Mass Measurement in Boosted Decay Chains w/ Missing Energy

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base on the work with Jay Hubisz
arxiv:1009.1148

LEPP PARTICLE THEORY SEMINAR
SEP 15 2010

Plan

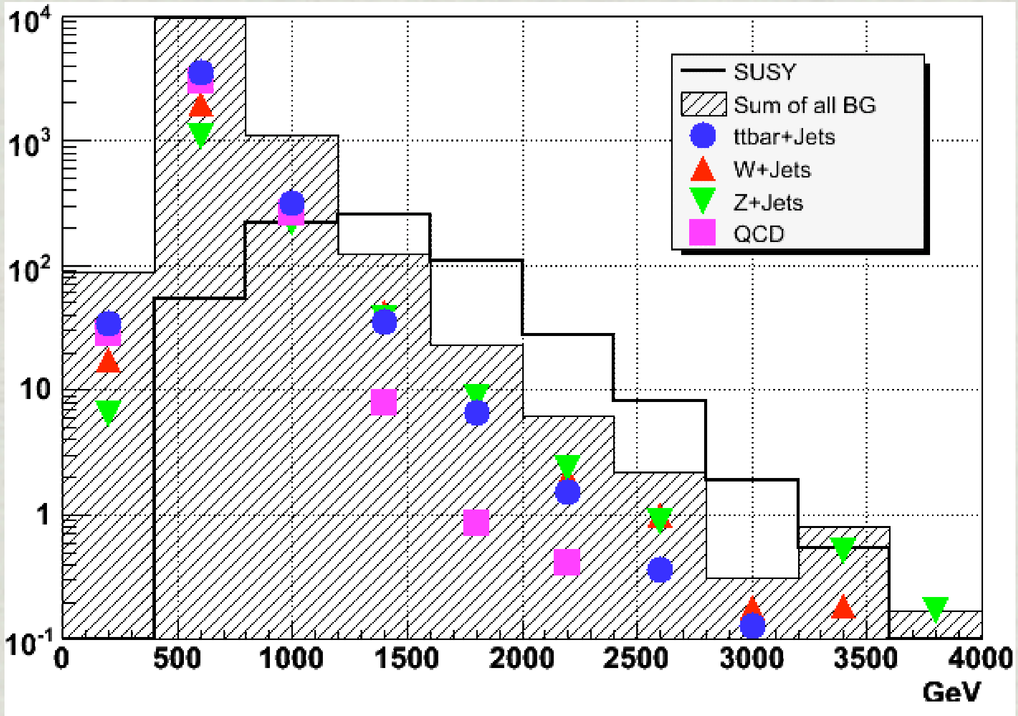
- Warm-up for mass measurement w/ missing energy
  - The Problem
  - The knowns methods
- Boosted decay chain, Collinearity
- MET-cone method
- 1D projection of MET-cone: m_{test} variable
  - Definition, analytic solution and endpoints
  - Numerical results
- Test consistency
- Conclusion

Missing energy events in new physics

- Missing energy event is not unusual -- neutrino in SM

  e.g. $W \rightarrow e\nu$

- We are interested in the missing energy from new physics
- Dark matter motivation : exist (meta)stable exotic particle
- New symmetry to protect it from decay
  - Z_2 parity --> pair production of stable exotics at LHC
  - SUSY, UED ...



Mass reconstruction is important

- Crucial for understanding the underlying physics
  - distinguish different physical models

The dark matter Connection

- The mass of the missing particle determines the relic density

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{\alpha^2}.$$

- Comparison with direct detection and indirect detection

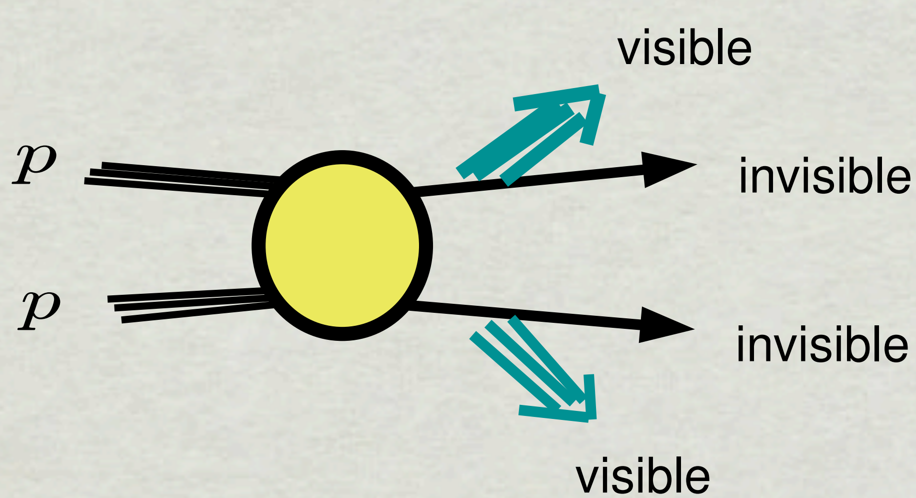
Baltz, Battaglia, Peskin and Wizansky, hep-ph/0602187.

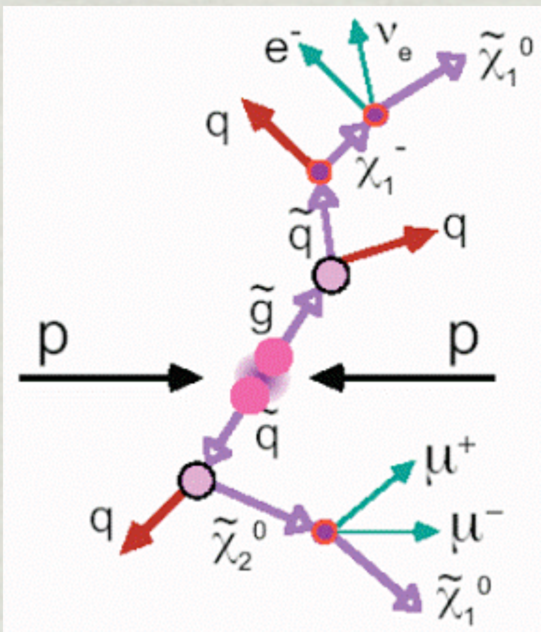
Determine the Dark Matter Mass

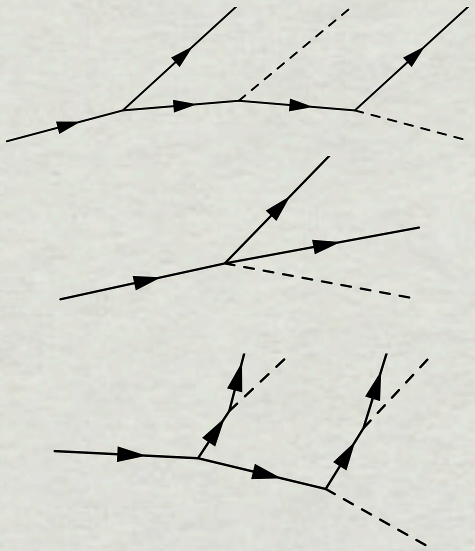
-- *challenging at the LHC*

THE DIFFICULTY:

- Two missing particles in each event
- Unknown parton frame leads to less constrained kinematics
- Interpretation of the signal as a particular physics process
  maybe complicated -- different underlying topologies or a mixture of them







Kinematic Approaches

- Demand that at least some particles are sufficiently close to their mass shells that their energy-momentum Lorentz invariant can be used to constrain their masses

Advantage: do not need to know many details of the underlying physical model (gauge group, spin etc)

Three main catergories:

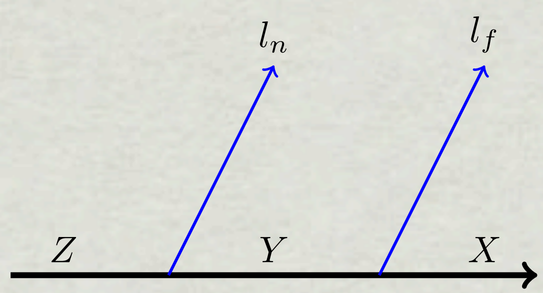
see a recent review:
Barr and Lester, arXiv:1004.2732[Hep-ph]

- ☆ Invariant Mass endpoint — BACHACOU, HINCHLIFFE AND PAIGE
- ☆ Polynomial method/Mass relation method — KAWAGOE, NOJIRI AND POLESELLO; CHENG, GUNION, HAN AND MCELRATH
- ☆ M_{T2} variable and Kink — LESTER AND SUMMERS; W.S. CHO, K.CHOI, Y.G.KIM, C.B.PARK; K.KONG, K. MATCHEV, M.PARK
  - ☆ Other variations: subsystem MT2, Mct,

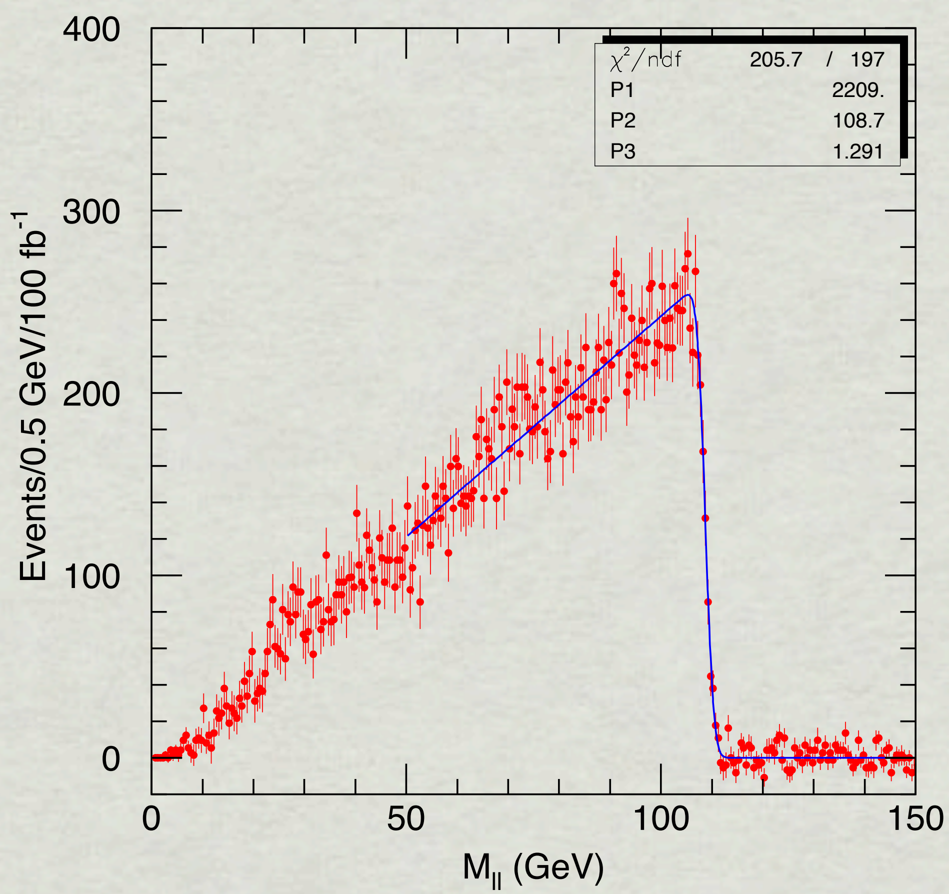
......

