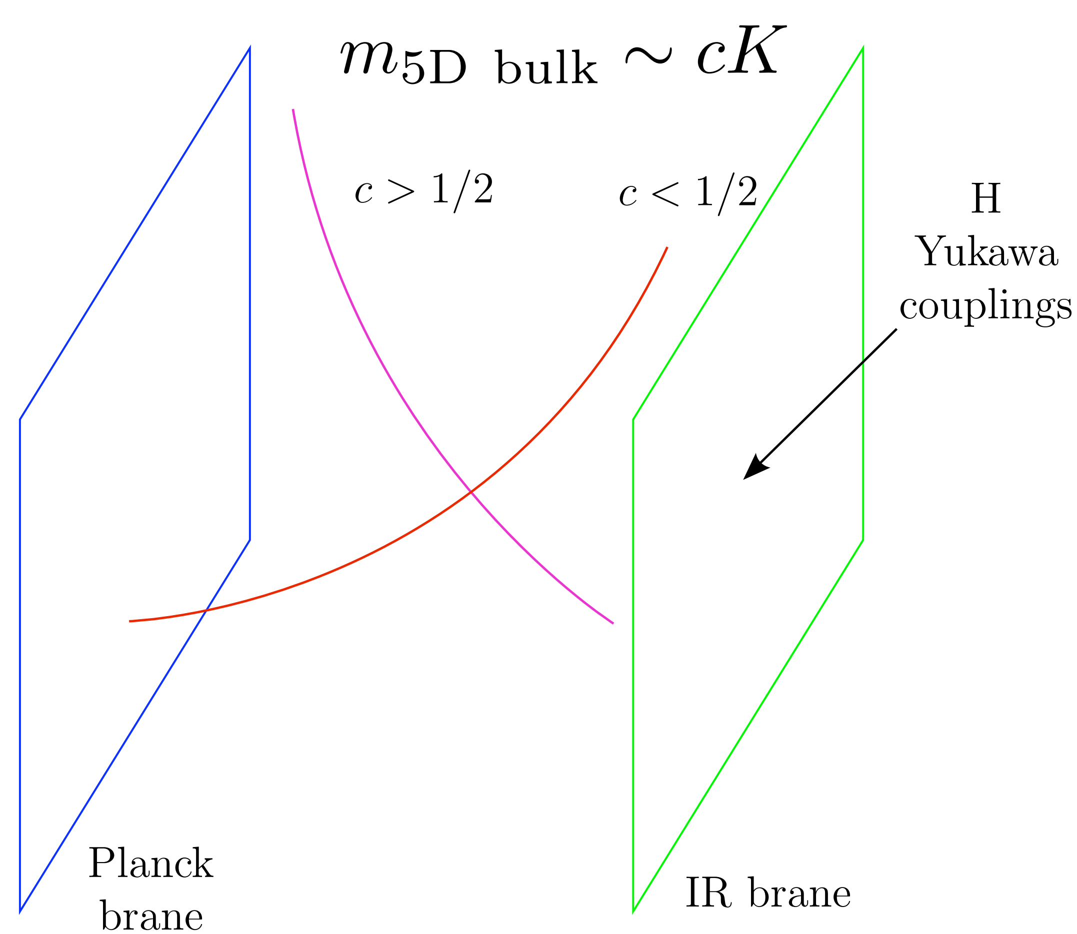

- 5-form fluxes
- 3-form fluxes
- D7-brane worldvolume flux

More Gauge Fixing

10D MW spinors: $\bar{\theta}\Gamma_{a_1\cdots a_n}\theta = 0 \text{ for } n \neq 3, 7$

Consider kinetic term: [turning off warping, fluxes]

$$\tau_{\rm D7}\int d^8\xi\, e^{\Phi}\bar{\theta}\Gamma^{\alpha}\partial_{\alpha}\theta$$

A constant MW spinor $\Gamma^{\alpha}\partial_{\alpha}\eta = 0$ minimizes the action

so does $S = \tau_{\rm D7}\int d^8\xi\, e^{\Phi}\left(f^2\bar{\eta}\not{\partial}\eta + f\bar{\eta}\not{\partial}f\eta\right)$

Ambiguity in EOM: $\Gamma^{\alpha}\left(\partial_{\alpha} - \partial_{\alpha}\ln f\right)\theta = 0$

Analogous to static gauge, choose superspace coordinates of D7 in non Grassmann-odd directions.

Bandos, Sorokin

More Gauge Fixing

After gauge fixing, the EOM becomes:

$$P_{-}^{\mathrm{D7}}\left(\Gamma^{\alpha}\mathcal{D}_{\alpha}^{E}+\frac{1}{2}\mathcal{O}^{E}\right)\Theta=0$$

Bandos, Sorokin

Warp factors cancel out in 4D SUSY variations:

$$\begin{aligned}\delta_{\epsilon}Y^{i} &= \bar{\epsilon}\Gamma^{i}\theta\\ \delta_{\epsilon}A_{\alpha} &= \bar{\epsilon}\Gamma_{\alpha}\theta\end{aligned}$$

Such gauge fixing should apply to non-SUSY setup.

