Towards Multi-field Inflation with a Random Potential

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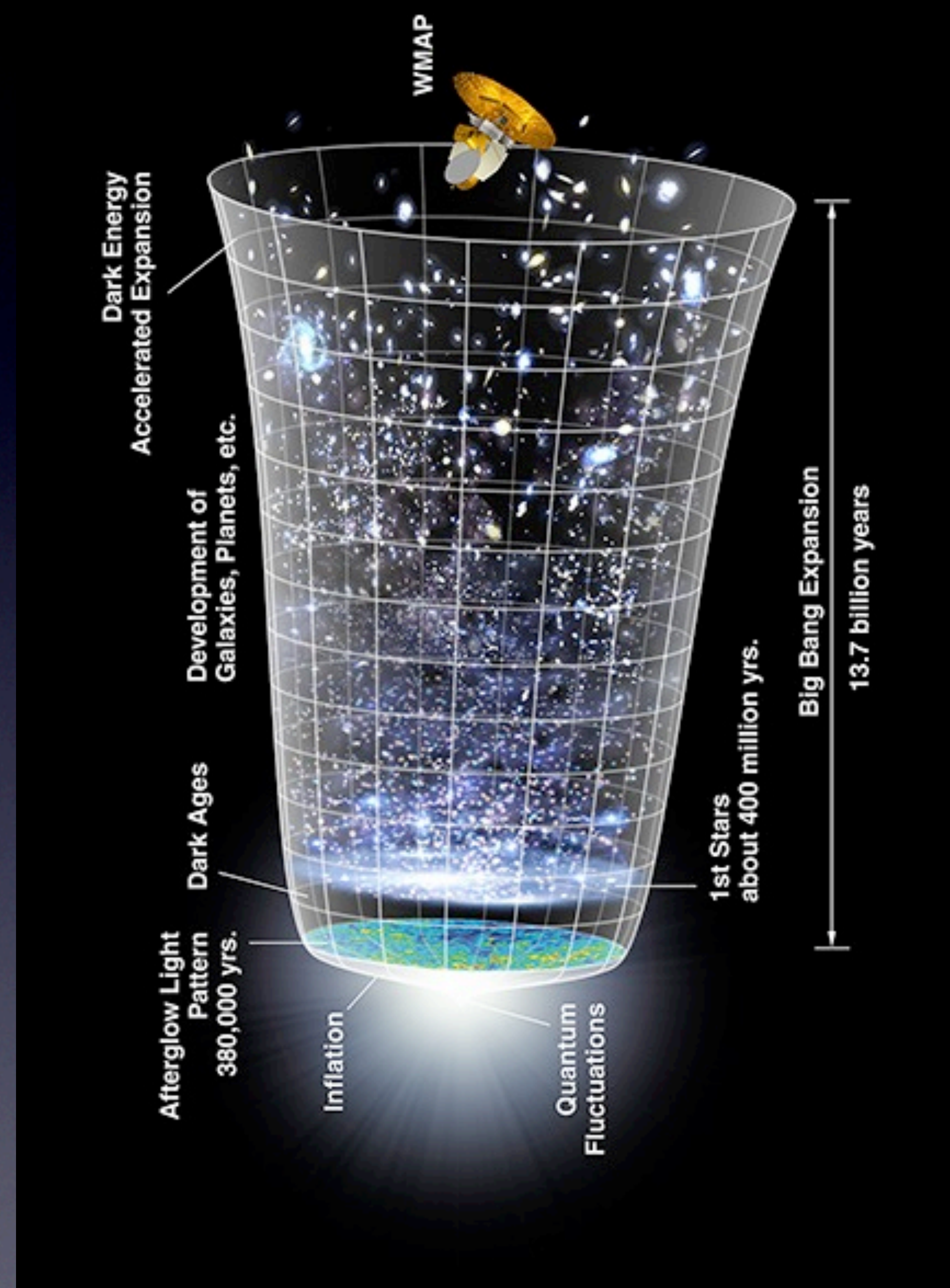
Based on "H. Tye, JX, Y. Zhang, arXiv:0812.1944"
and work in progress

Outline

- Motivation from string theory
- A scenario with random potential
- Primordial perturbations
- Power spectrum, (Non-Gaussianity)
- Anything observable ?
- Conclusions

Inflation is an elegant explanation to:
The flatness problem
The horizon problem
....