Invariant Mass endpoint

- Simple decay chain



$$m_{\ell\ell}^{max} = \sqrt{(M_Z^2 - M_Y^2)(M_Y^2 - M_X^2)}/M_Y$$



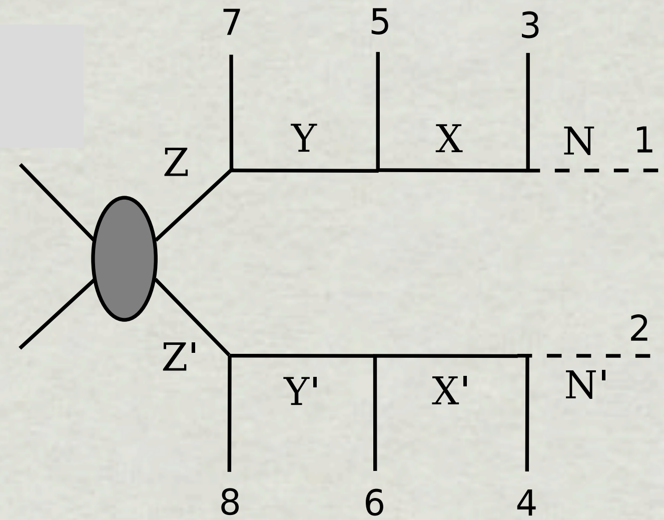
- Only probe mass differences

- Need long decay chain to get enough constraints suffer from combinatoric ambiguities

BACHACOU, HINCHLIFFE AND PAIGE

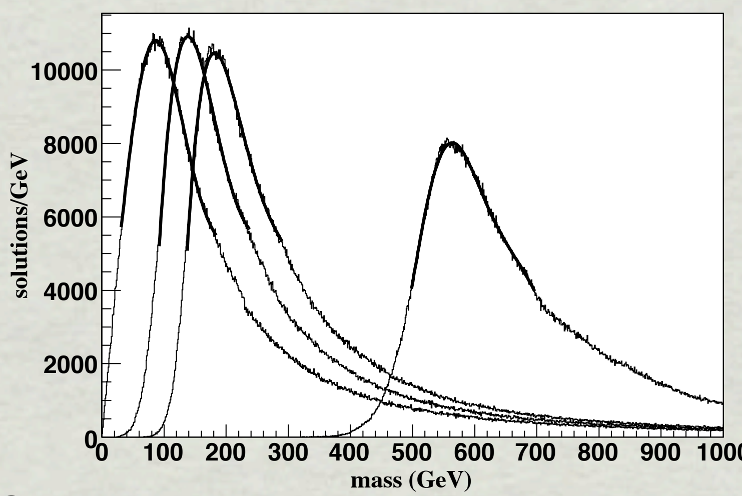
Polynomial method(Mass relation)

- Using On-shell conditions event-by-event



$$\text{constraints} \geq \text{unknowns}$$

$$10n. \qquad 4 + 8n$$

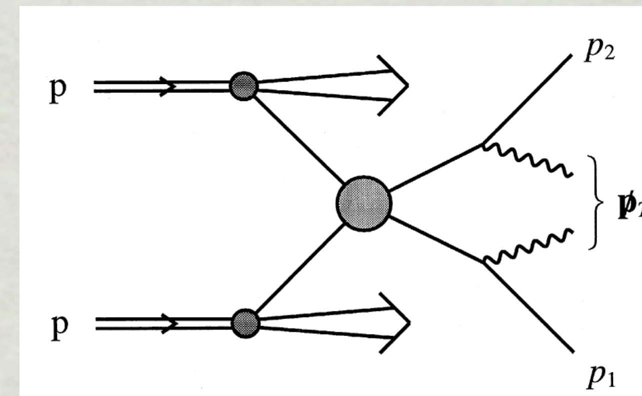


- For n>2, over-constrained system
- Very restrictive kinematics
- Require long decay chains -- at least four on-shell particles in each chain

KAWAGOE, NOJIRI AND POLESELLO;
CHENG, GUNION, HAN AND MCELRATH

Cambridge M_{T2} method

- Massive new particles pair produced $pp \to \tilde{l}\tilde{l}$

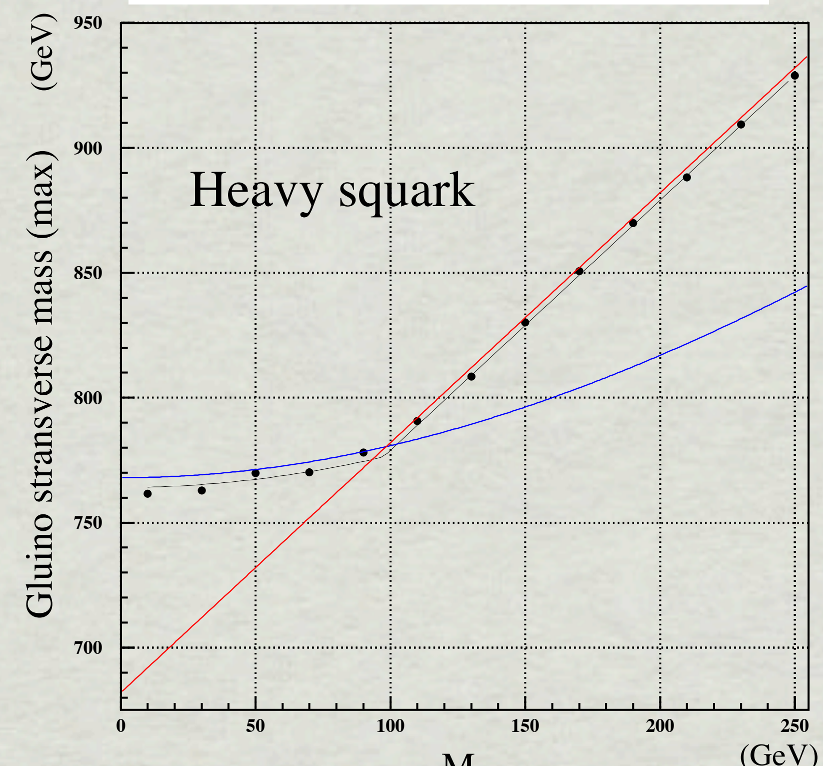


$$m_{\tilde l}^2 \geq M_{T2}^2 \equiv \min_{\not p_1 + \not p_2 = \not p_T} \left[\max\{m_T^2(\mathbf{p}_{Tl^-}, \not p_1), m_T^2(\mathbf{p}_{Tl^+}, \not p_2)\}\right]$$

minimization over all possible trial LSP momentum

$$m_l^2 + m_{\tilde\chi}^2 + 2\left(E_{Tl}E_{T\tilde\chi} - \mathbf{p}_{Tl}\cdot\mathbf{p}_{T\tilde\chi}\right)$$

- M_{T2} endpoint func. of trial LSP mass
- M_{T2} kink --> LSP mass
  - for simple 2-body decay, no clear kink
  - for multi-body decay, combinatoric dilute the kink



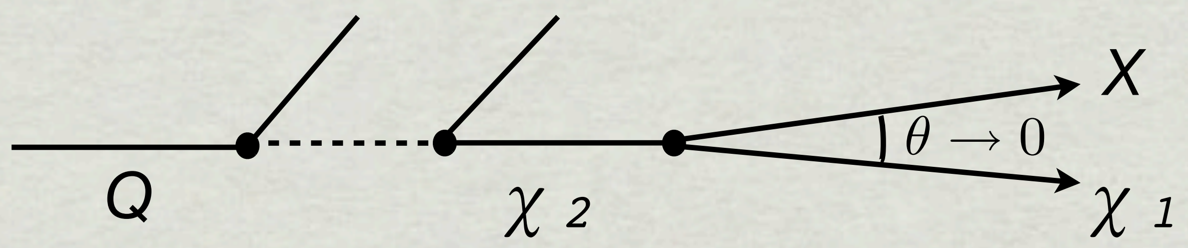
LESTER AND SUMMERS

W.S. CHO, K.CHOI, Y.G.KIM, C.B.PARK

Having multiplet methods is crucial

Any new orthogonal ideas?

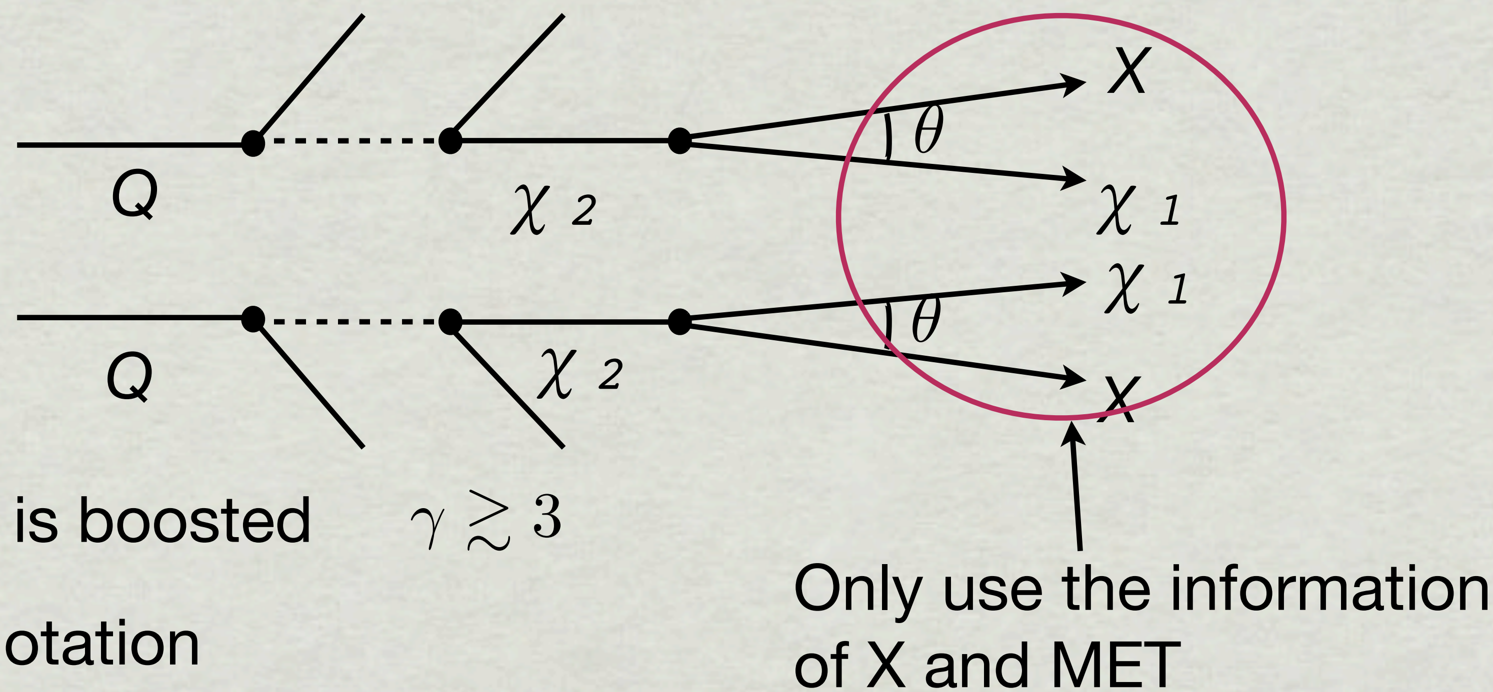
- In many new physics models: there are both heavy(~TeV) exotics as well as light(~100GeV) ones
  - *SUSY little hierarchy*



- Boosted decay is generic
- Can we get additional handle if missing particle is approximately collinear with visible particles ?

