Duality Cascade in the Sky

(R.Bean, X.Chen, G.Hailu, S.H.Tye and JX, to appear)

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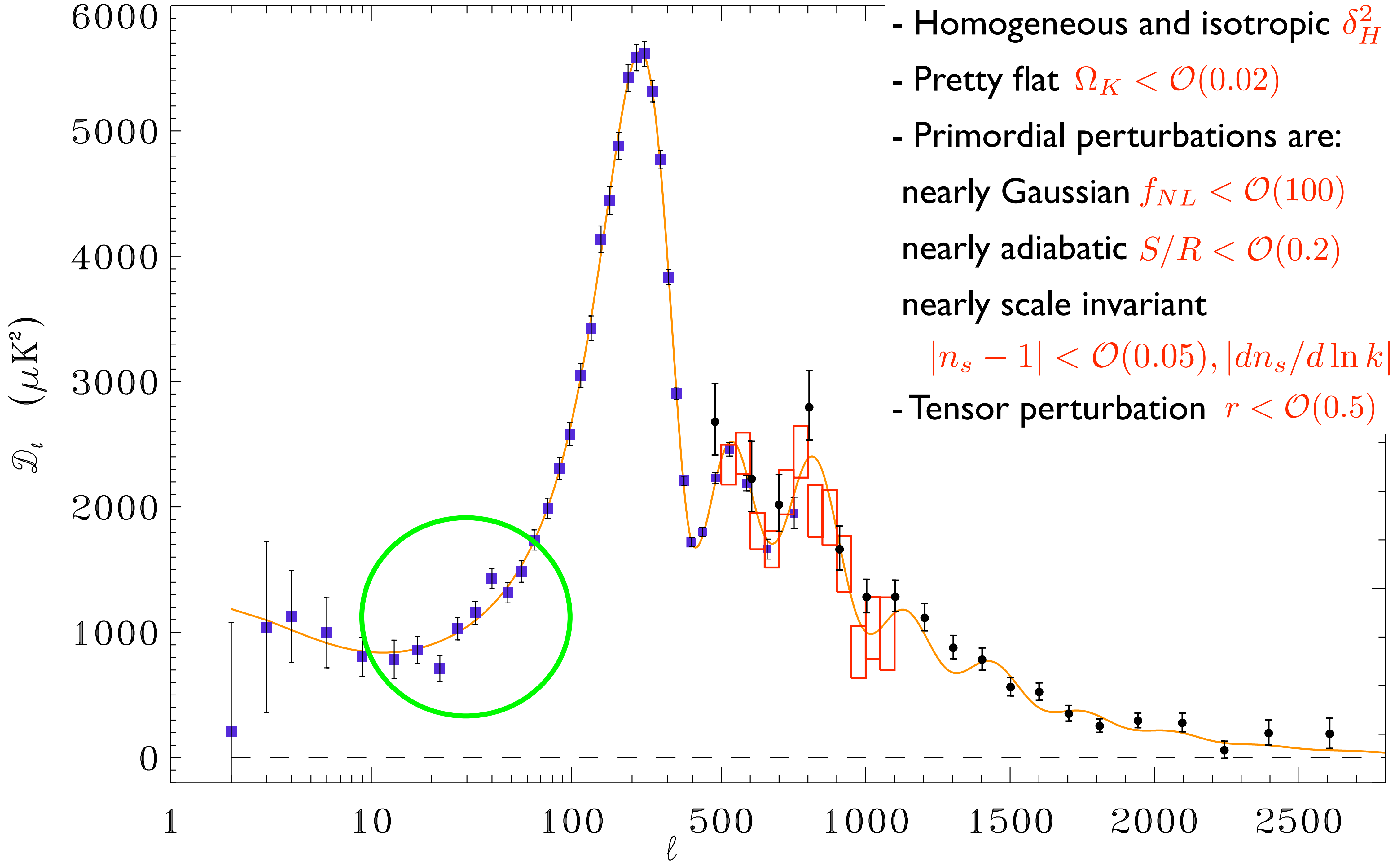
Our Current Understanding of the Early Universe



- Homogeneous and isotropic $\delta_H^2 = 10^{-10}$
- Pretty flat $\Omega_K < \mathcal{O}(0.02)$
- Primordial perturbations are:

nearly Gaussian $f_{NL} < \mathcal{O}(100)$

nearly adiabatic $S/R < \mathcal{O}(0.2)$

nearly scale invariant

$|n_s - 1| < \mathcal{O}(0.05), |dn_s/d\ln k| < \mathcal{O}(0.05)$

- Tensor perturbation $r < \mathcal{O}(0.5)$

- to explain the local feature in WMAP data, we need local feature in the slow-roll inflaton potential.

  (Adams, Ross, Sarkar 1997,
  Leach, Liddle, 2001
  Hunt, Sakar, 2004, 2007
  Adams, Creswell, Easther, 2001
  Peiris et al, 2003
  Covi et al, 2006, 2007)

$$V(\phi)(1+\delta(\phi))$$

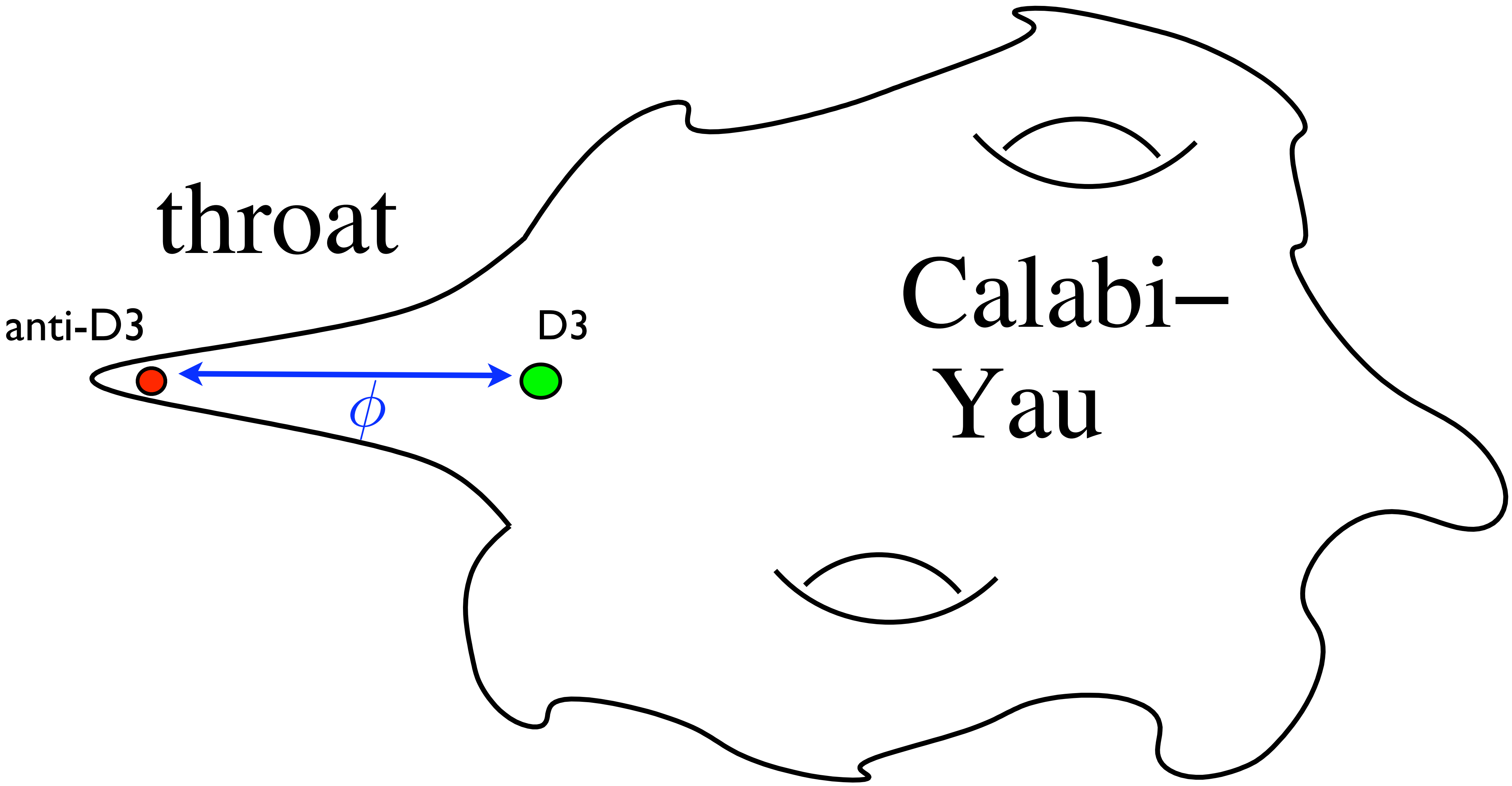
$$P_{\mathcal{R}} = \frac{H^2}{2\pi\dot{\phi}}$$

- local features in the potential also generates large non-gaussianity. (Chen, Easther, Lim, 2006)

$$f_{NL} \sim \epsilon,\ \eta'$$

- In brane inflation, local features arise both in slow-roll scenario and DBI scenario, due to gauge/gravity duality.