The primordial seeds of structure formation

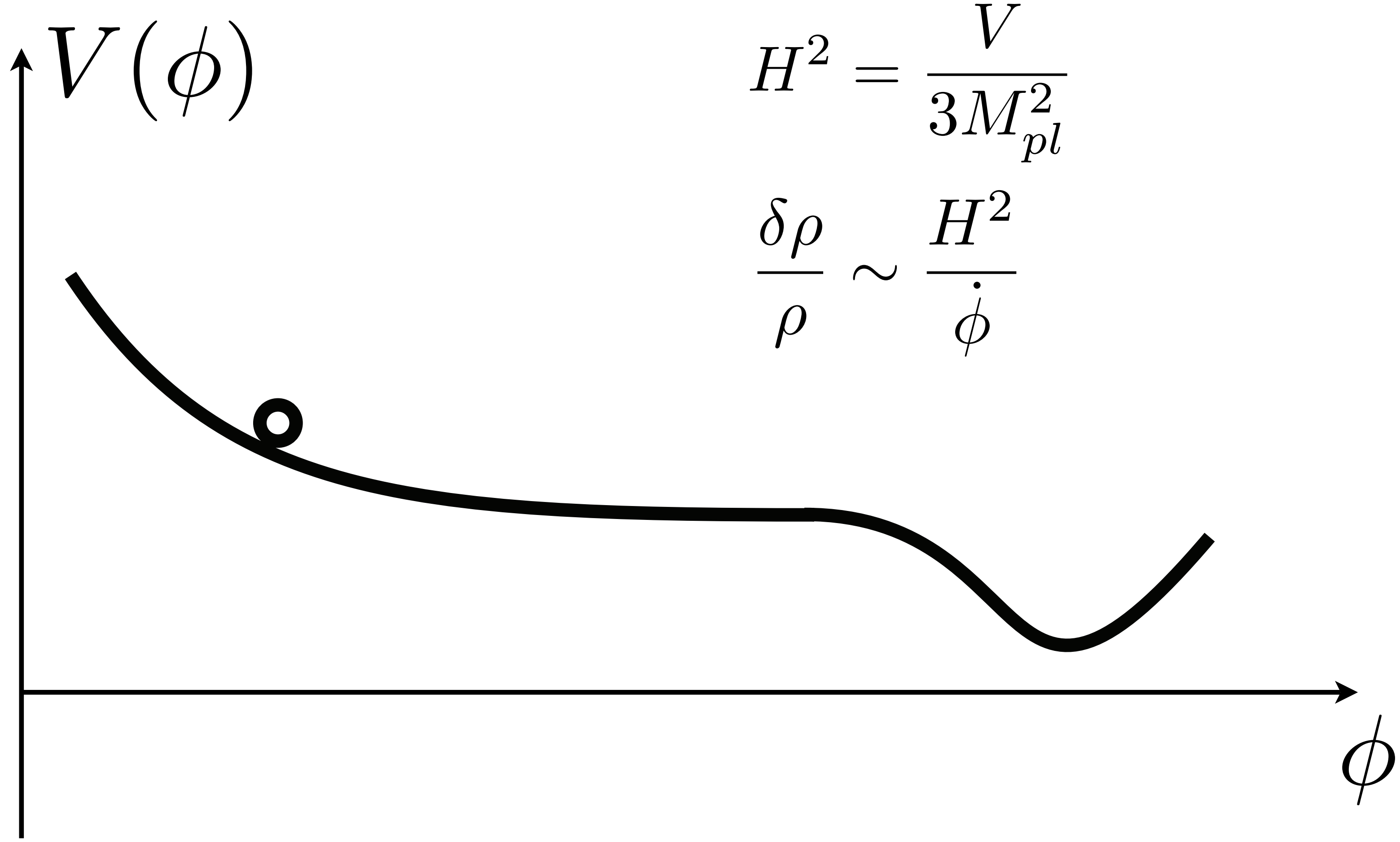


Inflation with a Single Field

one scalar field
+
flat potential

⇒

sufficient expansion
+
nearly scale invariant density perturbations



$$H^2 = \frac{V}{3M_{pl}^2}$$

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}}$$

A Multi-field Perspective

- Many (~100) scalar fields from string theory (moduli, dilaton...) + complicated potential
- Some moduli are stabilized, some participate in inflation with total number $D \gg 1$
- A convoluted inflaton path (random walk)

An Example of Random Potential

$$\begin{aligned} V(\rho_i, \phi_i) &= V_0(\rho_j) + \alpha_i \cos\left(\frac{\phi_i}{f_i}\right) \\ &+ \beta_{ij} \cos\left(\frac{\phi_i}{f_i} - \frac{\phi_j}{f_j}\right) \\ &+ U(\rho_i, \phi_i) \\ &+ \ldots \end{aligned}$$

$$\alpha_i = M_i^4 e^{-S^i_{\text{inst}}} \gg \beta_{ij}$$

periodic and regular

impurities & randomness

- ϕ_i are axion fields serving as inflatons
- $V_0(\rho_i)$ is the moduli potential, contains vacuum energy for inflation, relatively flat
- $U(\rho_i, \phi_i)$ couples moduli and axions, introduces randomness