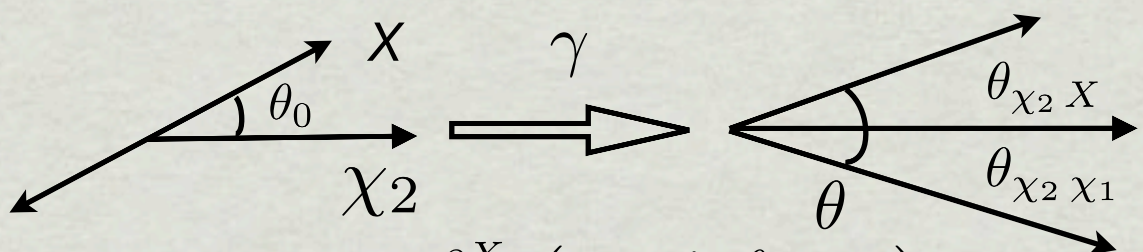
MET-cone method

- Based on the simple observation:
  - MET only allowed to vary a narrow region around visible momentum -- “MET-cone”
  - MET-cone boundary is sensitive to the underlying masses
    - for initial study: symmetric double decay chains
      - consider χ_2 is boosted $\gamma \gtrsim 3$
      - use SUSY notation



Collinearity of the decay

- parametrize the opening angle in the lab frame



β, γ velocity & boost factor of χ_2

β_0, γ_0 velocity & boost factor in the rest frame of χ_2

$$\tan\theta_{\chi_2 X} = \frac{\beta_0^X}{\gamma}\left(\frac{\sin\theta_0}{\beta_0^X\cos\theta_0 + \beta}\right)$$

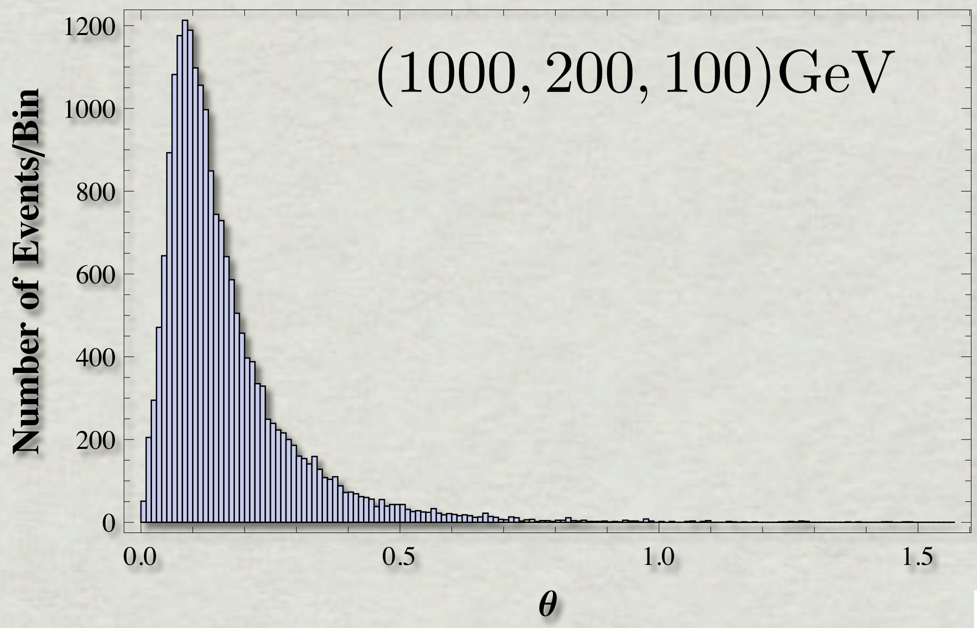
- Narrow range of variation $\beta_0^X < \beta$

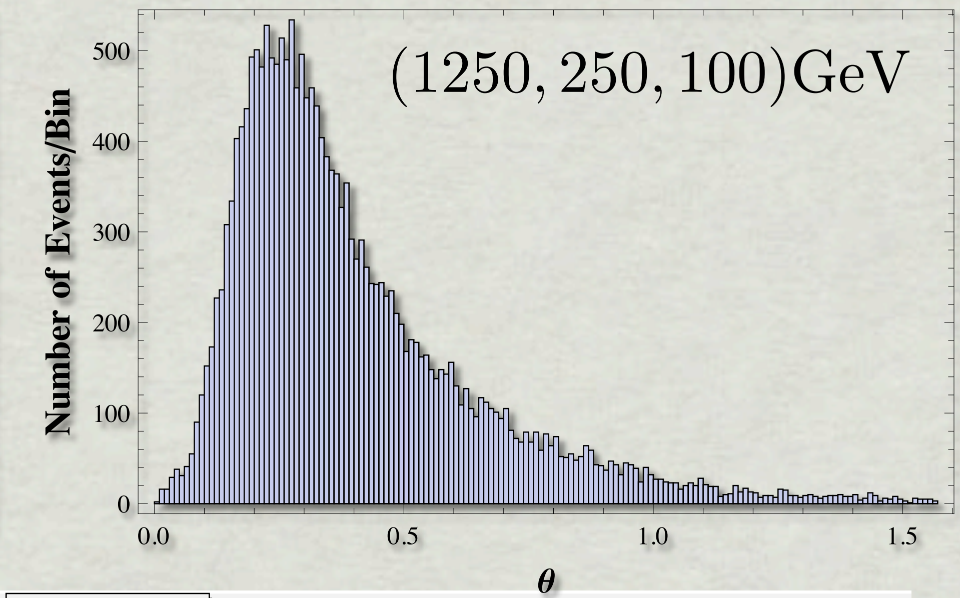
$$0 \le \tan\theta_{\chi_2 X} \le \frac{\beta_0^X}{\gamma\beta}\frac{1}{\sqrt{1-(\beta_0^X/\beta)^2}} \xrightarrow{\gamma\gg 1} \frac{\beta_0^X}{\gamma}\frac{1}{\sqrt{1-(\beta_0^X)^2}}.$$

- Two Cases
  - Large boost factor $\gamma \gg 1$
  - Moderate boost factor γ , but the decay products are non-relativistic in the rest frame of the decay $\beta_0 \ll 1$

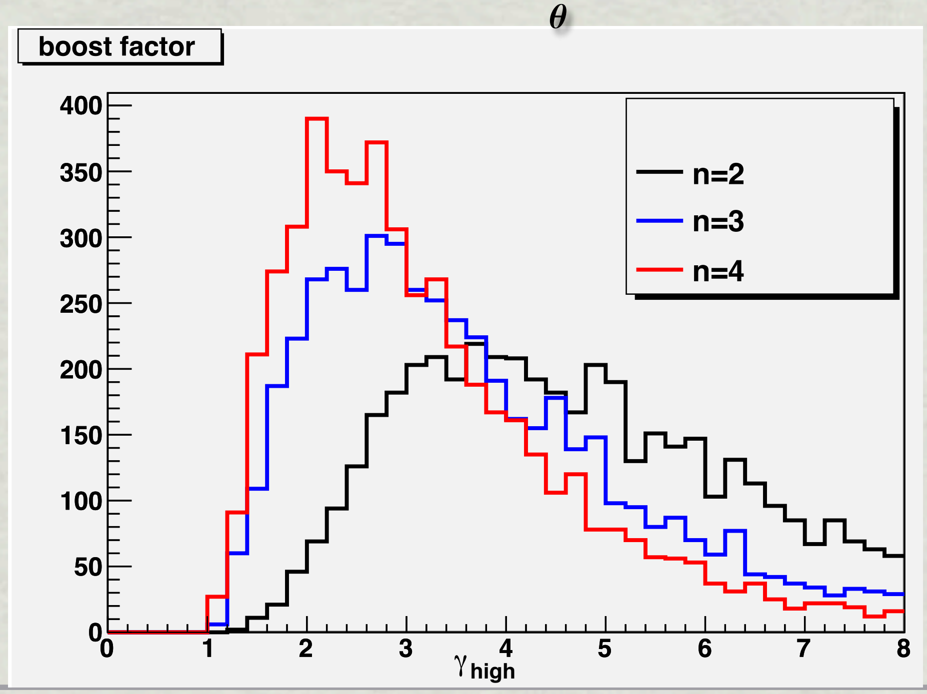
- For a given underlying physics, both boost factor and θ_0 vary according to the matrix element

$$\tilde{q}_L \to \chi_2 q \to \chi_1 \, Z \, q$$





- boost factor decrease with increased # of steps in the cascade



Correlation in the magnitude

- boost factors are correlated

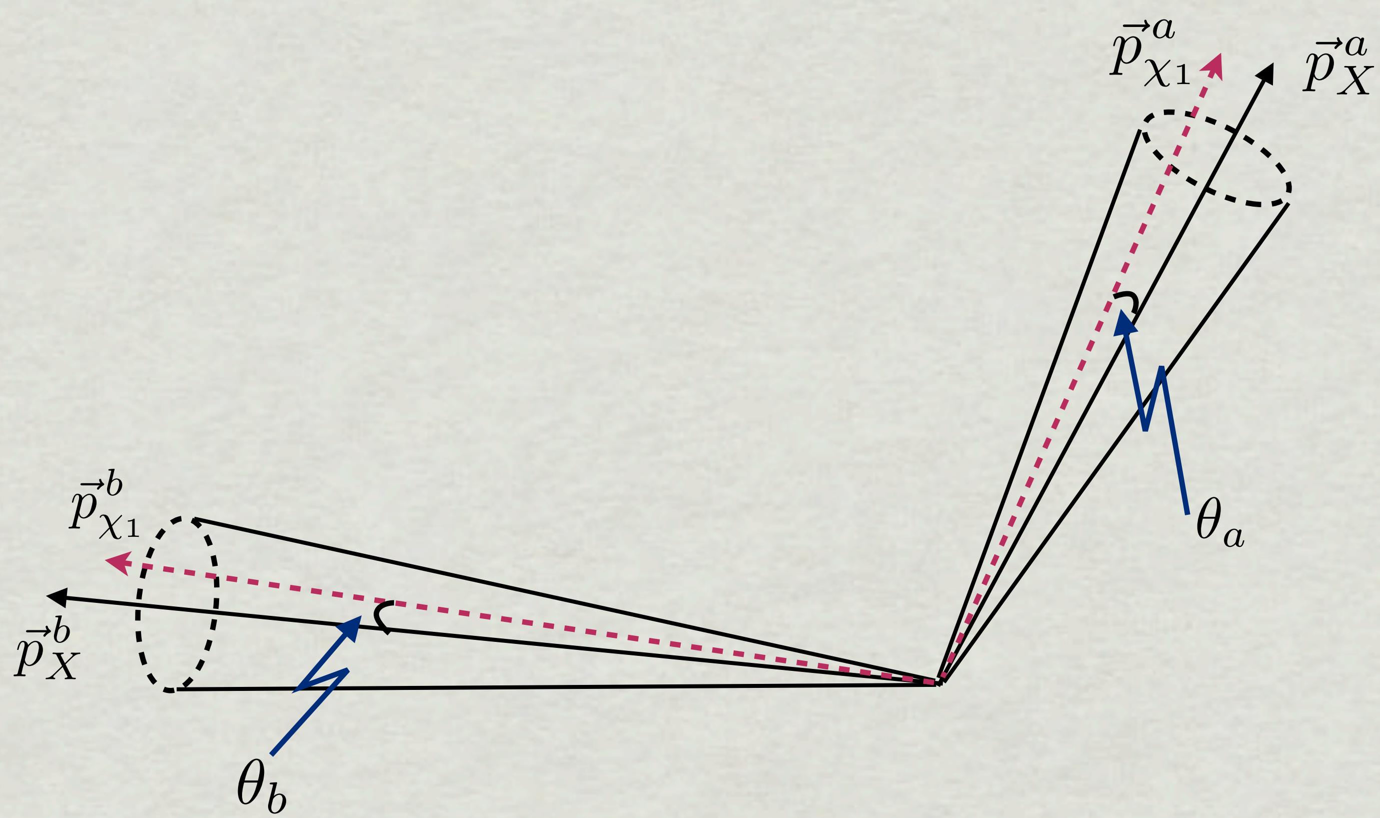
$$\begin{aligned}\gamma_{\chi_1} &= \gamma\,\gamma_0^{\chi_1}(1+\beta\,\beta_0^{\chi_1}\cos\theta)\\ \gamma_X &= \gamma\,\gamma_0^{X}(1-\beta\,\beta_0^{X}\cos\theta)\end{aligned}$$

$$\begin{aligned}p_{\chi_1} &= \gamma_{\chi_1}\beta_{\chi_1}m_{\chi_1}\\ p_X &= \gamma_X\beta_X m_X.\end{aligned}$$