Brane Inflation (Dvali & Tye)



Brane Inflation (KKLMMT)

Consider D3 Branes moving in the $AdS_5 \times X_5$ background

$$ds^2 = h^2(r)(-dt^2 + a(t)^2 d\mathbf{x}^2) + h^{-2}(r)(dr^2 + r^2 ds^2_{X_5}),$$

in the UV region, $ds^2_{X5} = ds^2_{T^{1,1}}$

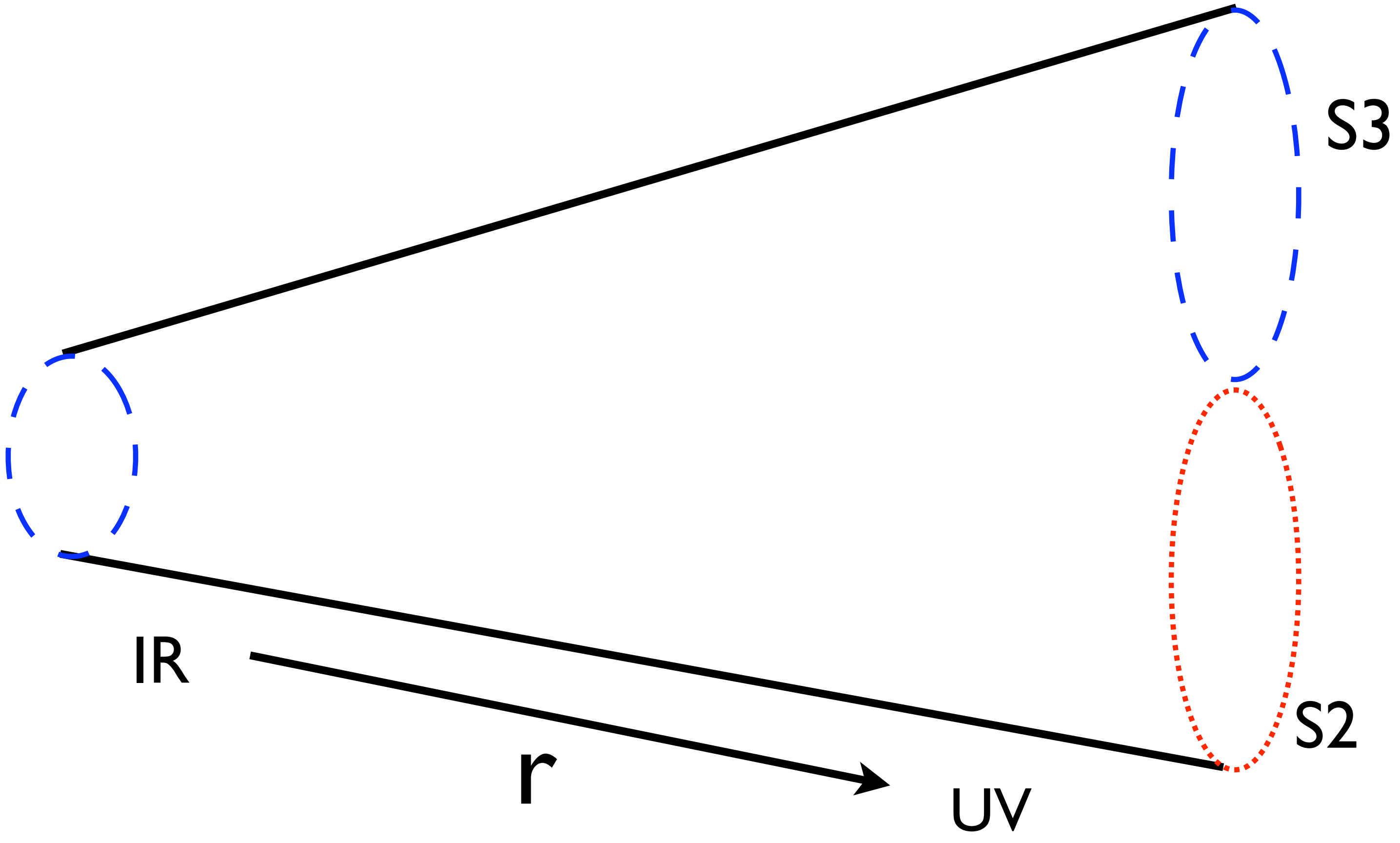
$$S = \int d^4x \sqrt{-g} \left[-e^{-\Phi} T(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + T(\phi) - V(\phi) \right]$$

$$T_3 \sim \frac{m_s^4}{g_s} = \frac{m_s^4}{\langle g_s \rangle} e^{-\Phi(r)}$$

$$T(\phi) = T_3 h^4(\phi)$$

$$\begin{aligned} V(\phi) &= \frac{\beta}{2} H^2 \phi^2 + V_{D\bar{D}}(\phi) \\ V_{D\bar{D}}(\phi) &= V_0 \left(1 - \frac{V_0}{4\pi^2 v} \frac{1}{\phi^4} \right) \\ V_0 &= 2T_3 h_A^4 \end{aligned}$$

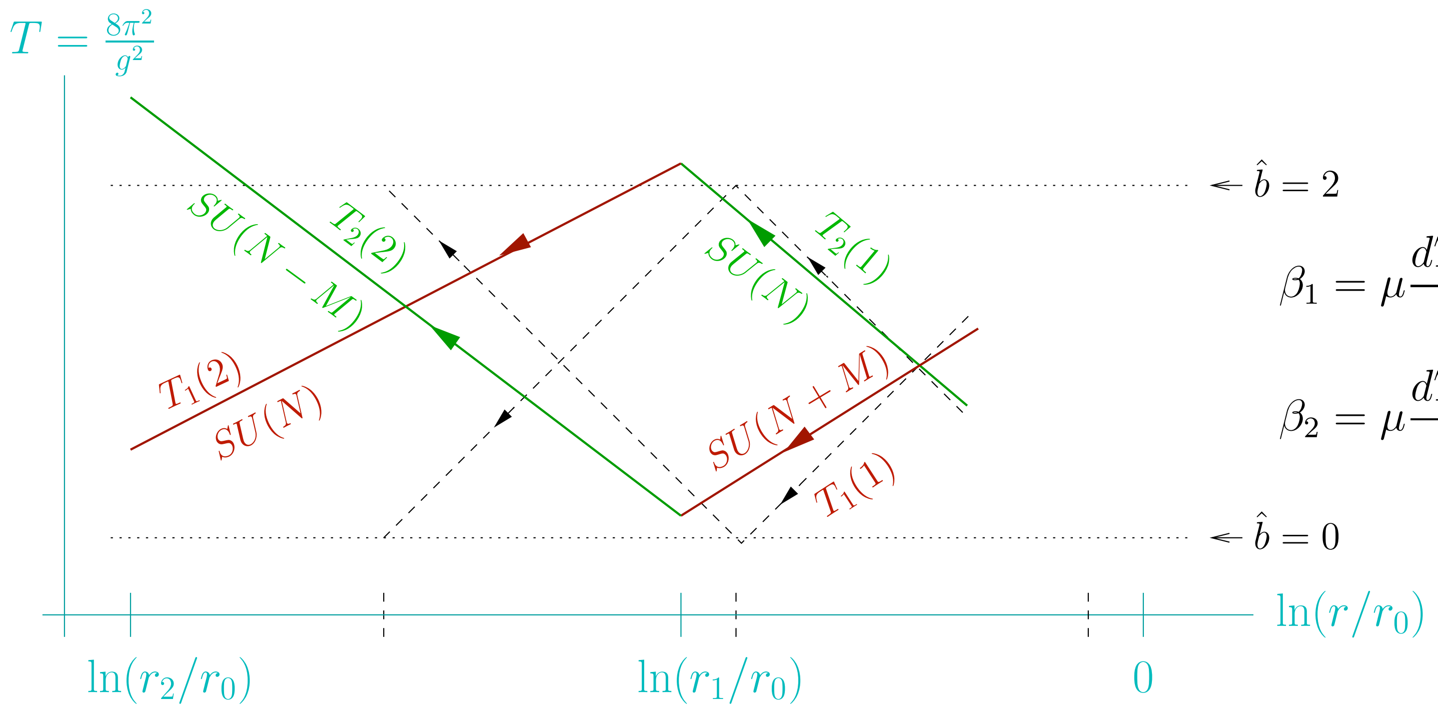
Klebanov-Strassler Throat



$$\frac{1}{2\pi\alpha'}\int_A F_3 = 2\pi M, \qquad \frac{1}{2\pi\alpha'}\int_B H_3 = -2\pi K$$

size of S3 at the bottom: $e^{-2\pi K/(Mg_s)}$

Gauge Gravity Duality

- gauge theory SU(N+M)XSU(N), N=KM, with bi-fundamental chiral fields A1, A2, B1, B2
- 

$$\beta_1 = \mu \frac{dT_1(1)}{d\mu} = 3(N+M) - 2N(1-\gamma_1(1)),$$

$$\beta_2 = \mu \frac{dT_2(1)}{d\mu} = 3N - 2(N+M)(1-\gamma_2(1)),$$

$$T_1 + T_2 = \frac{2\pi}{g_s e^{\Phi}},$$

$$T_1 - T_2 = \frac{2\pi}{g_s e^{\Phi}}(\hat{b} - 1) = \frac{2\pi}{g_s e^{\Phi}}(\bar{b}_2(\mathrm{mod}\, 2))$$

$$R = \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2} g_s^2 e^{2\Phi}(\partial C_0)^2 + \frac{1}{24} e^{-\Phi} H_3^2 + \frac{1}{24} g_s^2 e^{\Phi} \tilde{F}_3^2 .$$