Inflation with a Random Potential

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi_I - V(\vec{\phi}) \right)$$

$$3M_{pl}^2 H^2 = \frac{1}{2} \sum \dot{\phi_I}^2 + V \qquad \dot{H} = -\frac{1}{2M_{pl}^2} \sum \dot{\phi}_I^2$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\sum \dot{\phi}_I^2}{2M_{pl}^2 H^2} \qquad \dot{\sigma}^2 \equiv \sum \dot{\phi}_I^2$$

- $V(\vec{\phi})$ provides the energy for inflation
- $\dot{\sigma}^2 < H^2 M_{pl}^2 \sim V(\vec{\phi})$, so that $\epsilon < 1$
- The inflaton scatters while drifting down the potential, with drift velocity $\vec{v}$
- $\chi \equiv \frac{\dot{\sigma}^2}{\vec{v}^2} \gg 1$, the "refraction index"

The Fokker-Planck Description

$$\frac{\partial P}{\partial t} = -\nabla \cdot \left[\vec{v}(\vec{\phi})\, P \right] + \partial_I \partial_J \left[D^{IJ}(\vec{\phi})\, P \right]$$

The simplest case: $D^{IJ} = \lambda \delta^{IJ} \quad v(\vec{\phi}, t) = const$

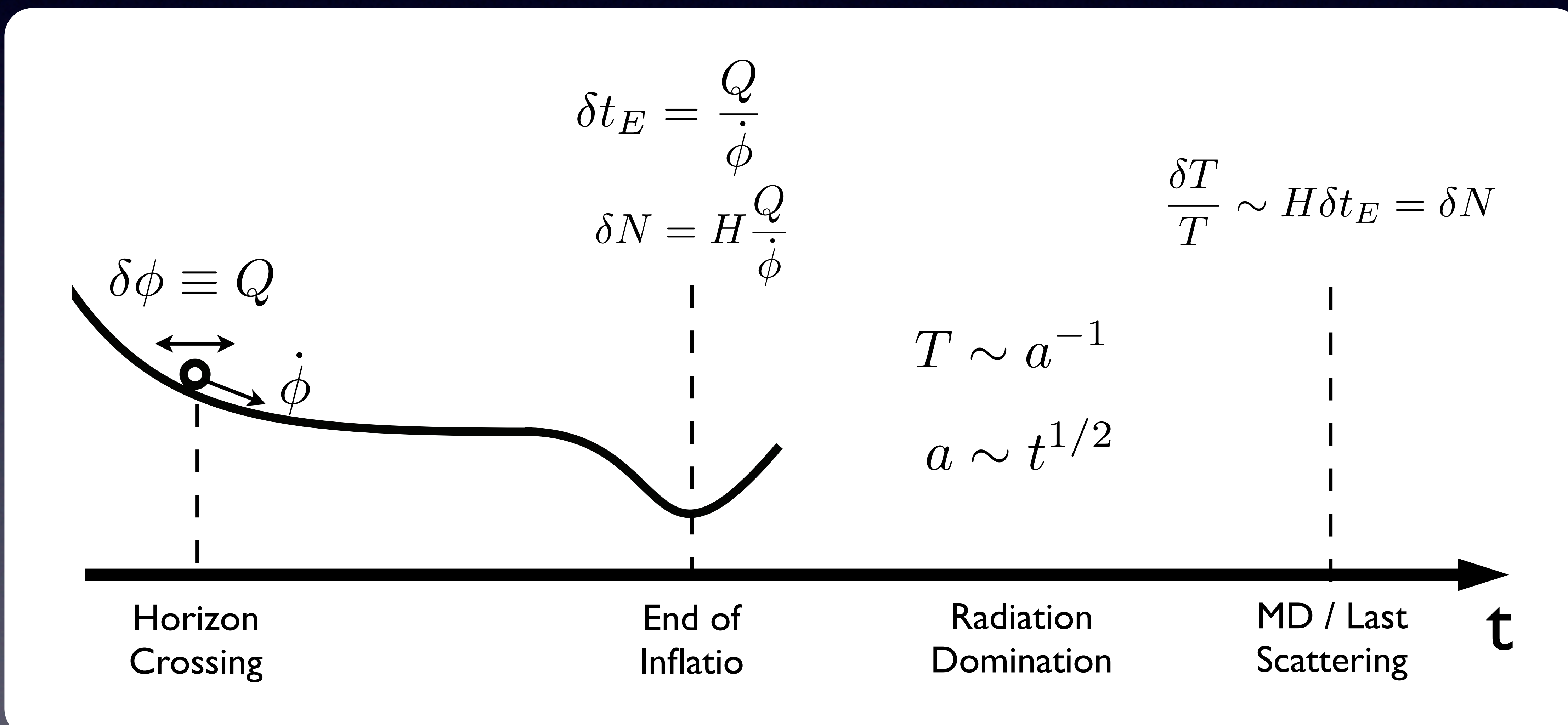
$$P(\vec{\phi}, t) = (4\pi\lambda\, t)^{-\frac{D}{2}} \, \exp\left(- \frac{|\vec{\phi} - \vec{v}\, t|^2}{4\lambda\, t} \right)$$