- In the limit $\beta_0 \ll 1$, two boost factors equal

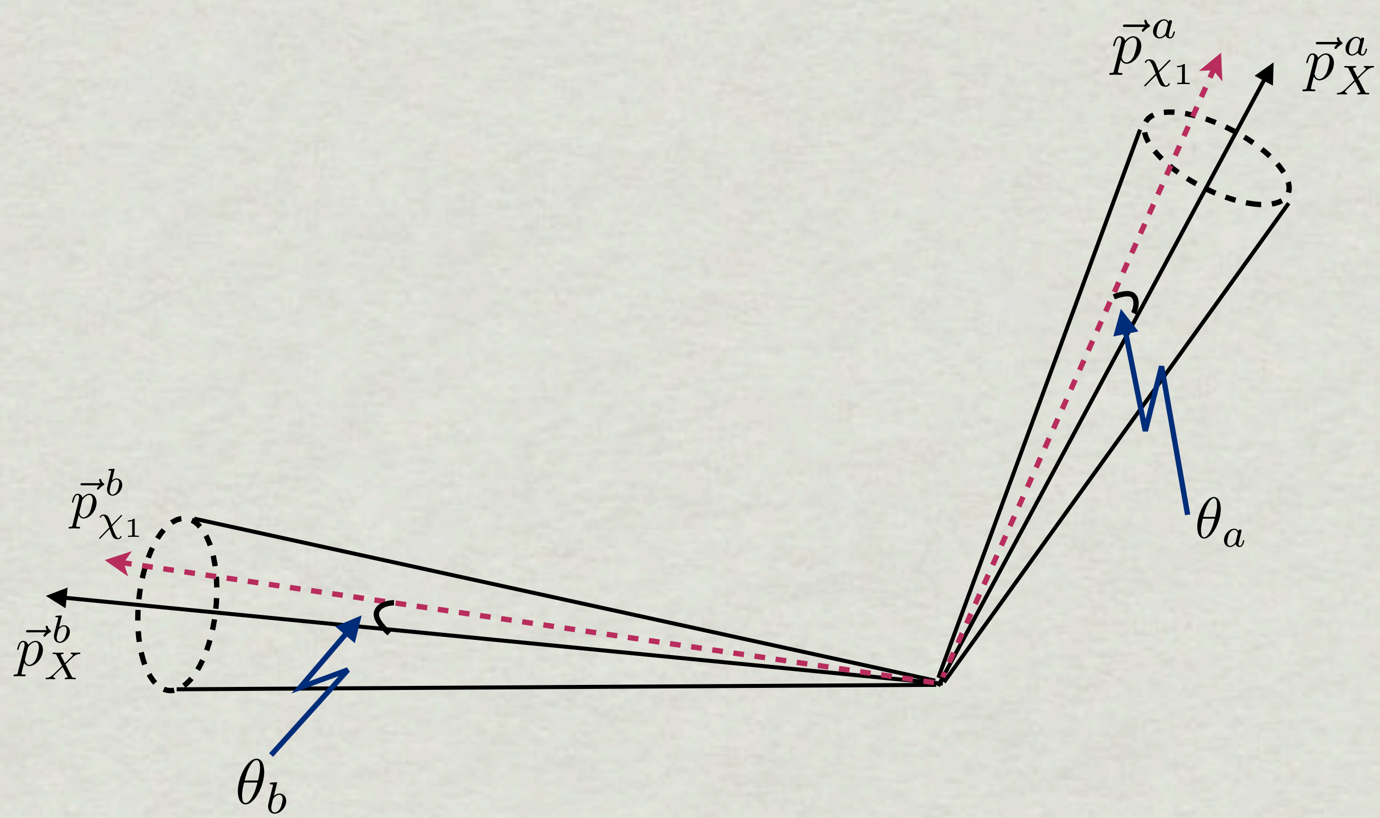
- the ratio mainly depend on θ_0, mildly dependence on the boost factor γ

MET-Cone



MET-Cone

- Sum over momenta of both decay chain



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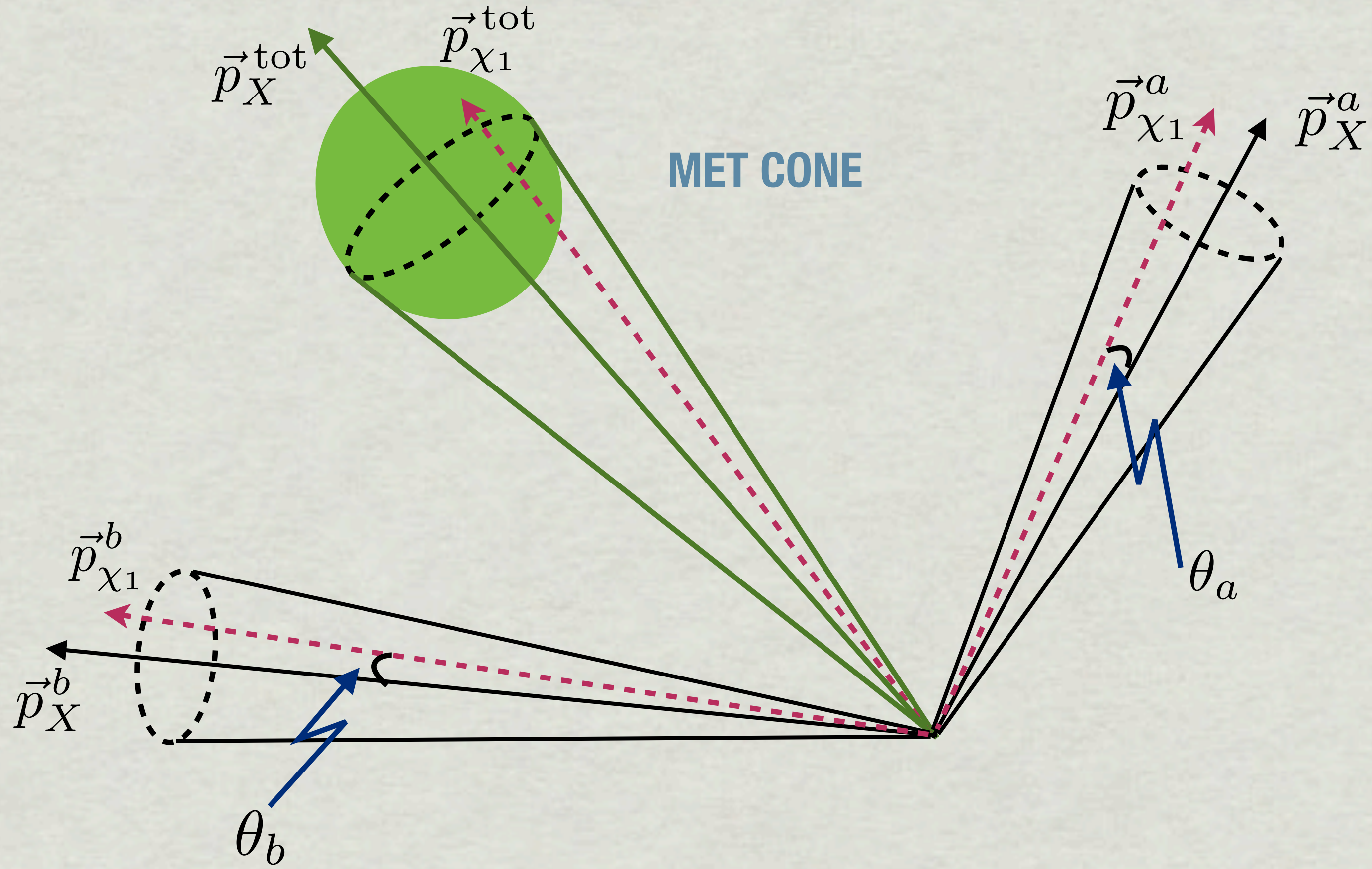

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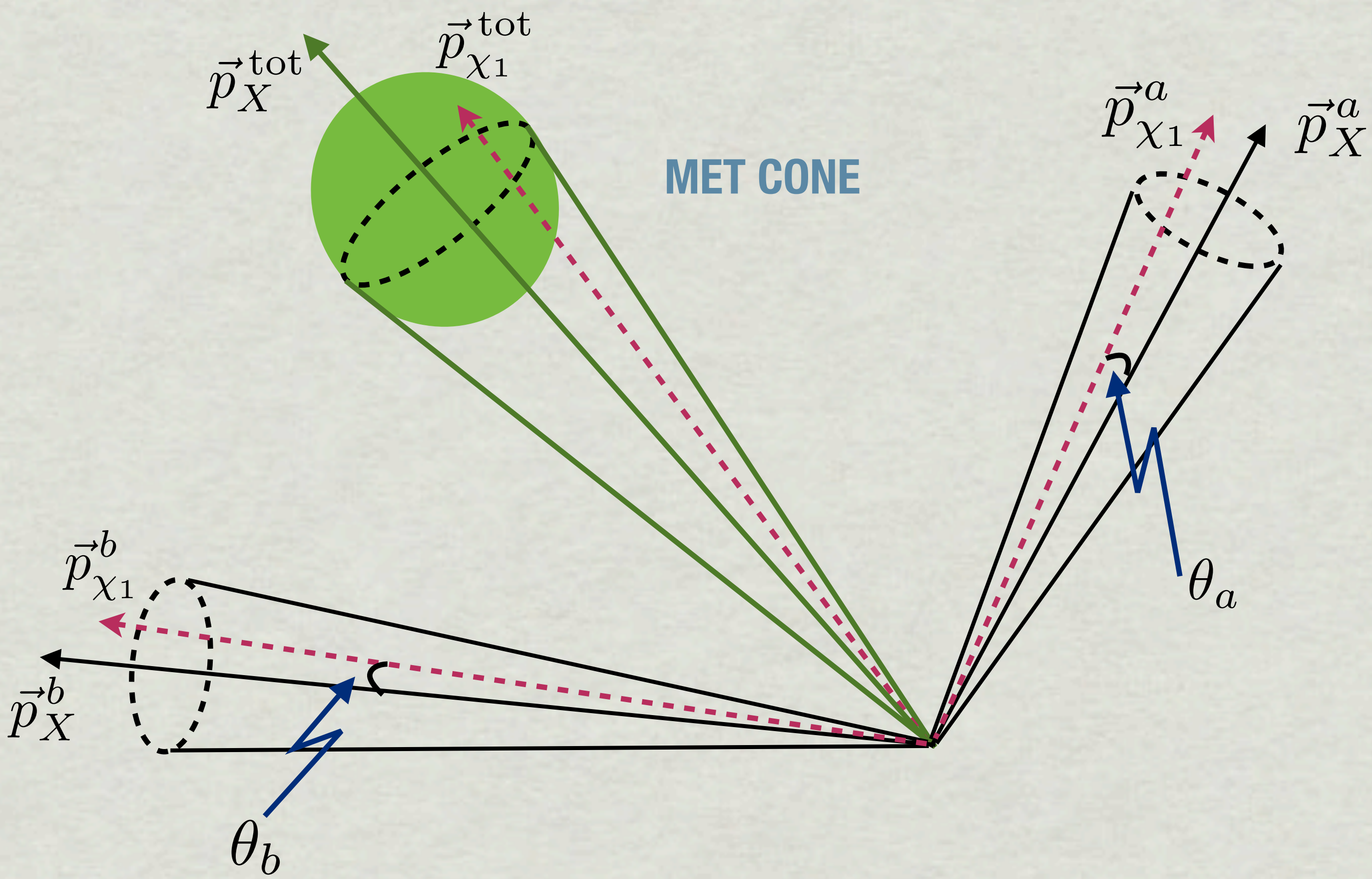


- For small β_0, the MET vector vary around $\vec{p}_X^{\,\mathrm{tot}}$, form a "Cone"

MET-Cone : a more precise definition

- For a given visible particle configuration, what is the allowed region of MET ?