The Slow-Roll Scenario with Running Dilaton

- expand the DBI action in non-relativistic limit

$$-e^{-\Phi}T(\phi)\sqrt{1-\frac{\dot{\phi}^2}{T(\phi)}}+T(\phi)-V(\phi)$$

$$=\quad \frac{1}{2}e^{-\Phi}\dot{\phi}^2-\boxed{\left[T(\phi)(e^{\Phi}-1)+V(\phi)\right]}$$

$T(\phi)=T_3h^4(\phi)$ sharp features in the warp factor translates into the effective potential

- $T(\phi)\sim\phi^4\gg\phi_A^4\sim V(\phi)$, too strong for slow-roll unless $\phi\gg M_{pl}$. However, $\frac{\Delta\phi}{M_{\rm pl}}\lesssim\frac{1}{\sqrt{KM}}\ll 1$

- Need $e^{-\Phi}\approx 1$, so that $T(\phi)(e^{-\Phi}-1)\ll V(\phi)$

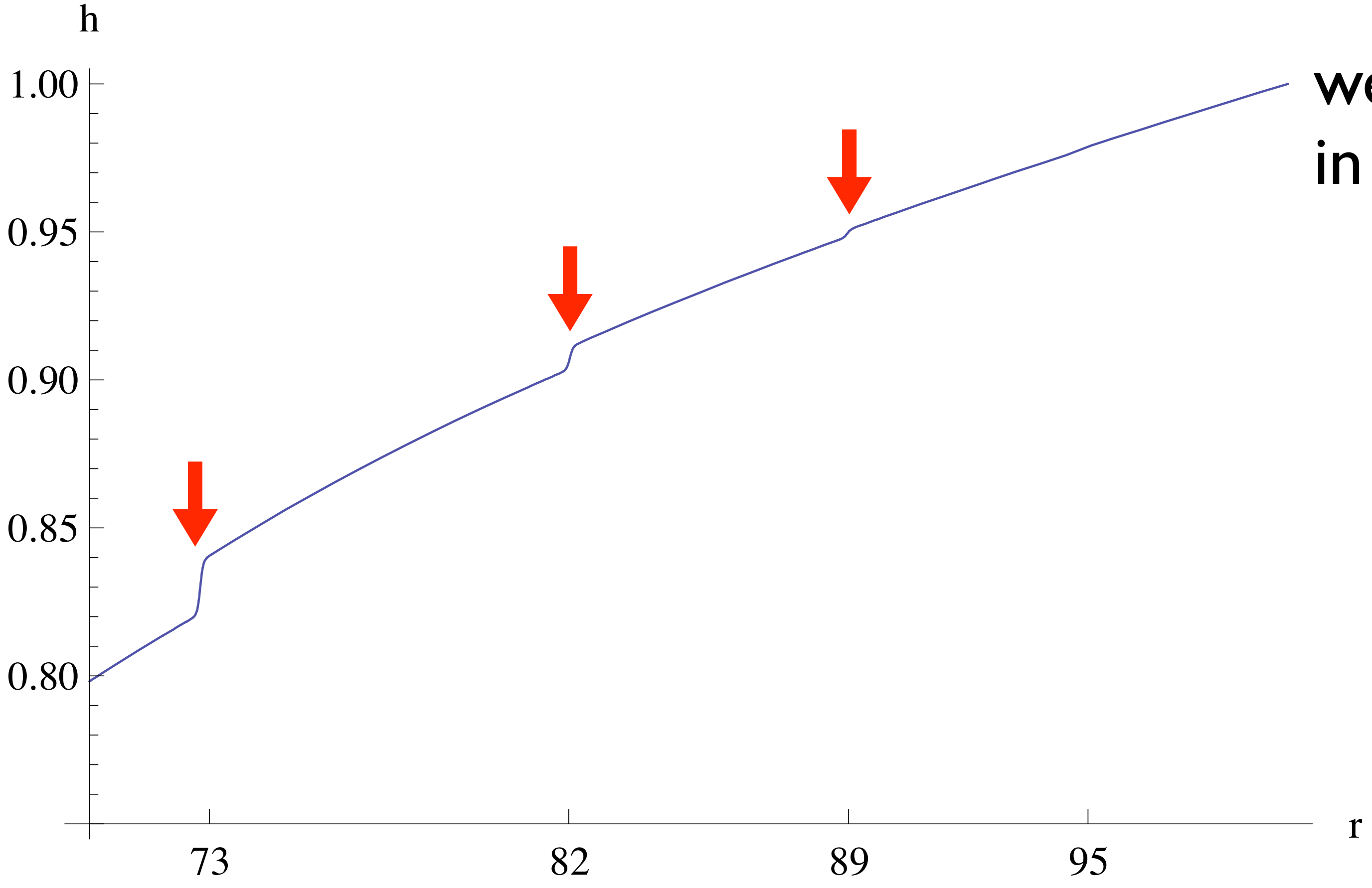
the warp factor

- A series of steps in the warp factor, spaced according to

$$\ln(r_{p+1}) - \ln(r_p) \simeq \frac{2\pi}{3g_s M}$$

$$h^4(r) \simeq \frac{r^4}{R_B^4}\frac{K}{p_l}(1+\Delta)\ , \quad \Delta = \sum_{p_i}^{K} \frac{3g_s M}{16\pi}\frac{1}{p^3}\left[1 + \tanh\left(\frac{r-r_p}{d_p}\right)\right]$$

$$R_B = \frac{27}{4}\pi g_s K M \alpha'^2$$



we expect sharp step features in the slow roll potential too

$$\begin{aligned} c &\equiv \frac{\Delta V}{V} \\ &= \frac{T(\phi)(e^{-\Phi}-1)}{V_0}\Delta \\ &\ll \Delta \end{aligned}$$

DBI Inflation

$$S = \int d^4x \sqrt{-g} \left[-e^{-\Phi} T(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + T(\phi) - V(\phi) \right]$$

- the exact equation of motion from the DBI action is

$$V(\phi) + T(\phi)(c_s^{-1} - 1) = 3H^2 \ ,$$

$$\ddot{\phi} - \frac{3}{2} \frac{T'(\phi)}{T(\phi)} \dot{\phi}^2 + 3H c_s^2 \dot{\phi} + T'(\phi) + c_s^3 [V'(\phi) - T'(\phi)] = 0$$

$$c_s = \gamma^{-1} = \sqrt{1 - \dot{\phi}^2 / T}$$

- $T(\phi)$ sets the speed limit, $\dot{\phi}^2 < T(\phi)$
- the brane moves relativistically, $c_s \ll 1, \gamma \gg 1$
- a sharp step in $T(\phi)$ → sharp change in c_s
- non-gaussian power spectrum $f_{NL} \sim c_s^{-2} \sim 10^2$

Observable effects (I): the power spectrum

- the power spectrum $\zeta(\tau,\mathbf{k}) = u(\tau,\mathbf{k})a(\mathbf{k}) + u^*(\tau,-\mathbf{k})a^\dagger(-\mathbf{k})$

$$v_k'' + \left(k^2 c_s^2 - \frac{z''}{z}\right) v_k = 0$$

$$v_k \equiv z u_k \ , \qquad z \equiv a\sqrt{2\epsilon}/c_s$$

$$\boxed{\begin{aligned} P_{\mathcal{R}}(k) &\equiv \frac{k^3}{2\pi^2}|u_{\mathbf{k}}|^2 \\ &= \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon c_s} \ , \end{aligned}}$$

$$v_k \sim e^{ik\tau},\ c_s^2 k^2 \gg z''/z \qquad v_k \sim z,\ c_s^2 k^2 \ll z''/z$$

- Define three parameters $\epsilon \equiv -\frac{\dot H}{H^2}\ , \quad \tilde\eta \equiv \frac{\dot\epsilon}{H\epsilon}\ , \quad s \equiv \frac{\dot c_s}{H c_s}\ .$