- $P(\vec{\phi}, t)$ is the field space probability density
- No density perturbations so far, this is the homogeneous background, no matter how convoluted the path is.

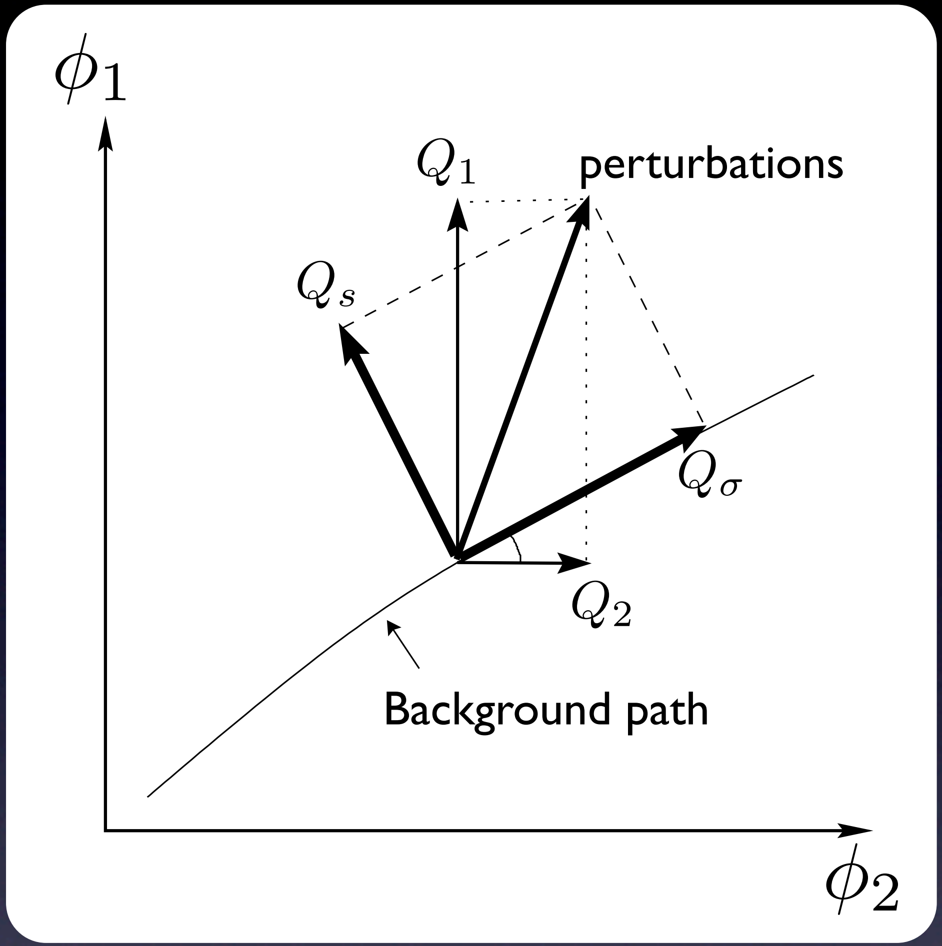
Primordial Perturbations

$$ds^2 = -(1-2\Phi)dt^2 + a(t)^2(1+2\Phi)dx_i\,dx^i$$

$$\zeta = \Phi - H\frac{\delta\rho}{\dot\rho}\ , \quad Q_I = \delta\phi_I - \frac{\dot\phi_I}{H}\Phi \qquad\qquad \zeta = -\frac{H}{\dot\phi}Q \quad (k \ll aH)$$