fix $\gamma_a^X, \gamma_b^X, \theta_{ab}^X$

$\theta_{\rm beam}, \phi_{\rm beam}$

$m_{\chi_1}, m_{\chi_2}, m_X$

Vary the rest frame angles with a flat prior

$\theta_{a,0}, \theta_{b,0}, \phi_{a,0}, \phi_{b,0}$

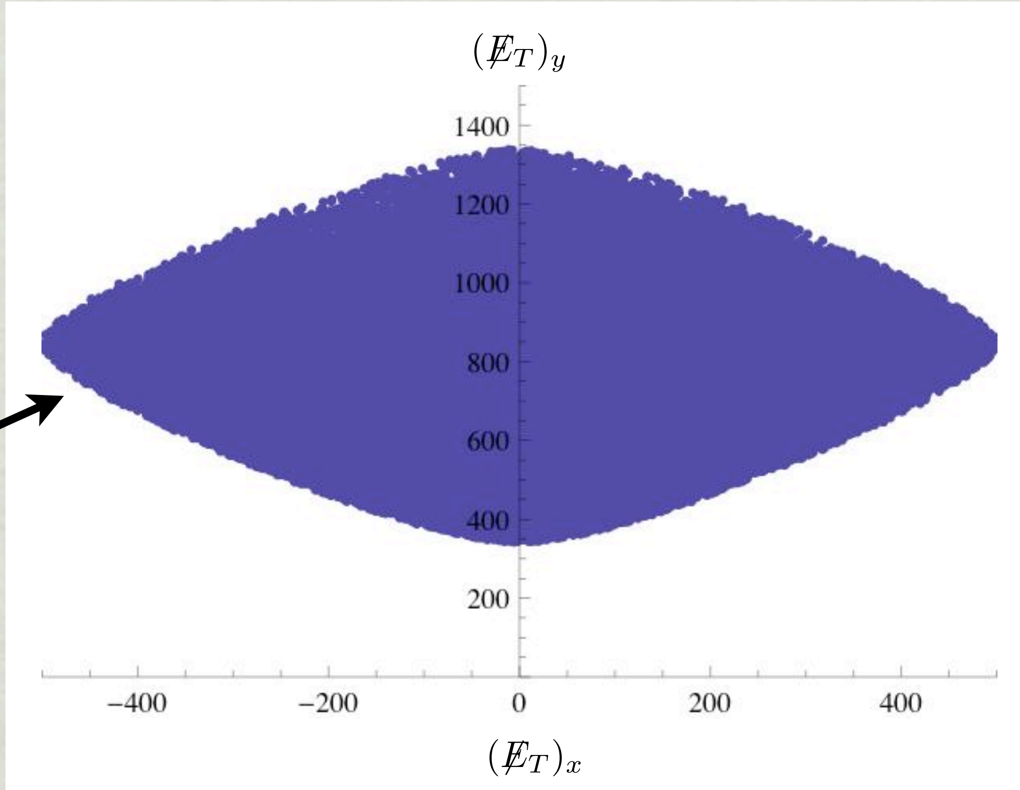
- A simple example:

$$\chi_2 \to \chi_1 Z$$

$$m_{\chi_2} = 200 \text{ GeV}, \ m_{\chi_1} = 100 \text{ GeV}.$$

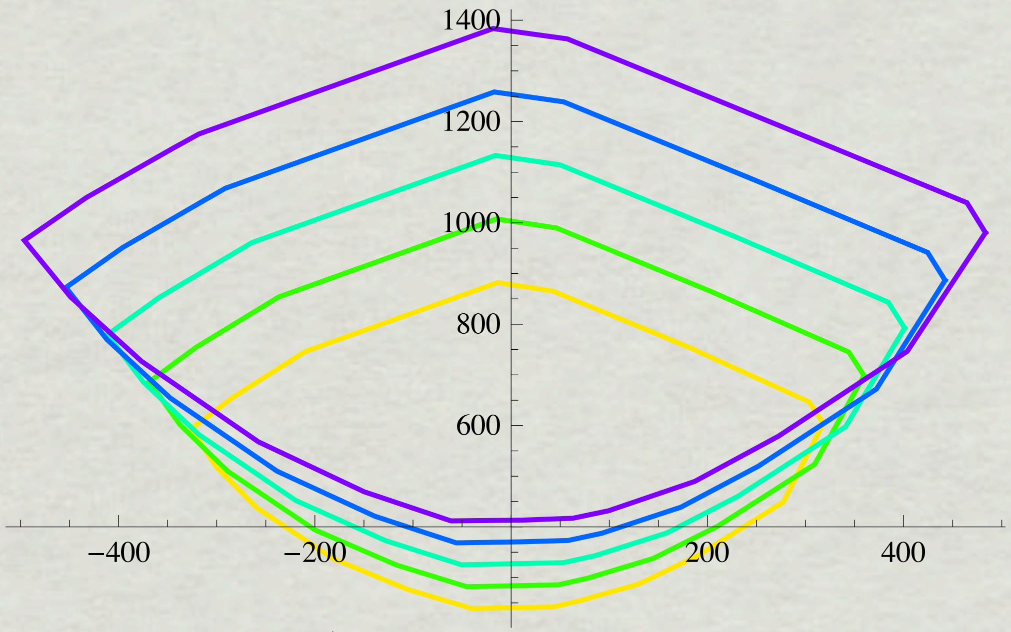
$$\gamma_{a,b}^X = 5 \quad \theta_{ab}^X = \pi/2 \quad \theta_{\rm beam} = 0$$

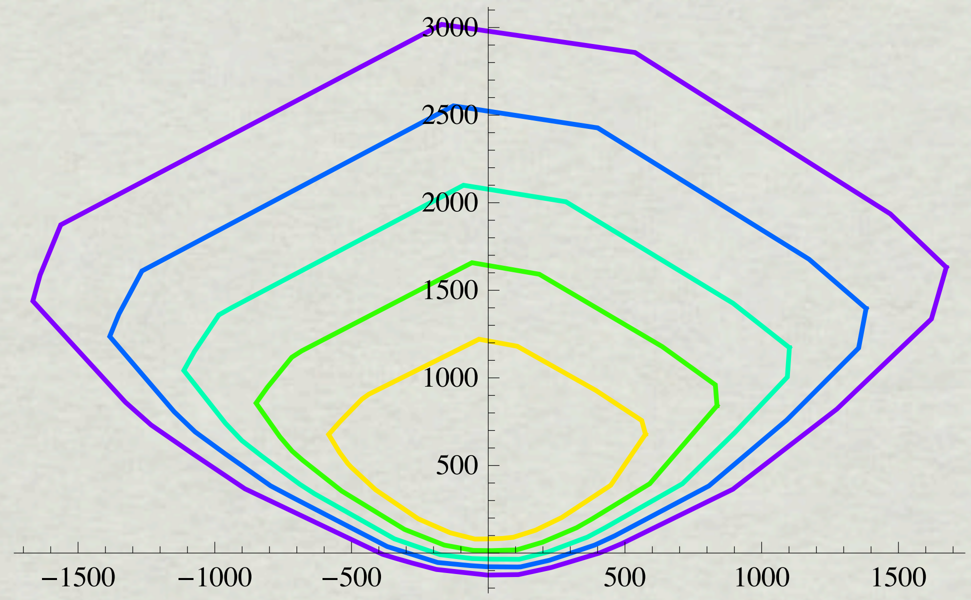
- Has definite boundary! MET must be inside if the correct masses were used



MET-Cone : mass dependence

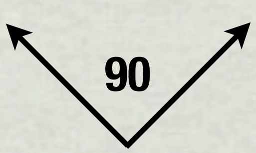
- the MET cone boundary is sensitive to the exotic masses in the decay





$(m_{\chi_2}, m_{\chi_1}, m_Z)$

(220-300, 120-200, 91)

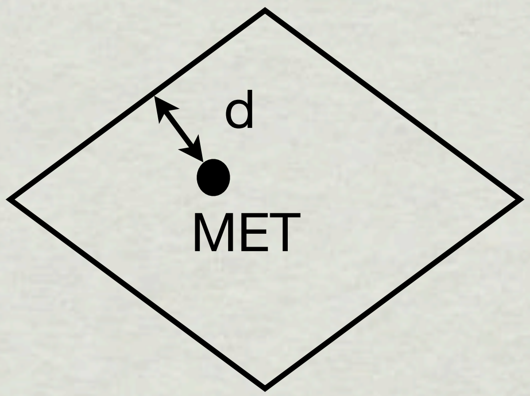


(220-300, 100, 91)

$\gamma_a^Z = \gamma_b^Z = 3.0$

MET-cone: application for mass measurement

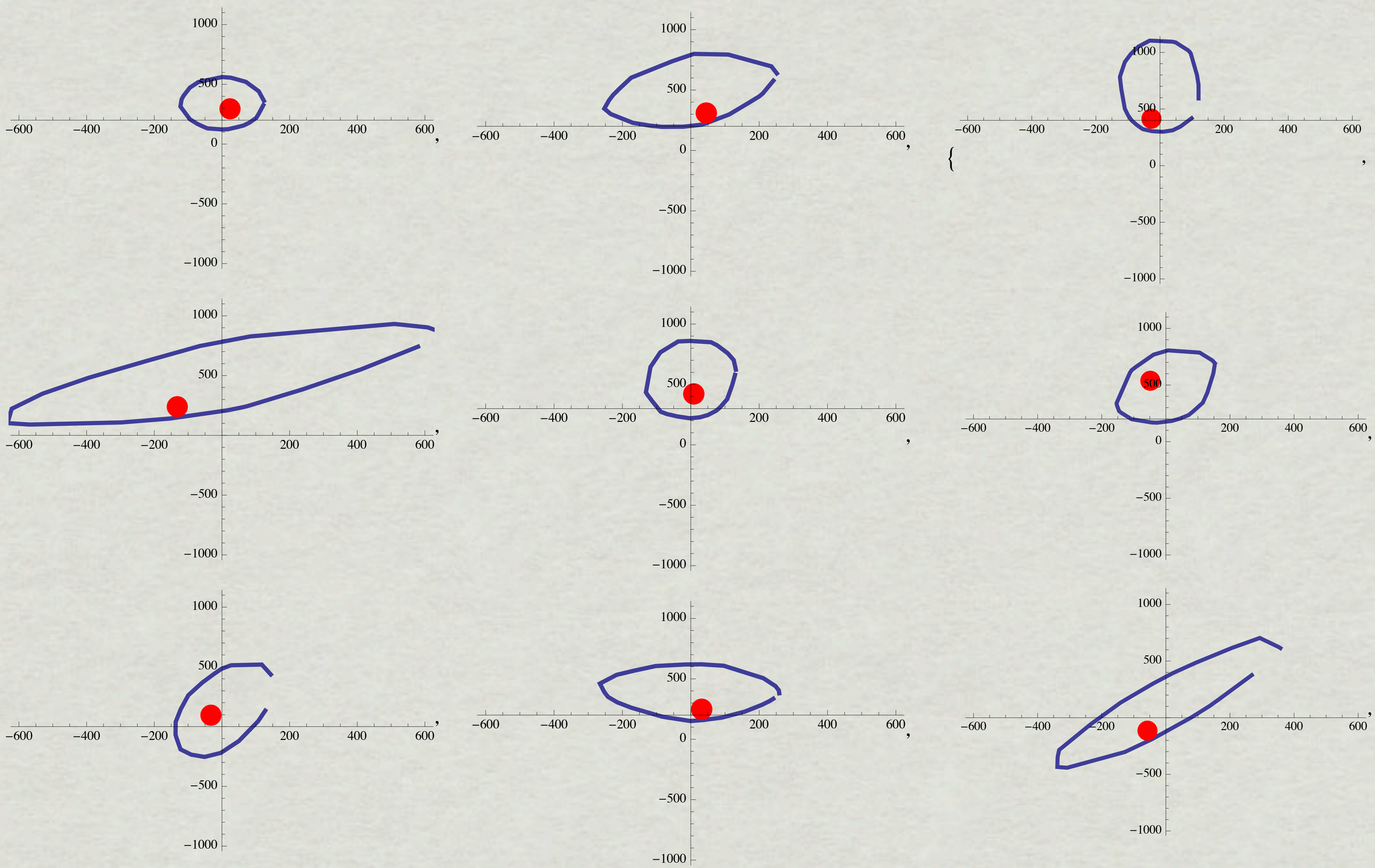
- For a set of events and trial masses, the MET-cone boundary can be determined by the Z configurations event-by-event.



- The correct masses are those that lead to the smallest MET-cone that enclose all the MET points

$$d_{\rm min} \to 0$$

- More systematically, compare the statistical likelihood of a MET data under different mass hypotheses.


−600 −400 −200 200 400 600
1000 500 0 −500 −1000

Quick Summary

- The MET-cone method is a completely new method
- Only need information of the visible particles in the final-step decay and MET
- Although motivated from boosted decay chain, the general idea of the method doesn't require boost.
- It should work best in the boosted case

More detailed numerical evaluation of this method is under investigation.

Numerically complication due to the event-by-event reconstruction of the envelope of the MET-cone.

Is there a simple way to access the power of MET-cone?