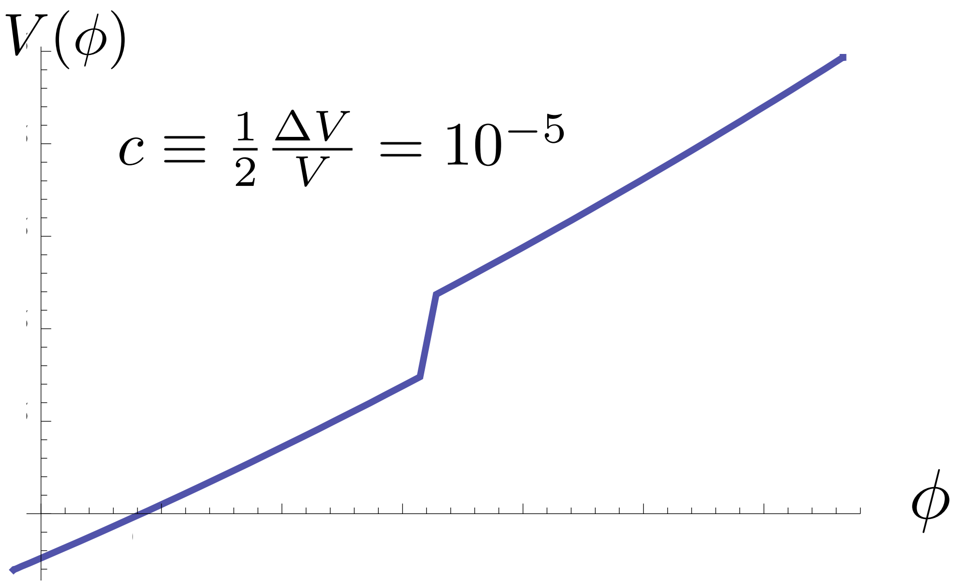
- the time dependent "mass" z''/z, encodes all the information of the background space-time

$$\frac{z''}{z} = 2a^2H^2\left(1 - \frac{\epsilon}{2} - \boxed{\frac{3\tilde\eta}{4}} - \frac{3s}{2} - \frac{\epsilon\tilde\eta}{4} + \frac{\epsilon s}{2} + \frac{\tilde\eta^2}{8} - \frac{\tilde\eta s}{2} + \frac{s^2}{2} + \frac{\dot{\tilde\eta}}{4H} - \boxed{\frac{\dot s}{2H}}\right)$$

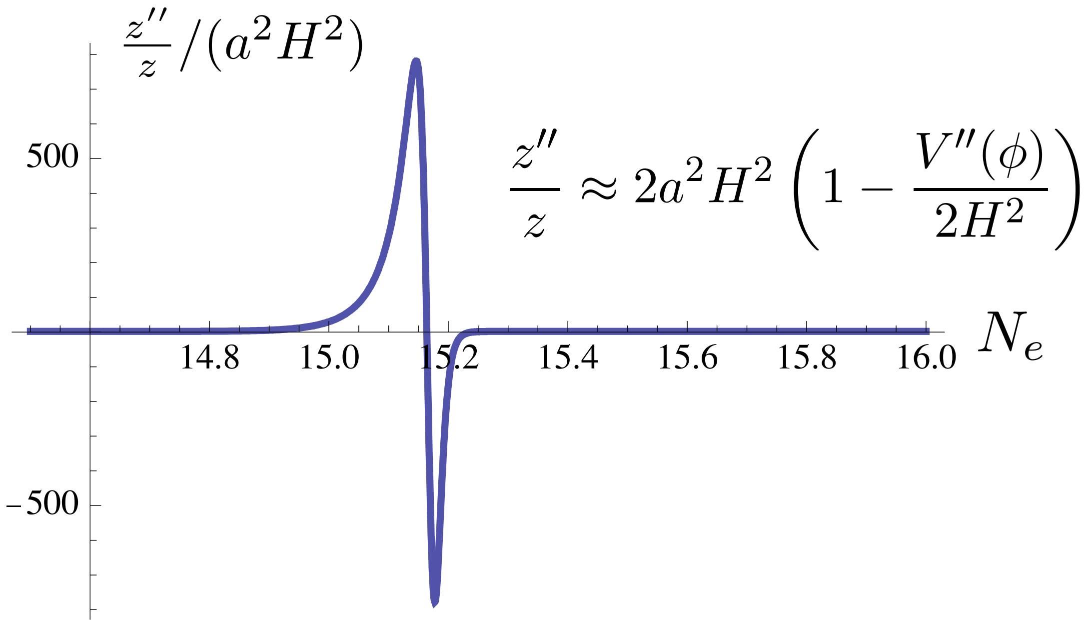
dominant in slow-roll

dominant in
DBI inflation

potential step in slow-roll

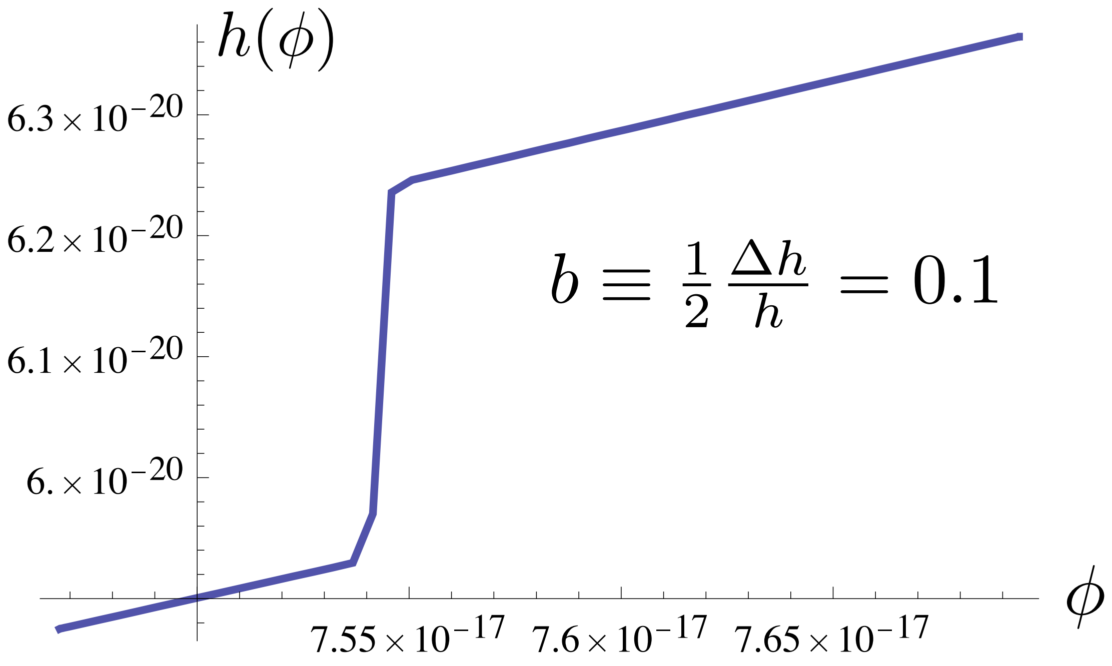


$$c \equiv \frac{1}{2}\frac{\Delta V}{V} = 10^{-5}$$

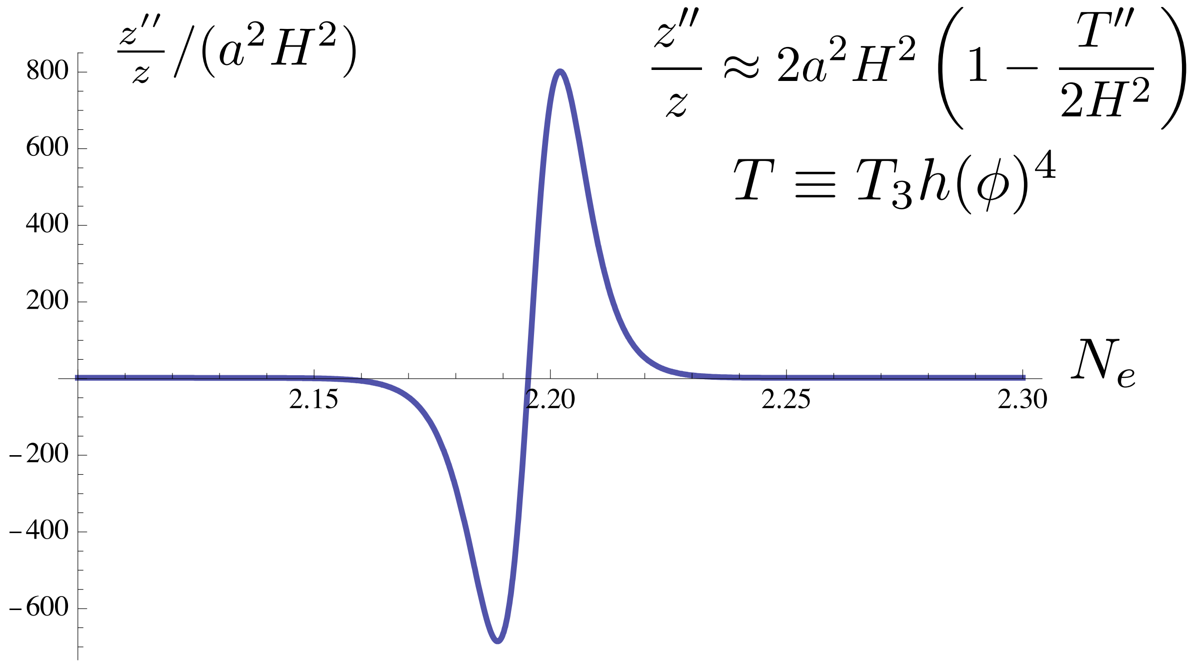


$$\frac{z''}{z} \approx 2a^2H^2\left(1 - \frac{V''(\phi)}{2H^2}\right)$$

warp factor step in IR-DBI



$$b \equiv \frac{1}{2}\frac{\Delta h}{h} = 0.1$$



$$\frac{z''}{z} \approx 2a^2H^2\left(1 - \frac{T''}{2H^2}\right)$$

$$T \equiv T_3 h(\phi)^4$$

Slow-roll power spectrum

- moving across the potential step generates efolds

$$\Delta N_e = H dt = H \frac{d}{\dot{\phi}} \sim \frac{d}{\sqrt{\epsilon}}$$

- $$\begin{aligned} \frac{z''}{z} &= 2a^2H^2\left(1 - \frac{V''}{H^2}\right) \\ &\sim 2a^2H^2\left(1 - \frac{c}{d^2}\frac{1}{H^2}\right) \\ &\sim 2a^2H^2\left(1 - \frac{c}{\epsilon}\frac{1}{\Delta N_e^2}\right) \end{aligned}$$