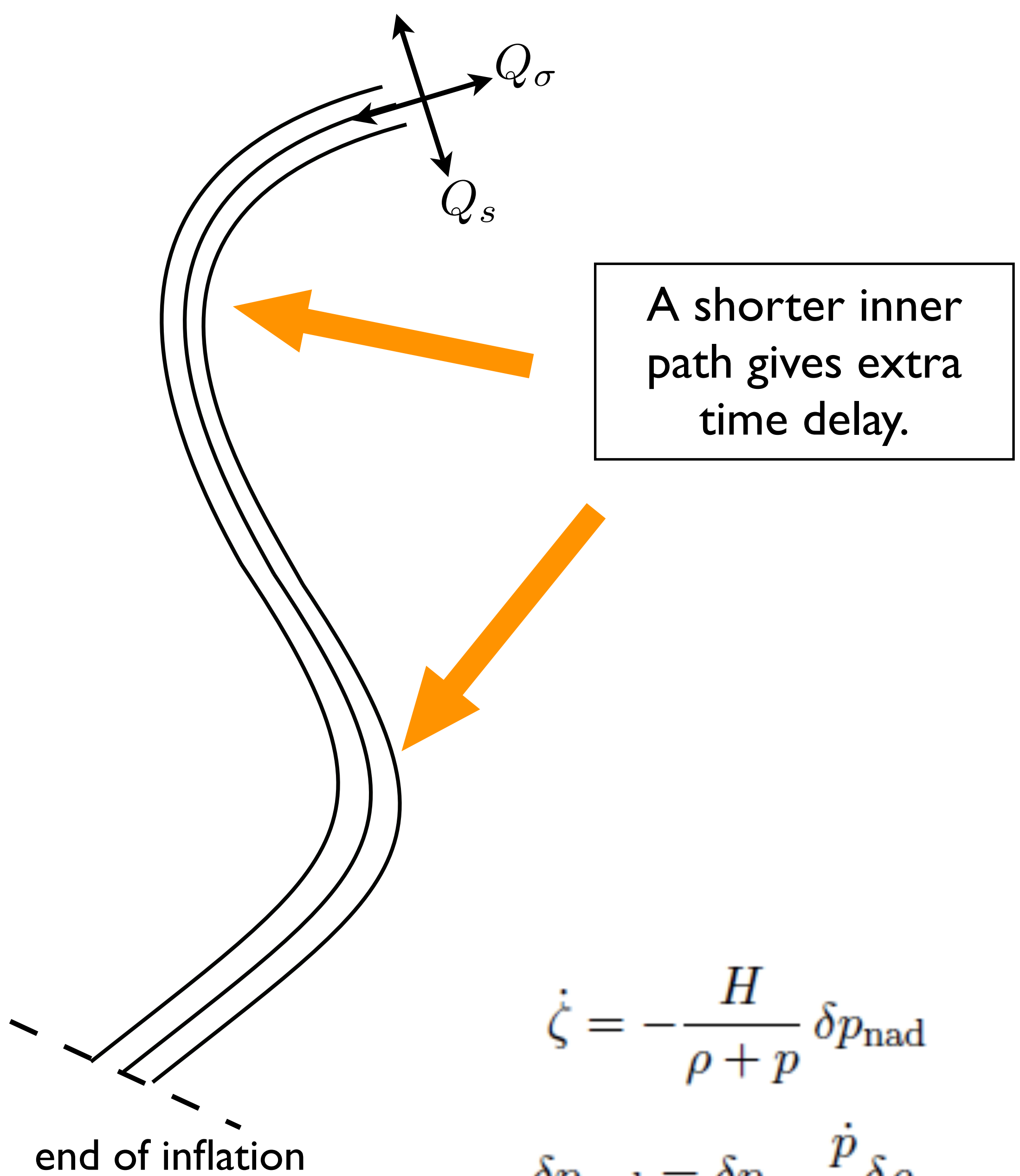
Adiabatic and Entropic Modes



[C. Gordon, D. Wands, B. A. Bassett, R. Maartens, astro-ph/0009131]

$$\zeta = -\frac{H}{\dot{\sigma}} Q_\sigma \ , \quad Q_\sigma \equiv Q_I e^I_\sigma$$

- One adiabatic mode Q_σ tangent to the inflaton path, leads to $\delta\rho \neq 0$.
- (D-1) Entropic modes Q_s orthogonal to the inflaton path, do not perturb the energy density $\delta\rho = 0$.
- Entropic perturbations only exist for multi-field inflation.



$$\dot{\zeta} = -\frac{H}{\rho + p}\,\delta p_{\rm nad}$$

$$\delta p_{\rm nad} = \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho$$

- In single field inflation, $\dot{\zeta} = 0$ after horizon crossing.
- A bending path convert Q_s into time delay, leading to super-horizon evolution of ζ.