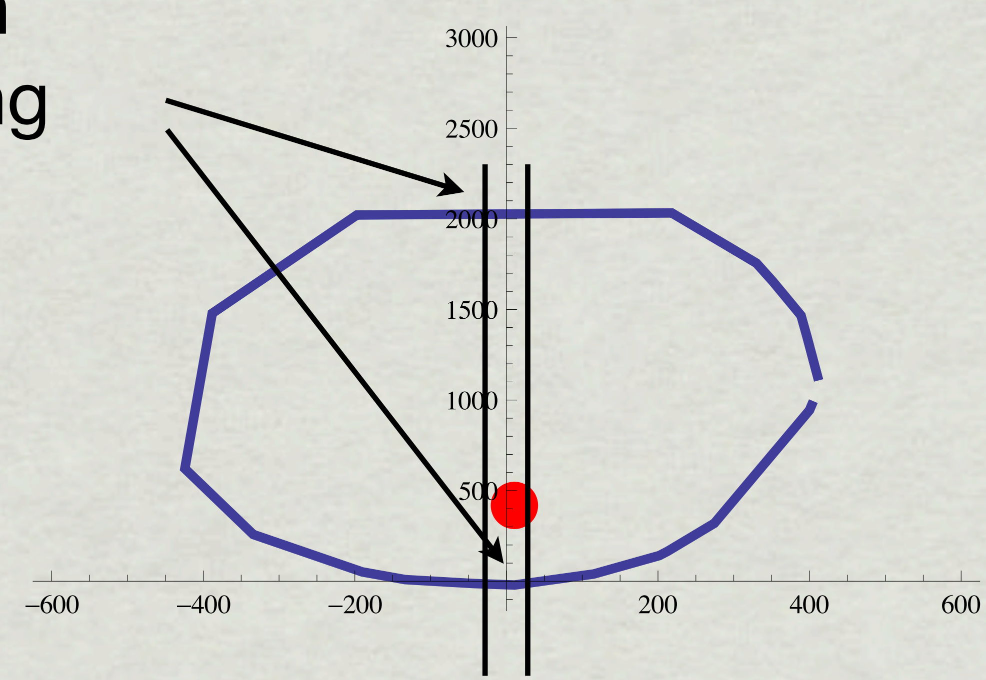
Is there a simple way to access the power of MET-cone?

Yes!

A 1D projection of the MET-cone

- focus on events where MET is in narrow window around y-axis (i.e. the direction of the total X momentum)

- Finite variation in the ratio between total X momentum and total missing momentum



- Define $m_{\chi_1}^{\text{test}}$:

$$\vec{p}_{\chi_1}^{\,T,\text{total}} = \vec{p}_X^{\,T,\text{total}}\, m_{\chi_1}^{\text{test}}/m_X$$

- Expect two endpoints for $m_{\chi_1}^{\text{test}}$

m_{test}: an alternative definition

- Introduce a test mass and a test missing momentum

$$\vec{\not{p}}^{\,a,b}_{\rm test} \equiv \vec{p}_X^{\,a,b}\,\frac{m_{\chi_1}^{\rm test}}{m_X},$$

- m_{test} is determined by minimizing

$$\Delta \not{E}_T^2(m_{\chi_1}^{\rm test}) = \left|\vec{\not{p}}^{\,T,\rm total}_{\rm test} - \vec{\not{p}}^{\,T}_{\rm exp}\right|^2$$

- The minimization condition + $\Delta \not{E}_T^{\rm min} \longrightarrow 0$

$$\Longrightarrow \quad \vec{p}_{\chi_1}^{\,T,\rm total} \to \vec{p}_X^{\,T,\rm total}\, m_{\chi_1}^{\rm test}/m_X \qquad \textbf{(\#)}$$

- In the limit $\beta_0 \to 0$,

$$\frac{p_{\chi_1}}{p_X} = \frac{\gamma_{\chi_1}\beta_{\chi_1}m_{\chi_1}}{\gamma_X\beta_X m_X} \to \frac{m_{\chi_1}}{m_X} \quad \Longrightarrow \quad m^{\rm test} = m_{\chi_1}$$

m_{test}: analytic solution

Solving the constraint Eq. (#)

Consider a simple case: Z's in the trans. plane.

$$\hat{n}^i_{\chi_1} = (\delta^i_j + \alpha^i_j)\hat{n}^j_X + \delta_i \hat{n}_\perp$$

$$\alpha^a_a = -2\sin^2(\theta_a/2) - \cot\theta_{ab}\sin\theta_a\cos\phi_a$$

$$\alpha^a_b = \cos\phi_a \sin\theta_a / \sin\theta_{ab}.$$

$$0 = \frac{\gamma^a_{\chi_1}\beta^a_{\chi_1}}{\gamma^a_X\beta^a_X} + \sum_{i=a,b}\frac{\gamma^i_{\chi_1}\beta^i_{\chi_1}}{\gamma^a_X\beta^a_X}\alpha^i_a - (a \to b) \implies \frac{\gamma^b_{\chi_1}\beta^b_{\chi_1}}{\gamma^b_X\beta^b_X} = \frac{\gamma^a_{\chi_1}\beta^a_{\chi_1}}{\gamma^a_X\beta^a_X}\Big(1 + \mathcal{O}(\theta_{a,b})\Big)$$

$$\frac{m^{\text{test}}_{\chi_1}}{m_{\chi_1}} = \frac{\gamma^a_{\chi_1}\beta^a_{\chi_1}}{\gamma^a_X\beta^a_X} + \sum_{i=a,b}\frac{\gamma^i_{\chi_1}\beta^i_{\chi_1}}{\gamma^a_X\beta^a_X}\alpha^i_a \implies \frac{m^{\text{test}}_{\chi_1}}{m_{\chi_1}} \approx \frac{\gamma^a_{\chi_1}\beta^a_{\chi_1}}{\gamma^a_X\beta^a_X}\left(1 + \alpha^a_a + \alpha^b_a\right)$$

$$\boxed{\begin{aligned} m^{\text{test}}_{\chi_1} &\approx m_{\chi_1}\frac{\gamma^{\chi_1}_0}{\gamma^X_0}\frac{1+\beta\,\beta^{\chi_1}_0\cos\theta^a_0}{1-\beta\,\beta^X_0\cos\theta^a_0} \\ &\times \left(1 - \cot\theta^X_{ab}\cos\phi^a\theta^a + \csc\theta^X_{ab}\cos\phi^b\theta^b\right)\end{aligned}}$$

Force to have equal mom. ratios for two sides of the decay chains

m_{test}: endpoints

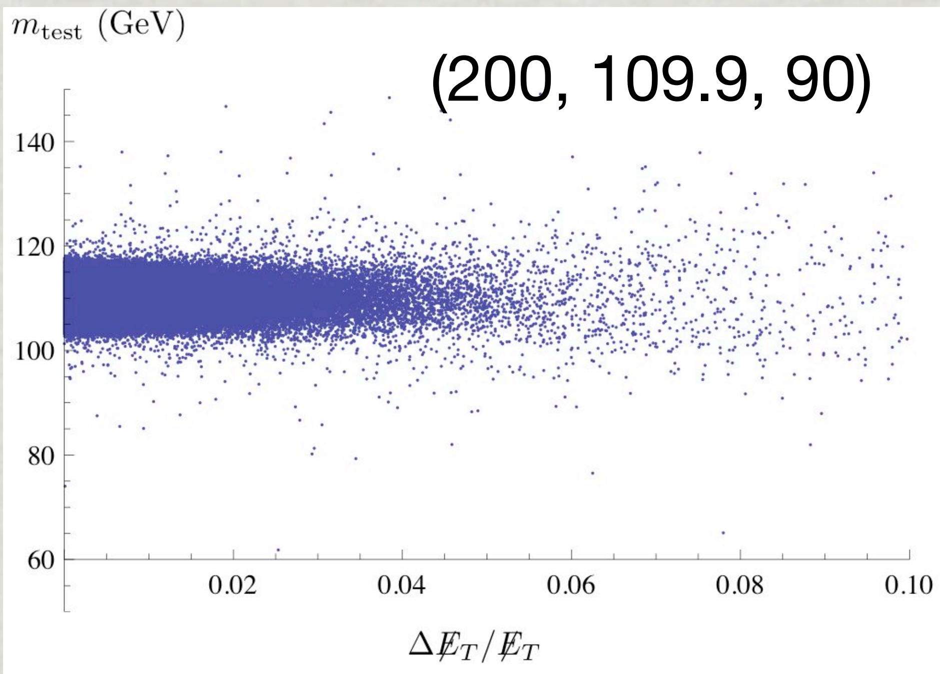
- In the limit $\gamma \to \infty, \beta_0$ fixed , the endpoint positions given by

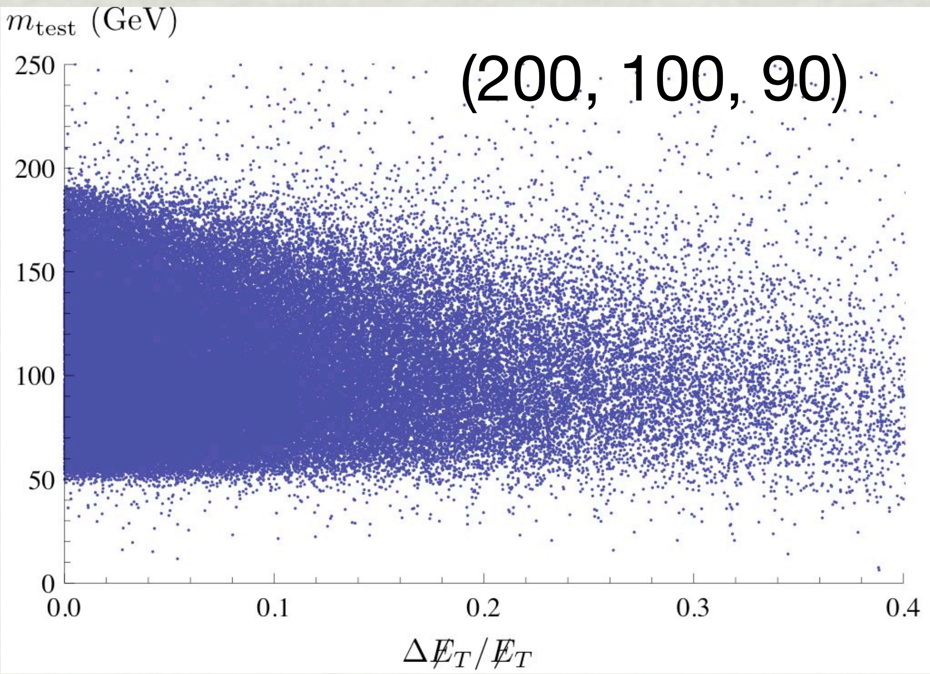
$$m_{\pm}^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^{X}} \frac{1 \pm \beta\, \beta_0^{\chi_1}}{1 \mp \beta\, \beta_0^{X}}$$

- $\beta_0 \to 0$, $m_{\pm}^{\text{test}} \to m_{\chi_1}$

- ***Punchline**: endpoints only depend on the masses*

  --> measure these endpoints experimentally can determine these masses

- 



non-collinear effects

- m_{test} not invariant under boost -- subjet to noncollinear correction

$$m_{\chi_1}^{\text{test}} \approx m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 + \beta\, \beta_0^{\chi_1} \cos\theta_0^a}{1 - \beta\, \beta_0^X \cos\theta_0^a}$$
$$\times \left(1 - \cot\theta_{ab}^X \cos\phi^a \theta^a + \csc\theta_{ab}^X \cos\phi^b \theta^b\right)$$

  - endpoints get smeared;
  - prefer small θ , not too small θ_{ab}^X

- If X's not in the trans. plane, extra projection needed
  -- more complicated in the above θ expansion

Quick Summary

- MET-cone method
- A simple 1D variable m_{test} for mass measurement
- How well this works in simulation?