Alternative κ-Fixing

Another κ-fixed choice: $P_{-}^{\mathrm{D7}}\Theta = 0$

$$\Theta^0_{6D} = \frac{Z^{-1/8}}{\sqrt{2}}\begin{pmatrix} \eta_- \\ i\eta_- \end{pmatrix} \quad \text{for} \quad \Gamma_{\text{Extra}}\eta_- = -\eta_- \qquad \text{Wilsonini}$$

$$\Theta^0_{6D} = \frac{Z^{3/8}}{\sqrt{2}}\begin{pmatrix} i\eta_+ \\ \eta_+ \end{pmatrix} \quad \text{for} \quad \Gamma_{\text{Extra}}\eta_+ = \eta_+ \qquad \text{gaugino + modulino}$$

More transparent what the zero modes correspond:

$$\Theta = Z^{-1/8}\Xi_- \quad \text{with} \quad P_+^{D3}\Xi_- = P_-^{D7}\Xi_- = 0$$

$$\Theta = Z^{3/8}\Xi_+ \quad \text{with} \quad P_-^{D3}\Xi_+ = P_-^{D7}\Xi_+ = 0$$

$$P_{\pm}^{D3} = \frac{1}{2}\left(\mathbb{I} \pm \Gamma_{(4)} \otimes \sigma_2\right)$$

The killing bispinors preserved by D3 should go like:

$$\epsilon \sim Z^{-1/8}\Xi_-$$

Generalizations

Calabi-Yau: $\nabla_m^{\rm CY}\,\eta_-^{\rm CY} = 0 \qquad \eta_+^{\rm CY} = (B_6\eta_-^{\rm CY})^*$

Killing bispinor: $\epsilon = \epsilon_{4D}\otimes Z^{-1/8}\begin{pmatrix}\eta_-^{\rm CY}\\ i\eta_-^{\rm CY}\end{pmatrix} - iB_4^*\epsilon_{4D}^*\otimes Z^{-1/8}\begin{pmatrix}i\eta_+^{\rm CY}\\ \eta_+^{\rm CY}\end{pmatrix}$

Gaugino: $\Theta = \theta_{4D}\otimes\frac{Z^{3/8}}{\sqrt{2}}\begin{pmatrix}i\eta_-^{\rm CY}\\ \eta_-^{\rm CY}\end{pmatrix} - iB_4^*\theta_{4D}^*\otimes\frac{Z^{3/8}}{\sqrt{2}}\begin{pmatrix}\eta_+^{\rm CY}\\ i\eta_+^{\rm CY}\end{pmatrix}$

Wilsonlini & Modulini: spinors annihilated by $\Gamma^a\nabla_a^{\rm CY}$:

$$\eta_W = W_a\Gamma^{z^a}\eta_-^{\rm CY} \quad \text{and} \quad \eta_m = m_{ab}\Gamma^{z^a z^b}\eta_-^{\rm CY}$$

harmonic (1,0) and (2,0) forms on S_4

$$\Theta_{6D} \sim Z^{-1/8},\ Z^{3/8} \text{ respectively}$$

Also generalized to 3-form fluxes and varying dilaton.

Magnetized D7-branes

Introducing chirality, dual to intersecting branes

BPS: $\mathcal{F} = -*_{\mathcal{S}_4}\mathcal{F}$

c.f. Blumenhagen, Cvetic, Langacker, GS

The Dirac action:

$$S^{\text{fer}}_{\text{D7}} = \tau_{\text{D7}} \int \mathrm{d}^8\xi\, \mathrm{e}^{\Phi} \sqrt{|\det\, M|}\, \bar{\Theta} P^{\text{D7}}_{-}(\mathcal{F}) \left(\Gamma^{\mu}\mathcal{D}_{\mu} + (\mathcal{M}^{-1})^{ab}\Gamma_a\left(\mathcal{D}_b + \frac{1}{8}\Gamma_b\mathcal{O}\right)\right)\Theta$$

where $M = P[G] + e^{-\Phi/2}\mathcal{F}$ $\quad \mathcal{M} = P[G] + e^{-\Phi/2}\mathcal{F}\sigma_3$

$$P^{\text{D7}}_{\pm}(\mathcal{F}) = \frac{1}{2}\left(\mathbb{I} \pm \Gamma^{\mathcal{F}}_{(8)} \otimes \sigma_2\right) \qquad \Gamma^{\mathcal{F}}_{(8)} = \Gamma_{(8)}\sqrt{\left|\frac{\det P[G]}{\det M}\right|}\left(\mathbb{I} - \frac{1}{2}e^{-\Phi/2}\not{\mathcal{F}} \otimes \sigma_3 + \frac{1}{8}e^{-\Phi}\not{\mathcal{F}}^2\right)$$