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}}\,\dot{e}_\sigma^I\,Q_I$$

Power Spectrum

- For single field inflation, one evaluates $\langle \zeta^2 \rangle$ at the time of horizon crossing t_*. $\dot{\zeta} = 0$ ensures the result does not change by the end of inflation.

- For multi-field inflation, generically $\dot{\zeta} \neq 0$,

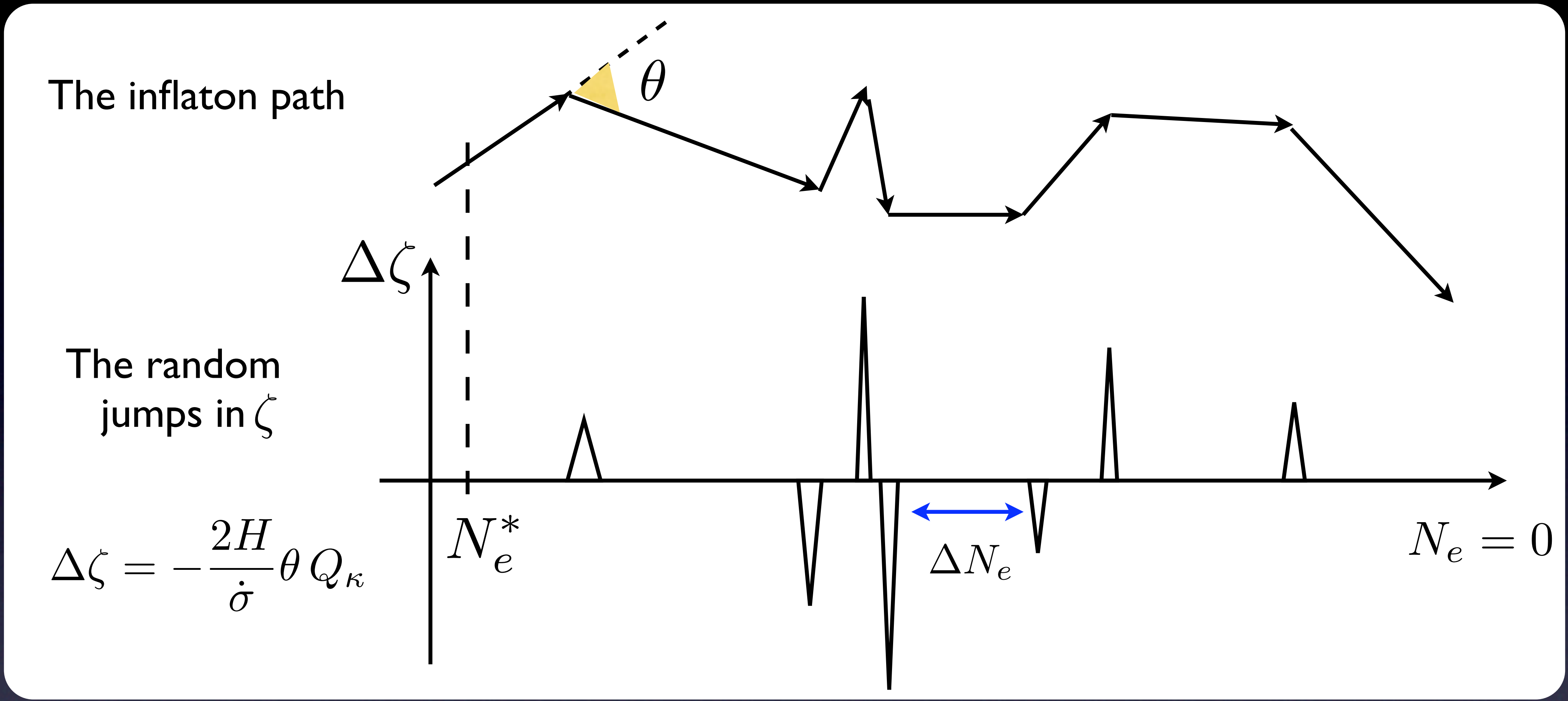
$$\langle \zeta^2 \rangle \Big|_{t_*} \neq \langle \zeta^2 \rangle \Big|_{t_E}$$

In principle, one can integrate

$$\dot{\zeta} = -\frac{2H}{\dot{\sigma}} \dot{e}^I_\sigma Q_I \qquad \dot{e}^I_\sigma = \frac{de^I_\sigma}{dt} = \frac{d\theta}{dt}$$

- However, the inflaton path is a random walk, so θ is a random variable !