Numerical study -- simulation

- Use MadGraph to generate 2--> 6 matrix element for SUSY squark production and decay

$$pp \to \tilde{q}_L \tilde{q}_L \to q\, \tilde{\chi}_1\, Z\, q\, \tilde{\chi}_1\, Z$$

- No detector effects included

- Parton-level cuts

p_T^Z	$\lvert\eta^Z\rvert$	$\lvert\eta^{Z,\mathrm{tot}}\rvert$	$\not{E}_T$	ϵ	$\cos\theta^{ab}$
> 50 GeV	< 3.0	< 1.0	> 200 GeV	0.15	< 0.5

PSEUDORAPIDITY OF TOTAL Z MOMENTUM

TWO Z'S OPENING ANGLE

Result

Model Mass Spectrum

	m_{χ_1}	m_{χ_2}	$m_{\tilde{q}_L}$	$(m_-^{\rm test})^{\rm theo}$	$(m_+^{\rm test})^{\rm theo}$
1	100	200	1000	54.6	183.2
2	100	250	1250	21.6	463.0
3	200	300	1000	117.9	339.2
4	200	350	1250	52.6	761.0

Model 1 :

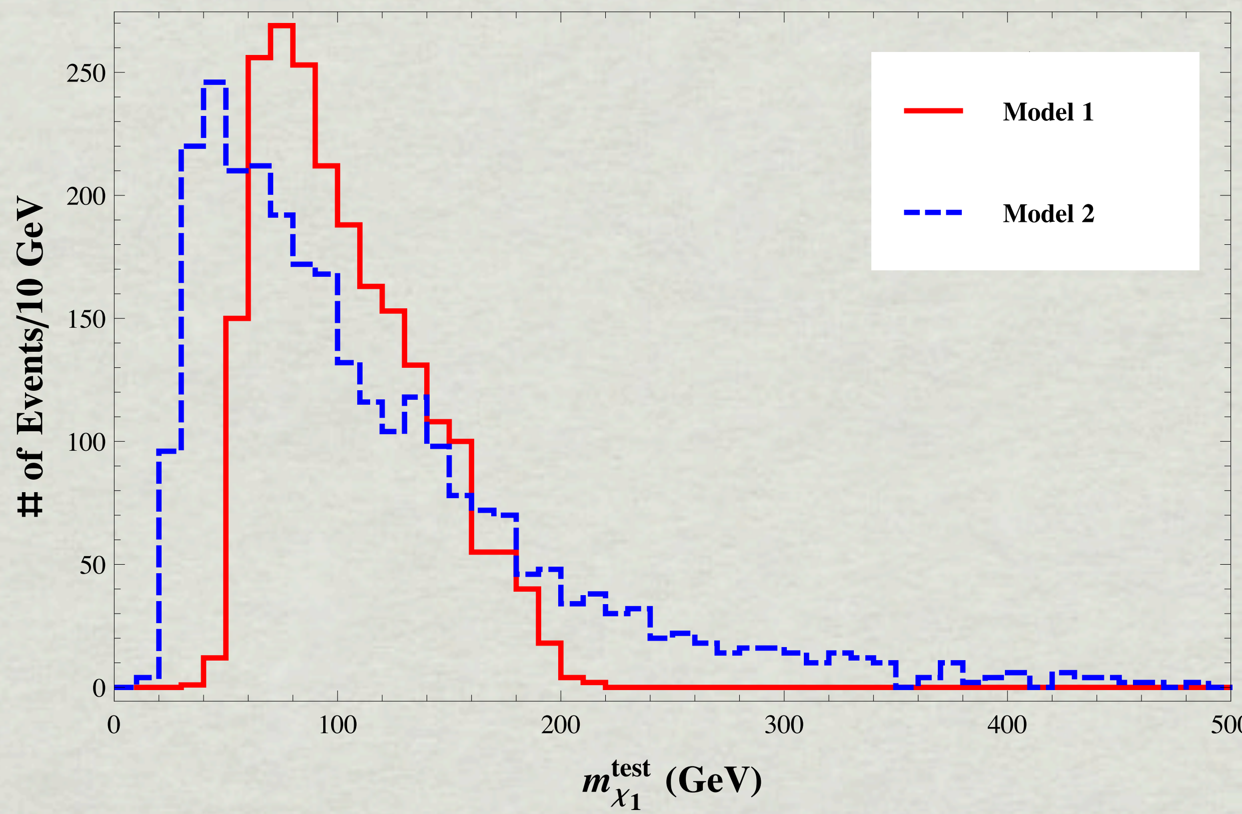
moderate boost + small β_0 --> small variation + sharp endpoints

Model 2 :

moderate boost + large β_0 --> larger variation + fuzzier endpoints

Model 3 & 4 :

even reduced boost

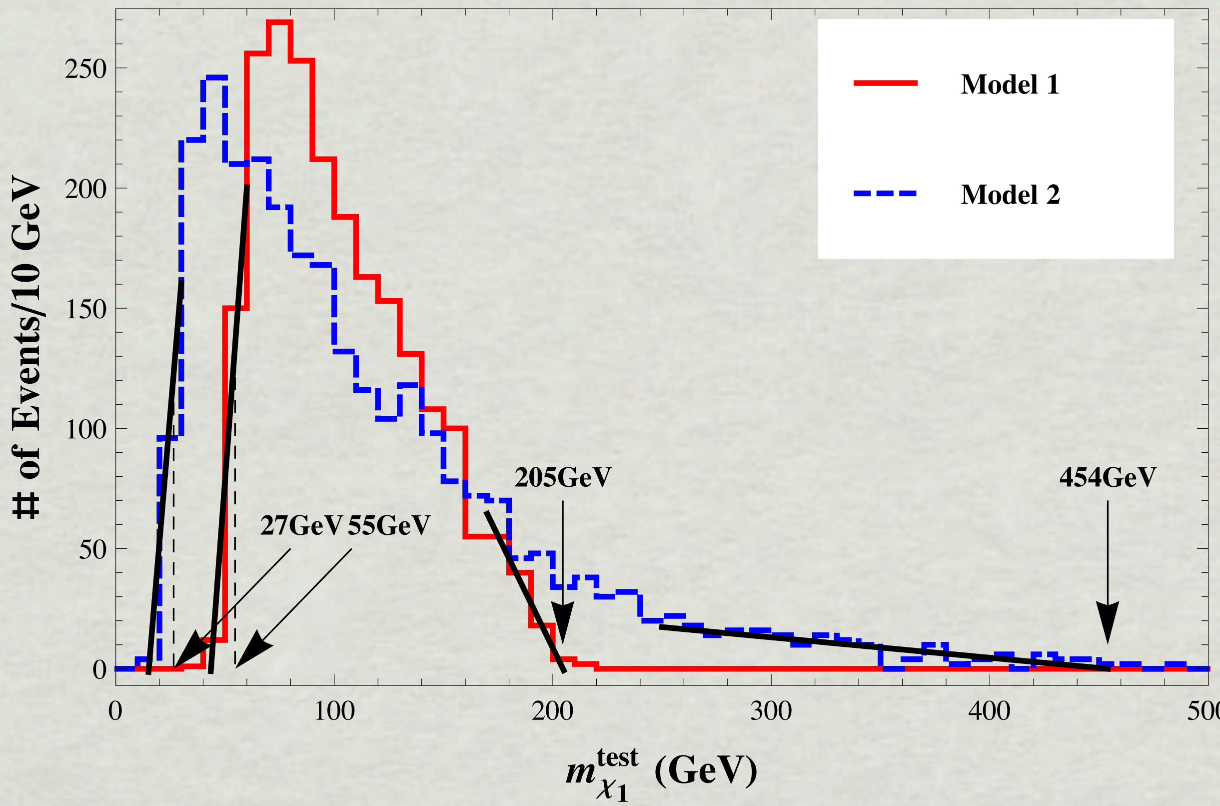


Fit of endpoints

- Use linear fits
  - Lower endpoint -- take half-max pt to reduce smearing effects
  - Upper endpoint -- intercept position
- Better fits are possible
- Larger sys. err. for Model 2 and 4

	$m_{\chi 1}$	$m_{\chi 2}$	m_-^{test}	m_+^{test}	$m_{\chi 1}^{meas}$	$m_{\chi 2}^{meas}$
1	100	200	55 ± 2	205 ± 3	106 ± 2	208 ± 3
2	100	250	27 ± 2	454 ± 20	110 ± 5	253 ± 5
3	200	300	112 ± 5	342 ± 10	195 ± 5	296 ± 5
4	200	350	49 ± 2	682 ± 16	183 ± 5	329 ± 5

Masses are in GeV with statistical error



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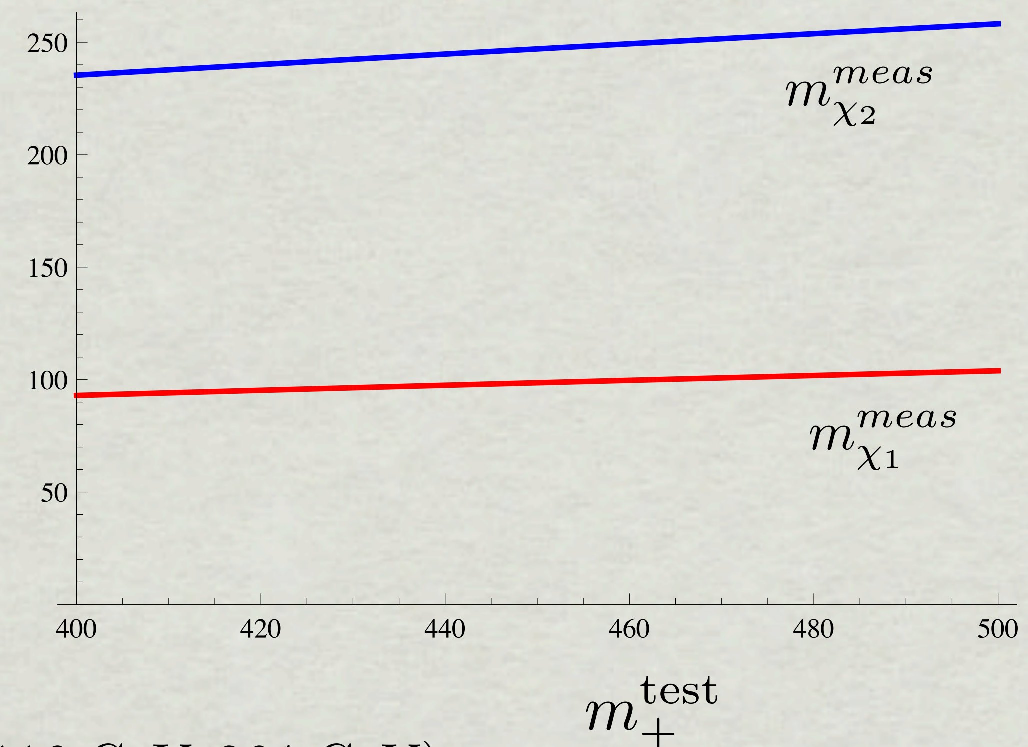


e.g. for Model 2:

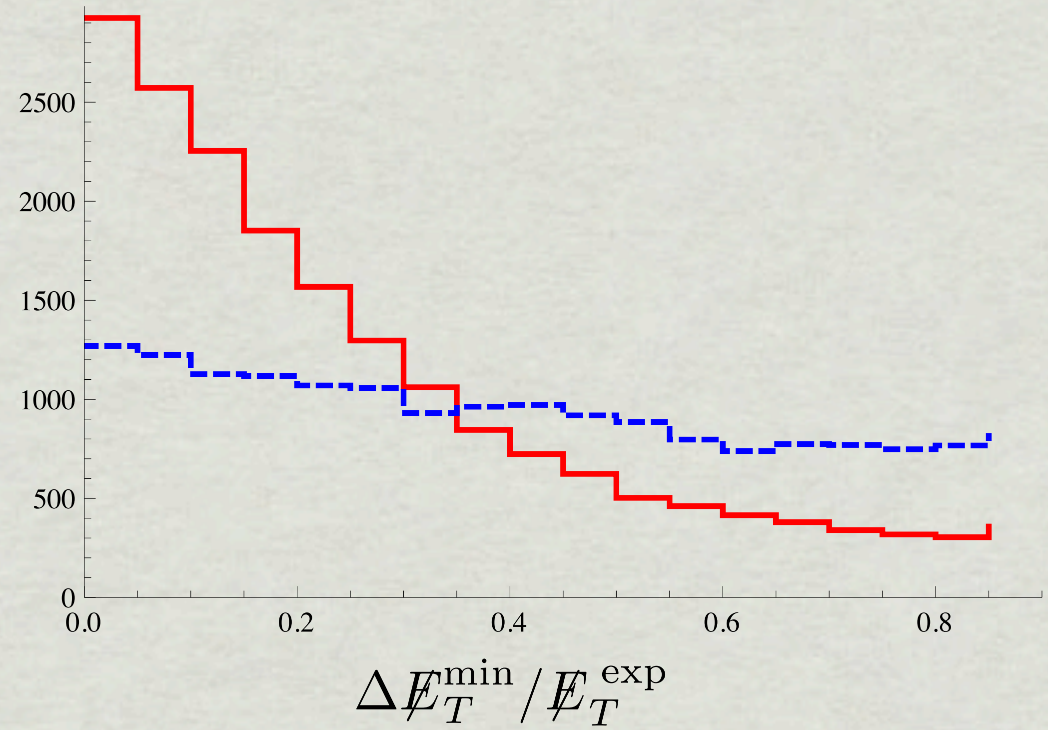
vary upper endpt 400 - 500 GeV

$$(m_{\chi_1}^{meas}, m_{\chi_2}^{meas}) = (103 \text{ GeV}, 241 \text{ GeV}) \text{ — } (116 \text{ GeV}, 264 \text{ GeV})$$

Is it a boosted decay chain?