$$\frac{c}{\epsilon} \sim 1,\ \Delta N_e \ll 1$$

↕

observable effects

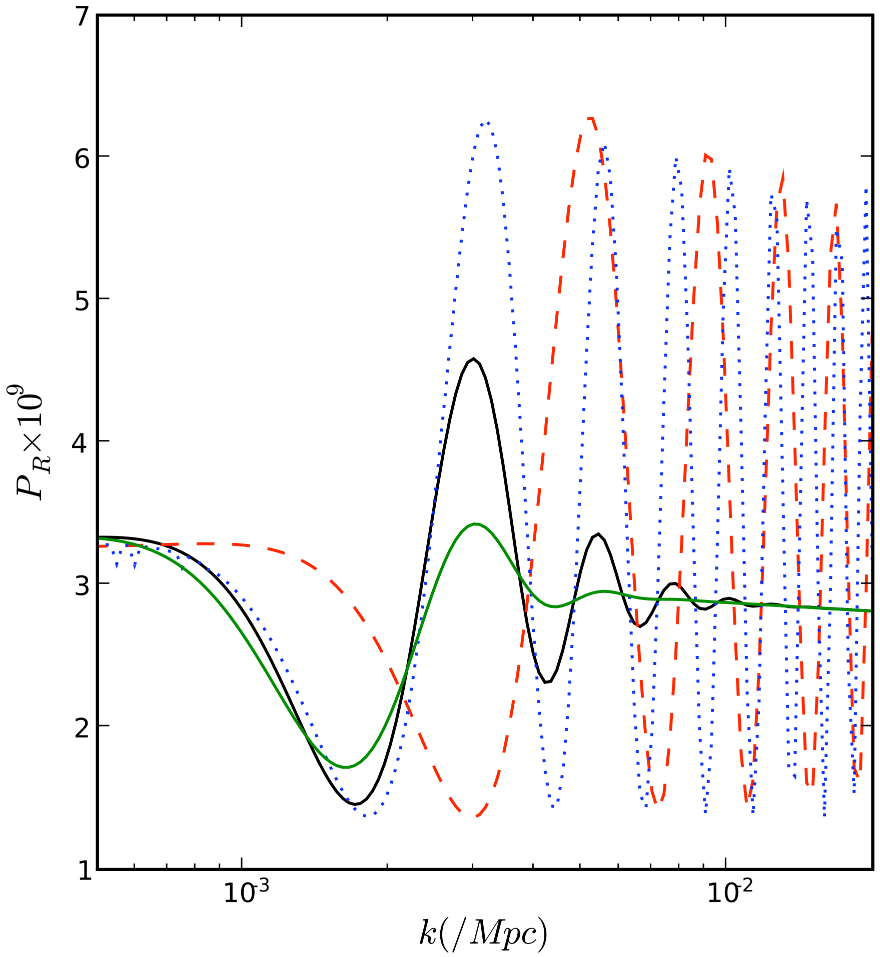
- in brane inflation models, ϵ is tiny

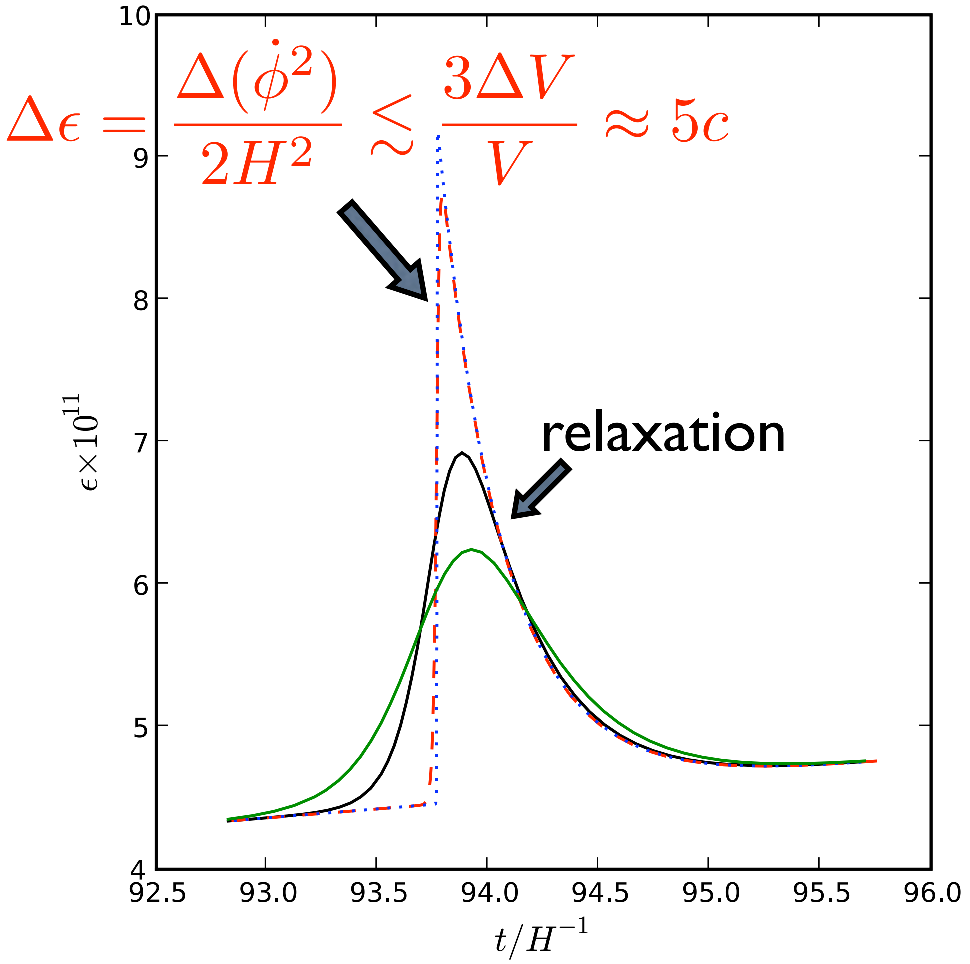
$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2c_s}\frac{\dot{\phi}^2}{H^2} \sim \left(\frac{d\phi}{dN_e}\right)^2$$

conservatively, take $(\Delta\phi)^2 = \frac{1}{KM} = 10^{-4}, \Delta N_e = 10^2,\ \epsilon \sim 10^{-8}$

In KKLMMT, $\epsilon \sim 10^{-11}$ → we are able to detect a potential step with $c \sim 10^{-11}$

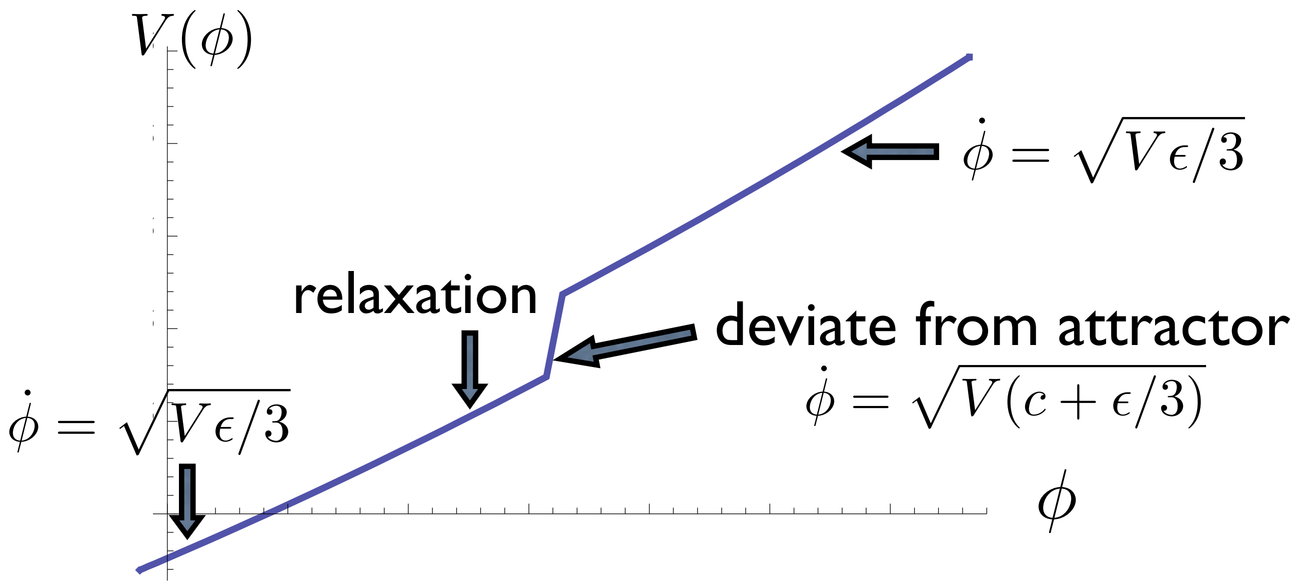
Numerical power spectrum in slow-roll





$$\Delta\epsilon = \frac{\Delta(\dot{\phi}^2)}{2H^2} \lesssim \frac{3\Delta V}{V} \approx 5c$$