Random Jumps in ζ



- From t_* to t_E, there are $N_e^*/\Delta N_e$ random jumps, giving

$$\sum \Delta\zeta \sim \frac{2H}{\langle\dot\sigma\rangle}\Theta\, Q_\kappa \sqrt{\frac{N_e^*}{\Delta N_e}}$$

$$\Theta^2 \equiv \langle\theta^2\rangle$$

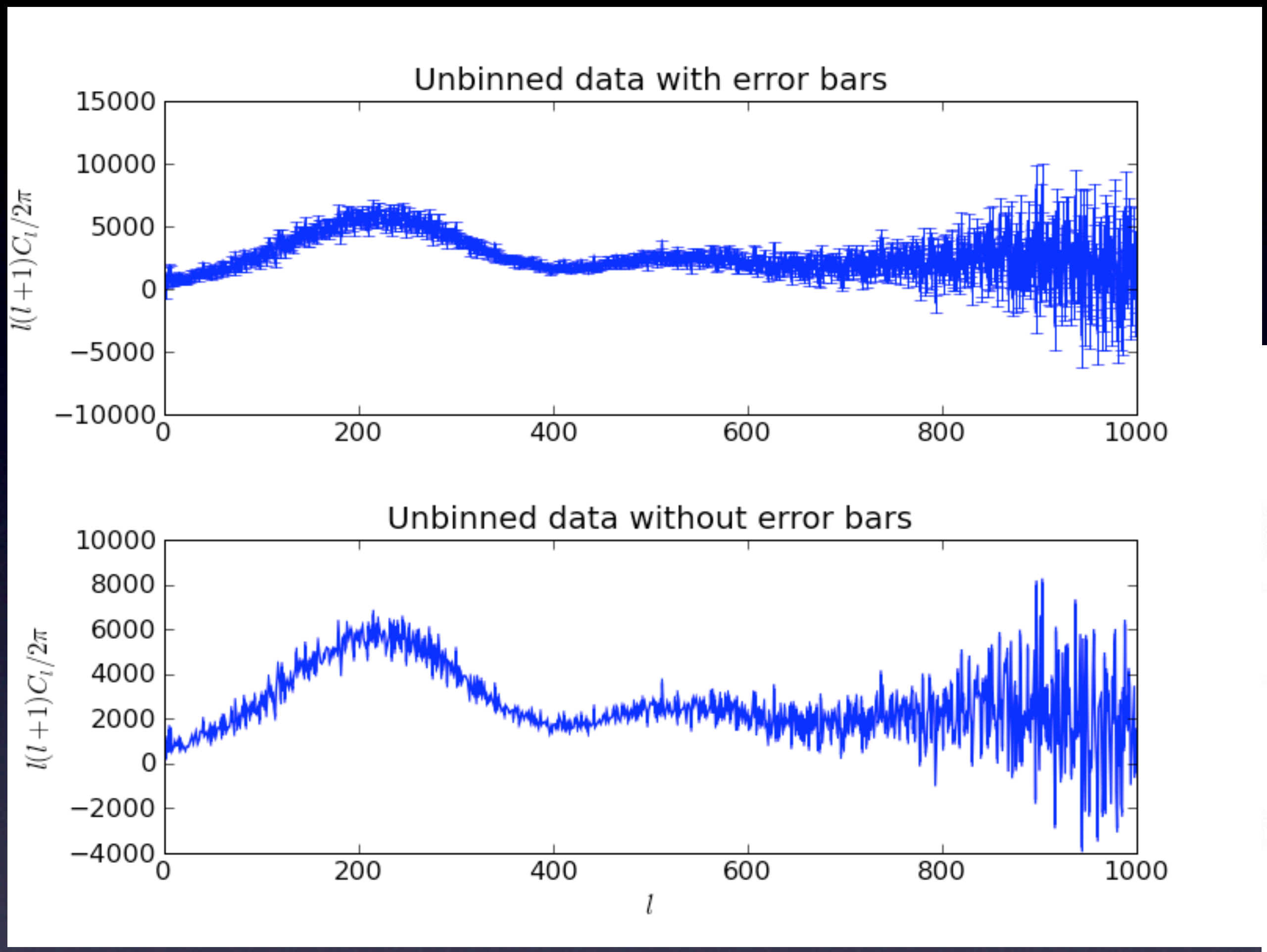
$$P_\zeta(k) \sim \left[\frac{H^4}{4\pi^2\dot{\sigma}^2} + \frac{H^4}{4\pi^2\langle\dot{\sigma}\rangle^2}\vartheta N_e\right]\Big|_{k=aH} \qquad \vartheta N_e \equiv 4(D-1)\Theta^2\frac{N_e}{\Delta N_e}$$