- $\Delta \not{E}_T^{\min} / \not{E}_T^{\exp}$ distribution

Note: extra information in this distribution is not yet used !



- Sharp endpoints in m_{test} distribution
- Measure upstream exotica masses : show how to determine the squark mass

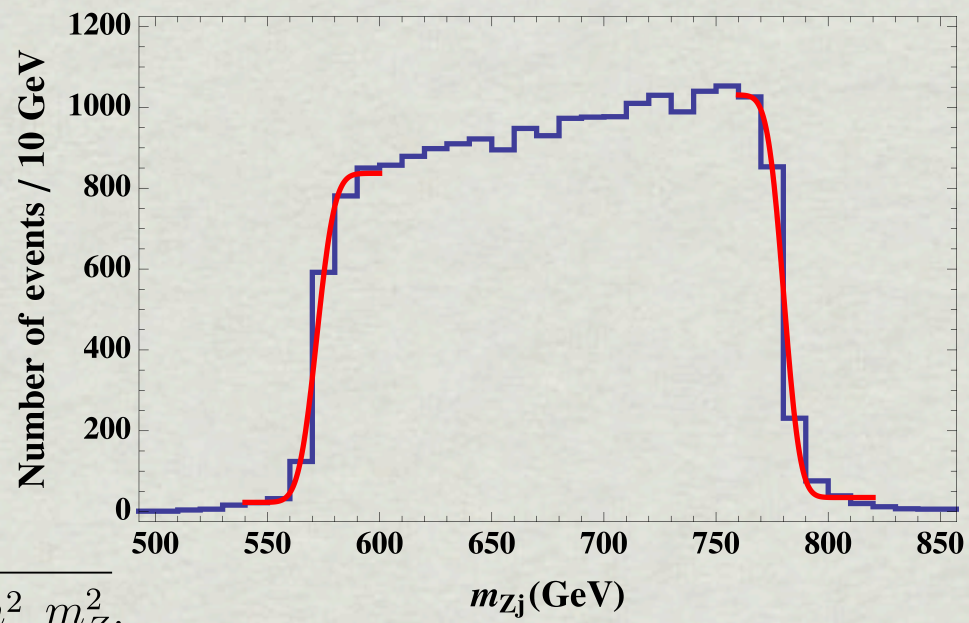
Squark Mass: Use Inv. Mass

- Z-jet invariant mass endpoints

$$(m_{Zj}^{min})^2 = \frac{m_Q^2 - m_{\chi_2}^2}{2m_{\chi_2}^2} A_-,$$

$$(m_{Zj}^{max})^2 = m_Z^2 + \frac{m_Q^2 - m_{\chi_2}^2}{2m_{\chi_2}^2} A_+$$

$$A_\pm \equiv (m_{\chi_2}^2 + m_Z^2 - m_{\chi_1}^2) \pm \sqrt{(m_{\chi_2}^2 - m_Z^2 - m_{\chi_1}^2)^2 - 4m_{\chi_1}^2 m_Z^2}.$$



- Fit with error function

$$\frac{a}{2}\,\mathrm{erf}\left[(x-b)/c\right] + d$$

$$m_{min}^{Zj} = 780.2 \pm 0.5 \text{ GeV}$$
$$m_{max}^{Zj} = 571.9 \pm 0.7 \text{ GeV}$$

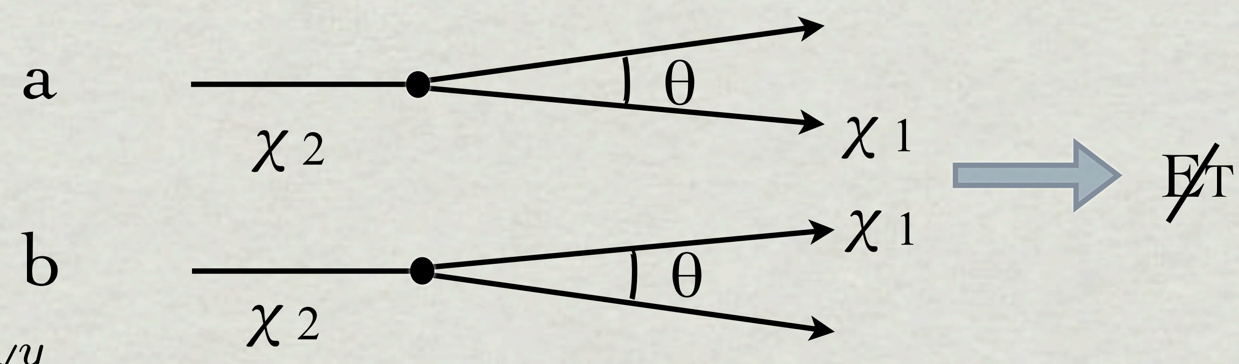
- Using upper endpoint and LSP/NLSP mass measured

$$m_{\tilde{q}} = 1002^{+38}_{-26} \text{ GeV}$$

Use CM energy Variable: $\sqrt{\hat{S}_{\min}}$

- Reconstruct missing particle momenta using collinear approx.

$$\vec{p}_{\chi_1,a} = k_a\, \vec{p}_{X,a}$$
$$\vec{p}_{\chi_1,b} = k_b\, \vec{p}_{X,b}$$



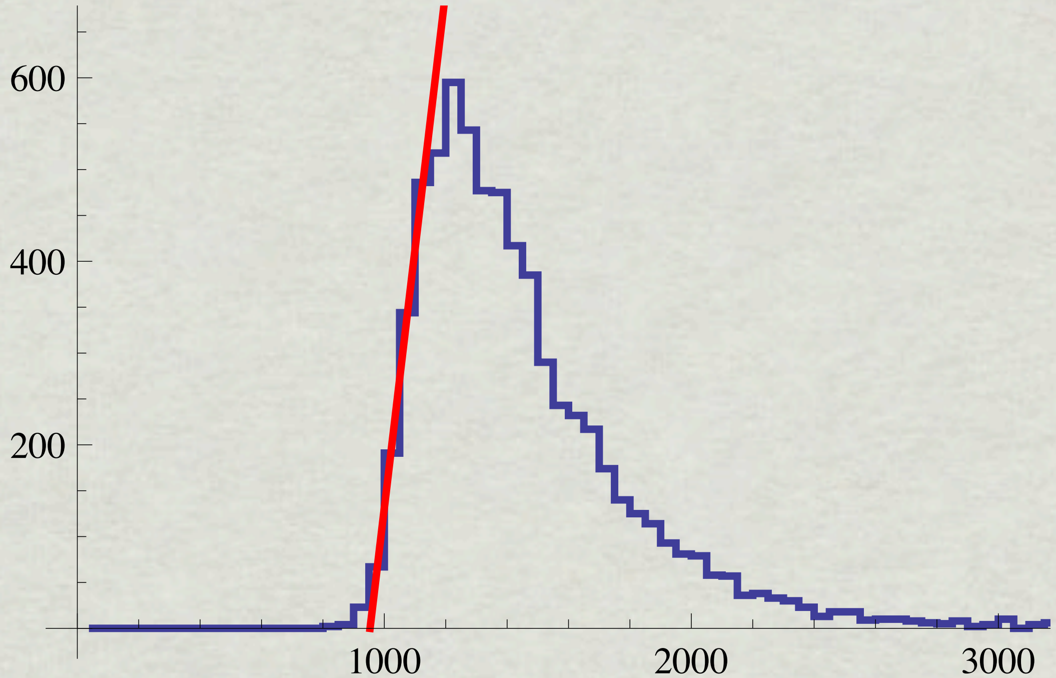
$$\longrightarrow \begin{cases} k_a = \dfrac{p^y_{X,b}\not{p}^x - p^x_{X,b}\not{p}^y}{-p^y_{X,a}p^x_{X,b}+p^x_{X,a}p^y_{X,b}} \\ k_b = \dfrac{-p^y_{X,a}\not{p}^x + p^x_{X,a}\not{p}^y}{-p^y_{X,a}p^x_{X,b}+p^x_{X,a}p^y_{X,b}} \end{cases}$$

- Reconstruct CM energy of the collision $s = \left(\sum_i p_i\right)^2$
- lower endpoint provide an estimate of the mass of mother particle

$$\hat{s} \geq 4m_Q^2$$

Use CM energy Variable:

☆ Use the measured LSP mass and cuts

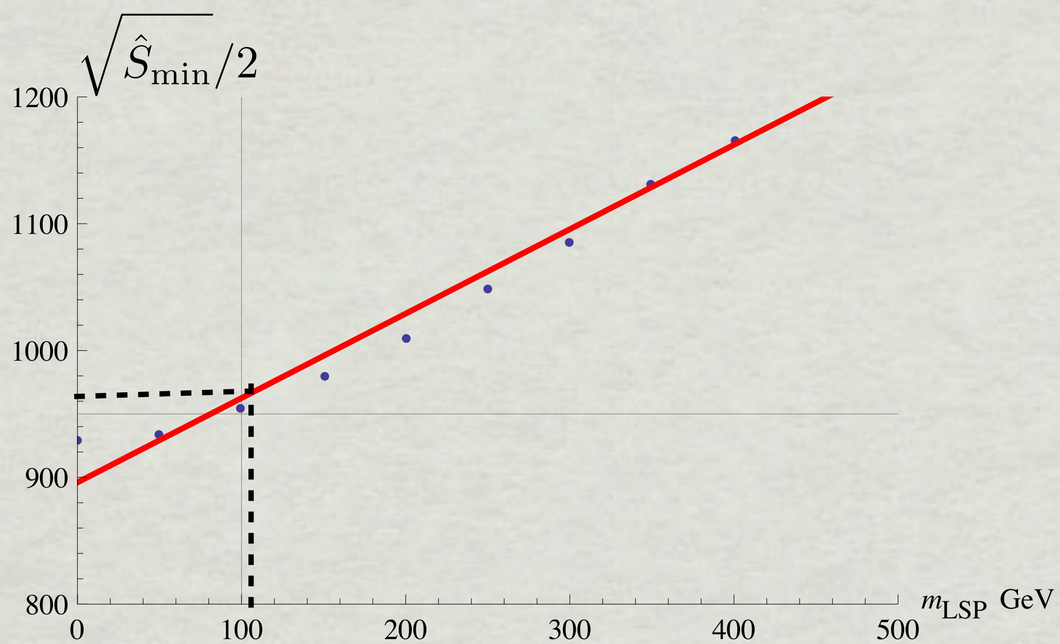


$\sqrt{\hat{s}}/2$

Lower endpoint ~ 960 GeV

- $p_T > 50$ GeV for jet
- $|\eta| < 3$ for jet
- missing E_T cut $E_T^{miss} > 100$ GeV

$p_Z > 300\,\text{GeV}$



Summary and Outlook

- LHC may discover new physics via large $\not{E}_T$, difficult for mass measurement - key information for studying cosmic relic dark matter

- MET-cone and m_{test} variable are useful tools for mass measurement in boosted events with $\not{E}_T$.

- Further explore the idea of MET-cone and develop a more general method that can apply for less-collinear events.

- More realistic collider study: include detector effects on MET, initial/final-state radiation etal