relaxation



$\dot{\phi} = \sqrt{V\epsilon/3}$

relaxation

deviate from attractor

$\dot{\phi} = \sqrt{V(c+\epsilon/3)}$

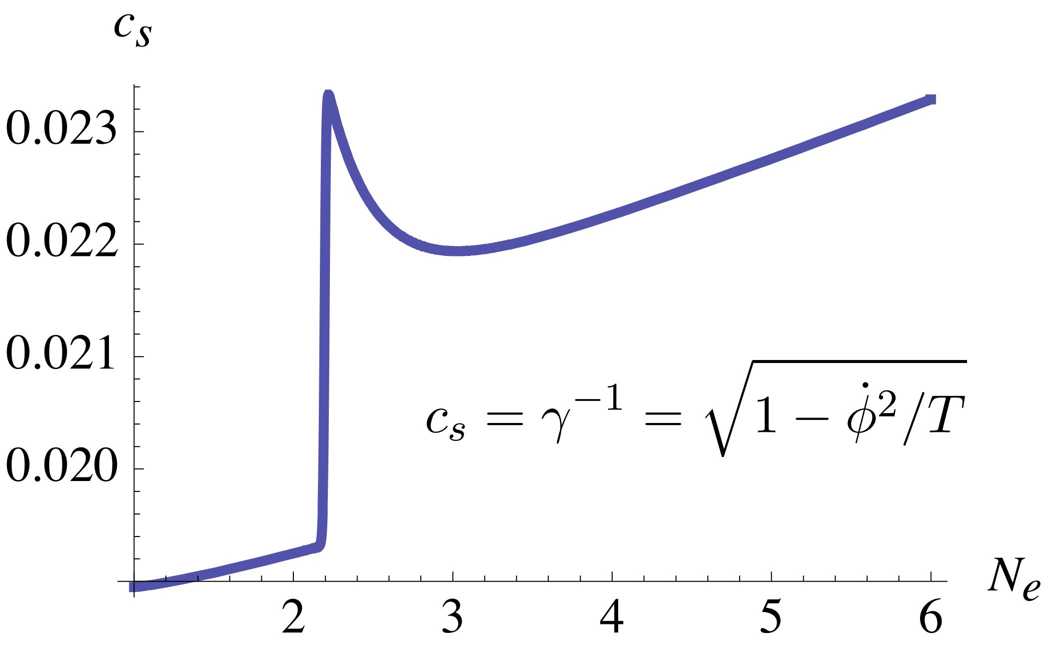
$\dot{\phi} = \sqrt{V\epsilon/3}$

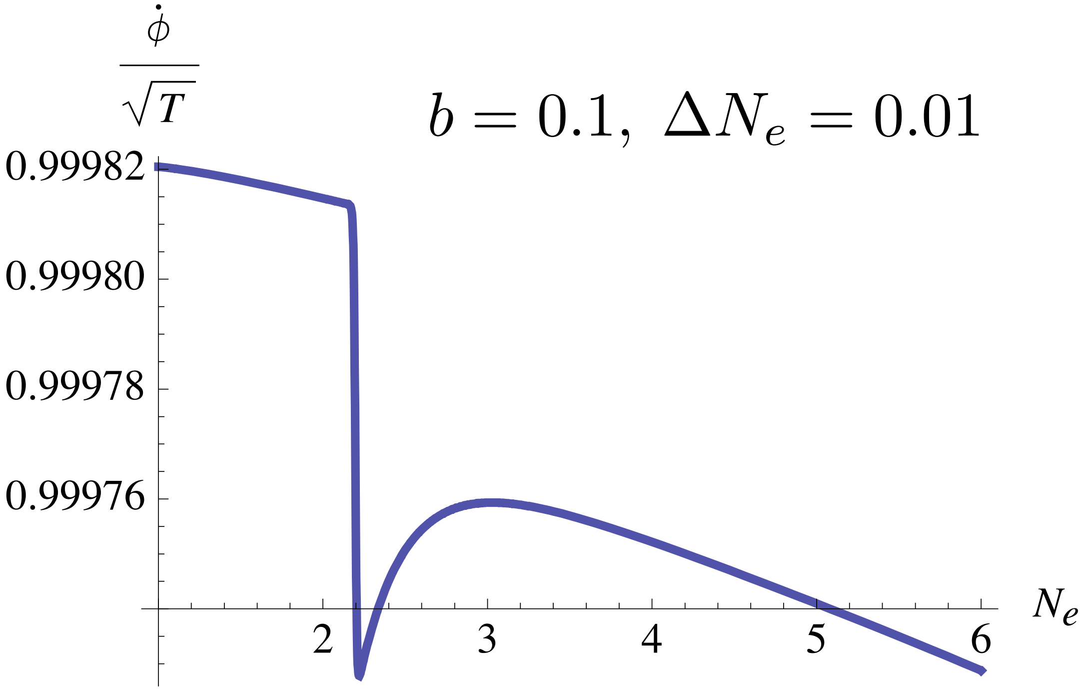
$$P_R = \frac{H^2}{2\pi\dot{\phi}}$$

$$\frac{\sqrt{c+\epsilon/3}}{\sqrt{\epsilon/3}} = \sqrt{1+3c/\epsilon}$$

IR-DBI power spectrum

- moving across the step in warp factor $\Delta N_e \equiv H\Delta t \approx H\frac{d}{\dot{\phi}} = \frac{d}{\sqrt{2c_s\epsilon}}$
- the sound speed changes sharply upon crossing the step