In the gauge: $P^{\text{D7}}_{-}(\mathcal{F})\Theta = 0$

$$\not{D}^{w} = \sqrt{\frac{\det M_{\mathcal{S}_4}}{\det g_{\mathcal{S}_4}}}\left[\not{\partial}^{\text{ext}}_{4} + (\mathcal{M}^{-1}_{\mathcal{S}_4})^{ab}\Gamma_a\left(\nabla^{\text{CY}}_b + \partial_b \ln Z\left(\frac{1}{8} - \frac{1}{2}P^{O3}_{+}\right)\right)\right]$$

Magnetized D7-branes

Worldvolume flux rotates the bispinor: $P_-^{\mathrm{D7}}(\mathcal{F})\Theta = 0$

$$\Theta = \begin{pmatrix} \Lambda(-\mathcal{F})^{1/2} & \\ & \Lambda(\mathcal{F})^{1/2} \end{pmatrix} \Theta' \qquad \text{with} \qquad P_-^{\mathrm{D7}}\Theta' = 0$$

Bergshoeff, Kallosh, Ortin, Papadopoulos

In general: $\Lambda(\mathcal{F}) \in Spin(4) = SU(2)_1 \times SU(2)_2$

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$$\begin{array}{c} SU(2)_1 \\ \bigcap \\ U(2) \end{array}$$

BPS: $\Lambda(\mathcal{F}) \in SU(2)_1$ $\quad U(2)$ holonomy group of S_4

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$\bigcap$

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$$P_-^{O3}\Theta' = 0 \quad (\mathbf{1},\mathbf{2}) \qquad \Theta = Z^{3/8}\left[\theta_{4D}\otimes\frac{1}{\sqrt{2}}\begin{pmatrix} i\eta_-^{\rm CY} \\ \eta_-^{\rm CY}\end{pmatrix} - iB_4^*\theta_{4D}^*\otimes\frac{1}{\sqrt{2}}\begin{pmatrix} \eta_+^{\rm CY} \\ i\eta_+^{\rm CY}\end{pmatrix}\right]$$

$$P_+^{O3}\Theta' = 0 \quad (\mathbf{2},\mathbf{1}) \qquad \Theta = Z^{-1/8}\frac{1}{4}(\mathcal{M}_{\mathcal{S}_4}^{-1})^{ab}\Gamma_a\Gamma_b\left[B_4^*\theta_{4D}^*\otimes\frac{1}{\sqrt{2}}\begin{pmatrix} i\eta_W \\ \eta_W\end{pmatrix} - i\theta_{4D}\otimes\frac{B_6}{\sqrt{2}}\begin{pmatrix} \eta_W^* \\ i\eta_W^*\end{pmatrix}\right]$$

Warped EFT

Transform to 4D Einstein frame: $\eta_{\mu\nu} \to \frac{\alpha'^3}{\mathcal{V}_{\mathrm{W}}}\eta_{\mu\nu}$

Gauge kinetic function:

$$f_{\mathrm{D7}} = \left(8\pi^3 k^2\right)^{-1} \int_{\mathcal{S}_4} \frac{\mathrm{dvol}_{\mathcal{S}_4}}{\sqrt{\hat{g}_{\mathcal{S}_4}}} \left(Z\sqrt{\hat{g}_{\mathcal{S}_4}} + iC_4^{\mathrm{int}}\right)\left(\alpha^0\right)^2$$

For Kahler potential, consider first the unwarped case.

D7-moduli:

Jockers, Louis

$$\sigma\left(x,y\right) = \zeta^A\left(x\right)s_A\left(y\right) + \bar{\zeta}^{\bar{A}}\bar{s}_{\bar{A}}\left(y\right) \qquad \left\{s_A\right\} : \mathcal{S}_4 \to \mathcal{S}_4'$$

have Einstein frame kinetic term:

$$i\tau_{\mathrm{D7}}\int_{\mathbb{R}^{1,3}} \mathrm{e}^{\Phi}\mathcal{L}_{A\bar{B}}\,\mathrm{d}\zeta^A \wedge *_4\mathrm{d}\bar{\zeta}^{\bar{B}}$$

$$\mathcal{L}_{A\bar{B}} = \frac{\int_{\mathcal{S}_4} m_A \wedge m_{\bar{B}}}{\int_{X_6} \Omega^{\mathrm{CY}} \wedge \bar{\Omega}^{\mathrm{CY}}}$$

$$\left\{m_A\right\} : m_A = \iota_{s_A}\Omega^{\mathrm{CY}}$$

Warped EFT

With warping, the D7 moduli kinetic term:

$$\mathcal{L}_{A\bar{B}} \to \mathcal{L}^{\mathrm{w}}_{A\bar{B}} = \frac{\int_{\mathcal{S}_4} Z\, m_A \wedge m_{\bar{B}}}{\int_{X_6} Z\, \Omega^{\mathrm{CY}} \wedge \bar{\Omega}^{\mathrm{CY}}}$$

Now combine with the closed string results.