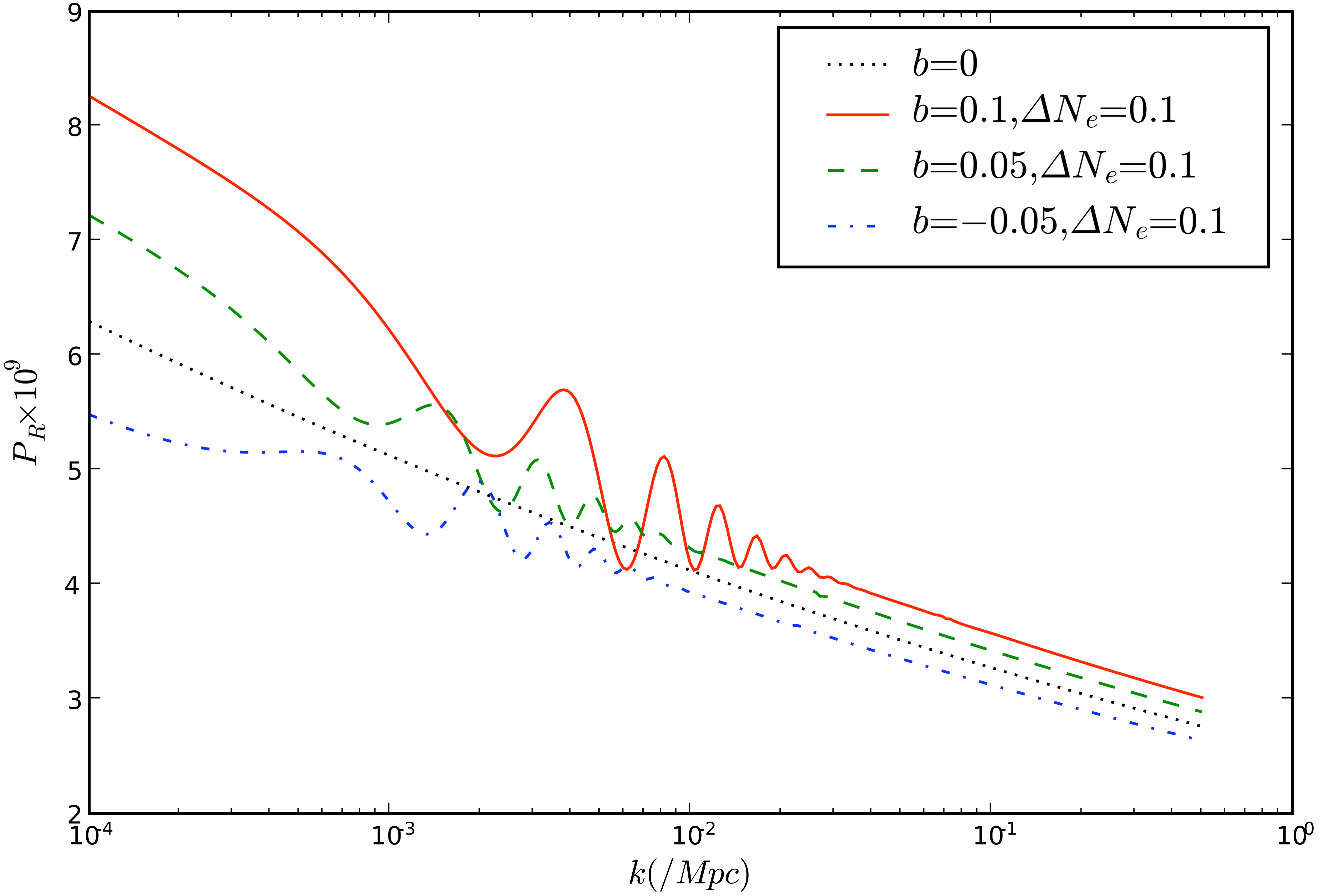




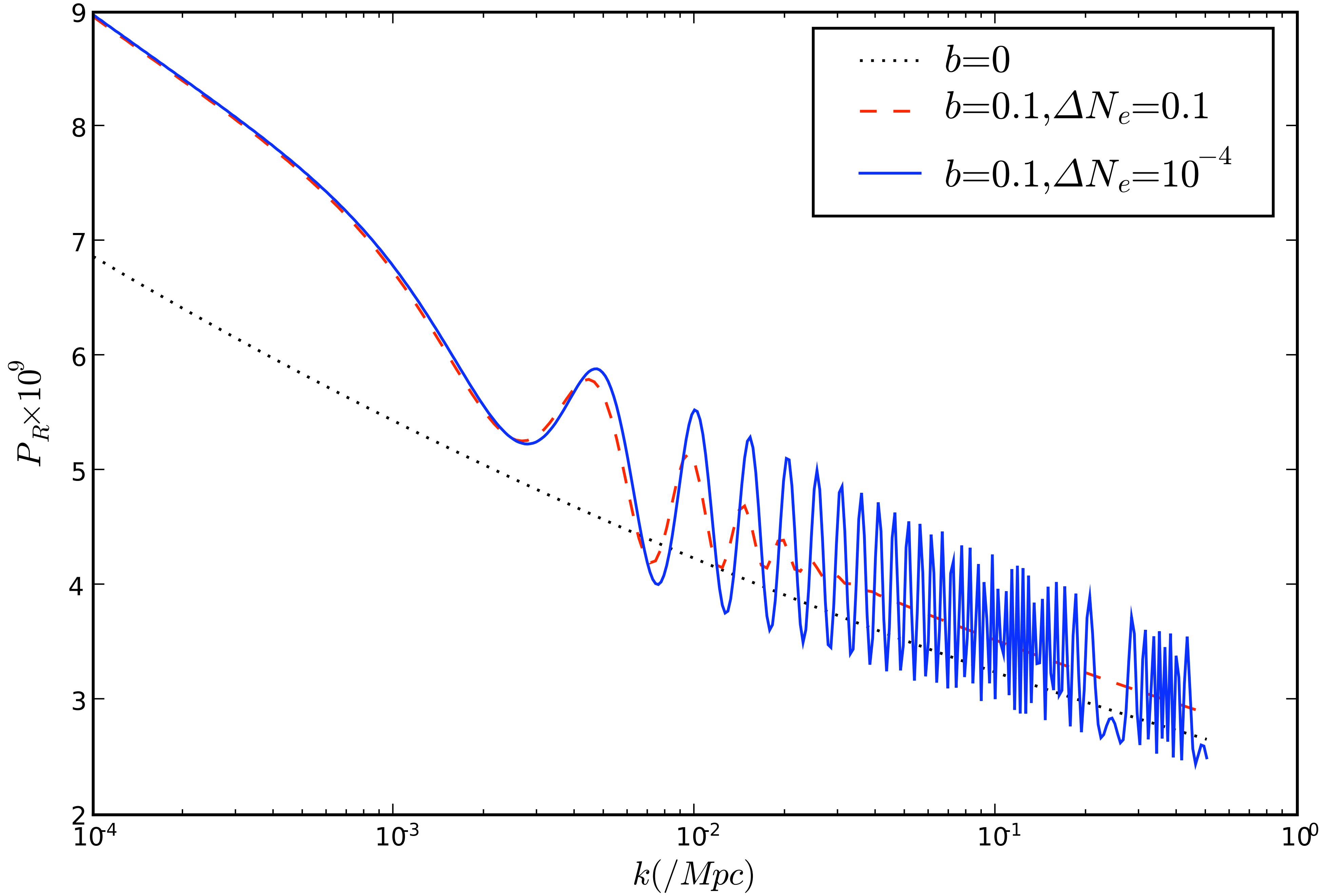
$$\rho = V(\phi) + T(\phi)(c_s^{-1} - 1) \qquad \frac{\Delta c_s}{c_s} = \frac{\Delta T}{T} = 2b$$

$$\frac{z''}{z} \approx 2a^2H^2\left(1 - \frac{\dot{s}}{2H}\right) \qquad s = \frac{\dot{c}_s}{Hc_s} = 3(1-c_s^2) + \frac{c_s\dot{V}}{TH} + \frac{\dot{T}}{TH}(1-c_s)$$

$$\frac{\dot{s}}{H} \approx 2c_s\epsilon\left(\frac{T''}{T} - \frac{1}{2}\frac{T'^2}{T^2}\right) \approx 2c_s\epsilon\left(\frac{T''}{T}\right) \sim b\,\frac{c_s\epsilon}{d^2} \sim \mathcal{O}\left(\frac{b}{\Delta {N_e}^2}\right)$$

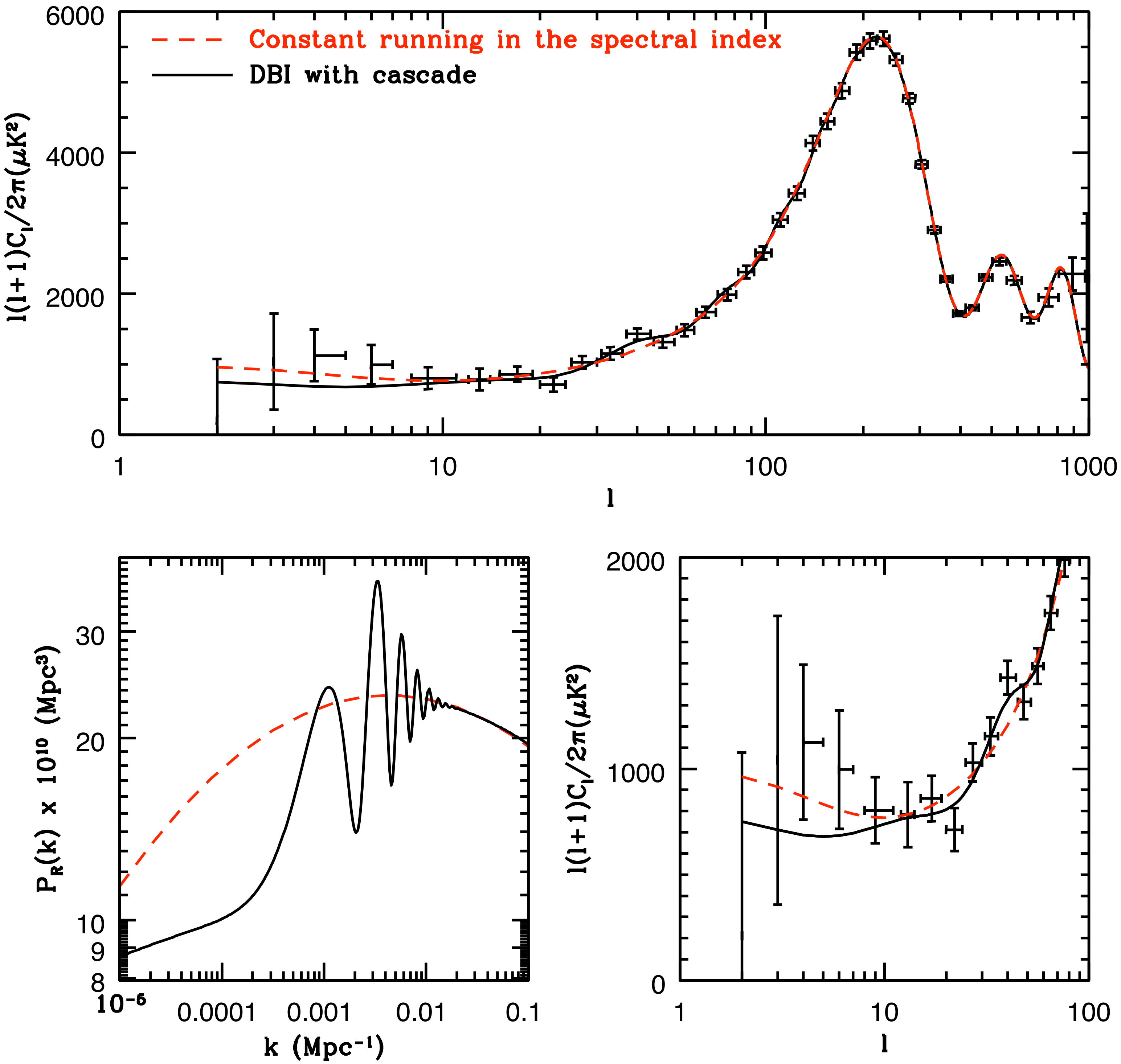


$$P_R = \frac{H^2}{2\pi\dot{\phi}} \qquad \dot{\phi} = \sqrt{T(\phi)}$$



$$\frac{k_{max}}{k_{min}} = \sqrt{\frac{z''}{z}} \sim \frac{b}{\Delta N_e}$$

$\frac{\Delta c_s}{c_s} = \frac{\Delta T}{T} = 2b$ always saturated ➡ oscillation amplitude weakly depends on step width



need b=-0.3, too large for steps
in duality cascade

Multiple Steps

- duality cascade gives a series of “K” steps, spaced according to

$$\ln(r_{p+1}) - \ln(r_p) \simeq \frac{2\pi}{3g_s M}$$

- feature on scale k in the power spectrum, shows up on angular scale l on WMAP

$$\frac{\pi}{l} \approx \frac{k^{-1}}{H_0^{-1}}$$

$$-\, dN_e \simeq d\ln k \simeq d\ln l \simeq H dt \simeq \frac{H}{\dot{\phi}} d\phi$$