From STUD, the axio-dilaton has kinetic term:

$$-\int_{\mathbb{R}^{1,3}} \mathrm{d}^4x\, \mathcal{K}_{\bar{t}t}\, \partial^\mu \bar{t}\, \partial_\mu t \quad \text{where} \quad \mathcal{K}_{\bar{t}t} = \frac{1}{8\,(\mathrm{Im}\tau)^2\, \mathcal{V}_{\mathrm{W}}} \int_{X^6} \mathrm{d}^6y\, Z\, Y_0^2$$

unaffected by warping.

In the unwarped case: $S = t - \kappa_4^2 \tau_{\mathrm{D7}} \mathcal{L}_{A\bar{B}}$ Jockers, Louis

$$\mathcal{K} \ni \ln\left[-i\left(S - \bar{S}\right) - 2i\kappa_4^2 \tau_{\mathrm{D7}} \mathcal{L}_{A\bar{B}} \zeta^A \bar{\zeta}^{\bar{B}}\right]$$

Warped EFT

The warped corrected Kahler potential for S:

$$\mathcal{K} \ni \ln\left[-i\left(S^{\mathrm{w}} - \bar{S}^{\mathrm{w}}\right) - 2i\kappa_4^2\tau_{\mathrm{D7}}\mathcal{L}^{\mathrm{w}}_{A\bar{B}}\zeta^A\bar{\zeta}^{\bar{B}}\right] \qquad S^{\mathrm{w}} = t - \kappa_4^2\tau_{\mathrm{D7}}\mathcal{L}^{\mathrm{w}}_{A\bar{B}}\zeta^A\bar{\zeta}^{\bar{B}}$$

Wilson lines:

$$A_a \mathrm{d}A^a = w_I\,(x)\,W^I\,(y) + \overline{w}_{\bar{I}}\,(x)\,\overline{W}^{\bar{I}}\,(y) \qquad \left\{W^I\right\} : (1,0)\ \mathrm{forms}$$

has kinetic term in the unwarped case: Jockers, Louis

$$i\frac{2\tau_{\mathrm{D7}}k}{\mathcal{V}}\int_{\mathbb{R}^{1,3}} \mathcal{C}^{I\bar{J}}_{\alpha} v^{\alpha} \mathrm{d}w_I \wedge *_4 \mathrm{d}\bar{w}_{\bar{J}}$$

$$J^{\mathrm{CY}} = v^a\omega_a$$

$$\mathcal{C}^{I\bar{J}}_{\alpha} = \int_{\mathcal{S}_4} P\left[\omega_\alpha\right] \wedge W^I \wedge \overline{W}^{\bar{J}}$$

With warping, we found the kinetic term is modified:

$$i\frac{2\tau_{\mathrm{D7}}k}{\mathcal{V}_{\mathrm{W}}}\int_{\mathbb{R}^{1,3}} \mathcal{C}^{I\bar{J}}_{\alpha} v^{\alpha} \mathrm{d}w_I \wedge *_4 \mathrm{d}\bar{w}_{\bar{J}}$$

Warped EFT

In the single modulus case, without warping:

$$-3\ln\left(T_\Lambda + \bar{T}_\Lambda - 6i\kappa_4^2\tau_{D7}k^2 C_\Lambda^{I\bar{J}} w_I \overline{w}_{\bar{J}}\right)$$ Jockers, Louis

This suggests to reproduce the warped kinetic terms:

$$T_\Lambda \to T_\Lambda^w$$

where $T_\Lambda^w v^\Lambda = \int Z\ J\wedge J\wedge J = \left(c + \frac{\mathcal{V}_W^0}{\mathcal{V}_{\rm CY}}\right)\mathcal{V}_{\rm CY}$ $\quad Z(x) = Z_0 + c(x)$

This is in agreement with closed string derivation:

$$-3\ln\left(-i(\rho-\overline{\rho}) + 2\,\frac{\mathcal{V}_W^0}{\mathcal{V}_{\rm CY}}\right)$$ where $Im(\rho) = c(x)$

Frey, Torroba, Underwood, Douglas

Summary

- Minimal Simple dS Solutions
- Warped EFT for Cosed strings
- Open String Wavefunctions & Warped EFT

THANKS