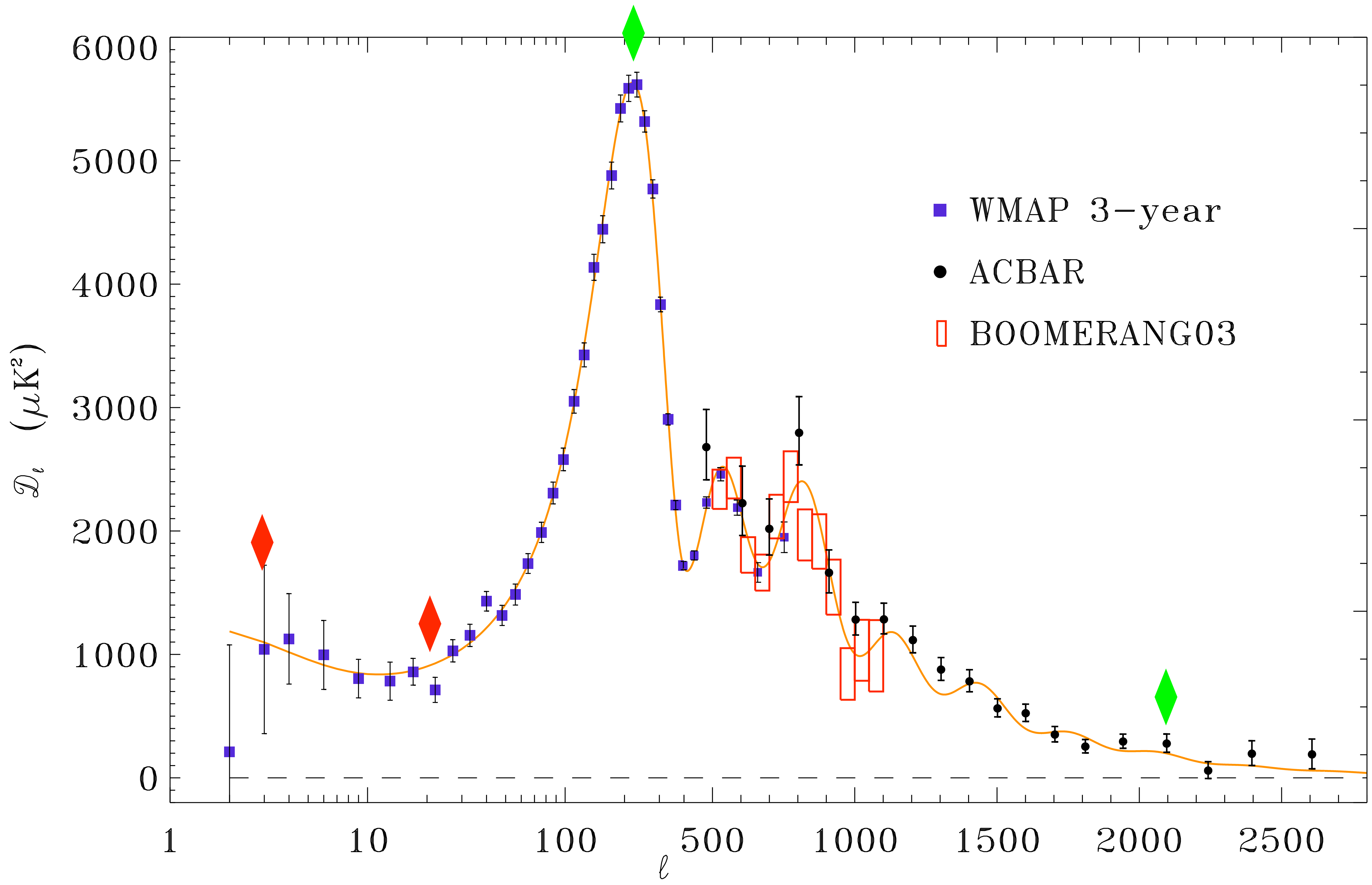
$$d\ln l \propto d\phi$$

$$\frac{\phi_p}{\phi_{p+1}} \approx \frac{\phi_{p+1}}{\phi_{p+2}} \approx \frac{\phi_{p+2}}{\phi_{p+3}} \approx \exp(-\frac{2\pi}{3g_s M}) \approx 1+\delta$$

$$\frac{\phi_p - \phi_{p+1}}{\phi_{p+1} - \phi_{p+2}} \approx 1+\delta$$

- take l=2, l=20 as two steps for example,

$$\ln(2) - \ln(20) = \ln(20) - \ln(l_3) \Rightarrow \; l_3 = 200, \; l_4 = 2000$$


6000
5000
4000
3000
2000
1000
0
$\mathscr{D}_\ell$ (μK^2)
1
10
100
500
1000
1500
2000
2500
ℓ
WMAP 3-year
ACBAR
BOOMERANG03

Observable effects (II): non-gaussianity

- the three-point correlation function

$$\zeta(\tau,\mathbf{k}) = u(\tau,\mathbf{k})a(\mathbf{k}) + u^*(\tau,-\mathbf{k})a^\dagger(-\mathbf{k})$$

- in slow-roll,

$$\langle\zeta(\tau_{end},\mathbf{k}_1)\zeta(\tau_{end},\mathbf{k}_2)\zeta(\tau_{end},\mathbf{k}_3)\rangle$$

$$= i\left(\prod_i u_{k_i}(\tau_{end})\right)\int_{-\infty}^{\tau_{end}} d\tau a^2 \boxed{\epsilon\tilde{\eta}'}\Big(u^*_{k_1}(\tau)u^*_{k_2}(\tau)\frac{d}{d\tau}u^*_{k_3}(\tau) + \mathrm{perm}\Big)$$

$$\times\ (2\pi)^3\delta^3(\sum_i \mathbf{k}_i) + \mathrm{c.c.}\ ,$$

small by construction
in usual slow-roll,
can be large locally at sharp features

- in DBI case, leading term is $\dfrac{a^3\epsilon}{2c_s^2}\dfrac{d}{dt}\left(\dfrac{\tilde{\eta}}{c_s^2}\right)\zeta^2\dot{\zeta}$

$$(a^3\epsilon/c_s^4)\zeta\dot{\zeta}^2 \Longrightarrow f_{NL} \sim 1/c_s^2.$$

$$\frac{d}{dt}\left(\frac{\tilde{\eta}}{c_s^2}\right) \Longrightarrow f_{NL}^{feature}$$

- slow roll case $f_{NL} = \mathcal{O}(\tilde{\eta})$

$$\Delta\epsilon \approx \Delta V/H^2 \approx 5c$$

$$\Delta t_{accel} \approx \Delta\phi/\dot{\phi} \approx d/\sqrt{cV} \ ,$$

$$\Delta\tilde{\eta} \approx \tilde{\eta} = \frac{\dot{\epsilon}}{H\epsilon} \approx \frac{7c^{3/2}}{d\epsilon} \ ,$$

data fitting give $c/\epsilon = 0.2$ $\sqrt{c}/d = \mathcal{O}(1)$ $f_{NL} = \mathcal{O}(1)$

- IR-DBI case $f_{NL}^{\text{feature}} \sim \frac{d}{dt}\left(\frac{\tilde{\eta}}{c_s^2}\right)\Delta t \sim \Delta\left(\frac{\tilde{\eta}}{c_s^2}\right)$

$$\tilde{\eta} \approx c_s s$$

$$f_{NL}^{\text{feature}} \sim \frac{\Delta s}{c_s} \sim \frac{1}{c_s}\frac{b}{\Delta N_e}$$

$$b = 0.01, c_s = 0.1, \Delta N_e = 0.01, \ \Rightarrow f_{NL}^{feature} = \mathcal{O}(10)$$

Conclusions

- Duality cascade predicts a series of steps in the warp geometry, the steps are equally spaced in ln(r). Steps are generic, with KS a calculable example.
- Generically, dilaton runs, and features in the warp geometry becomes features in slow-roll potential.
- In the slow-roll power spectrum, the sensitivity to the steps is controlled by c/ϵ . Brane inflation is highly sensitive to small features with $\epsilon \sim 10^{-11}$
- In DBI inflation, sharp features in warp factor may not be observed in the power spectrum, but gives detectable level non-guassianity.
- The steps features in the power spectrum are always accompanied by large non-guassianity on the same scale => chances to tell the feature from statistical fluctuation / cosmic variance