- The first term comes from the adiabatic mode. The second term is the contribution from the entropic modes.
- The first term is "wrong", it should exhibit fluctuations due to the randomness in $\dot{\sigma}^2$.
- The entropic contributions dominate,

$$P_\zeta(k) \sim \frac{H^4}{4\pi^2\langle\dot{\sigma}\rangle^2}\vartheta N_e \sim \frac{H^2}{8\pi^2 M_{pl}^2\epsilon}\vartheta N_e$$

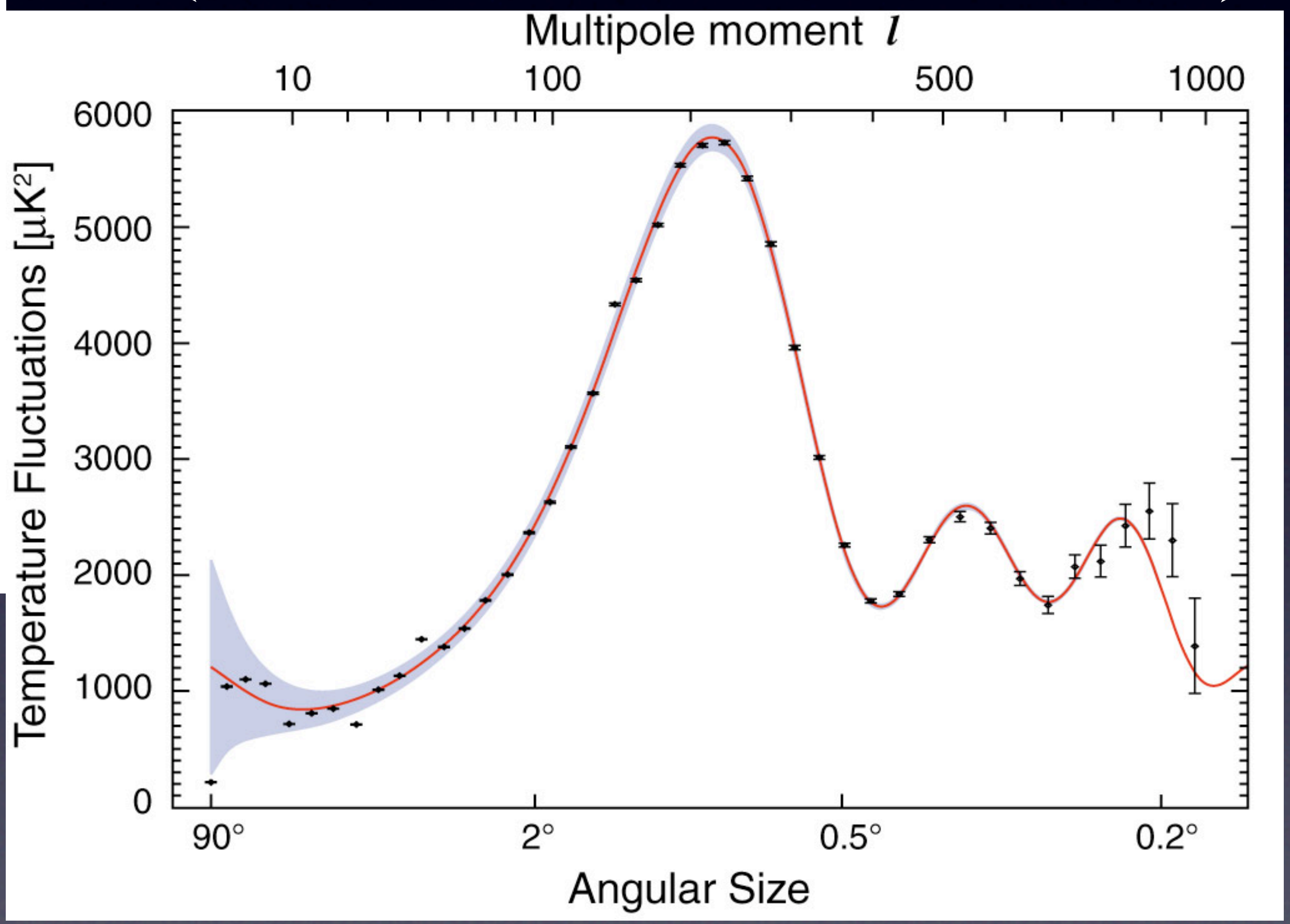
$$n_s - 1 \equiv \frac{d\ln P_\zeta}{d\ln k} \sim -2\epsilon - \eta - \frac{1}{N_e}$$

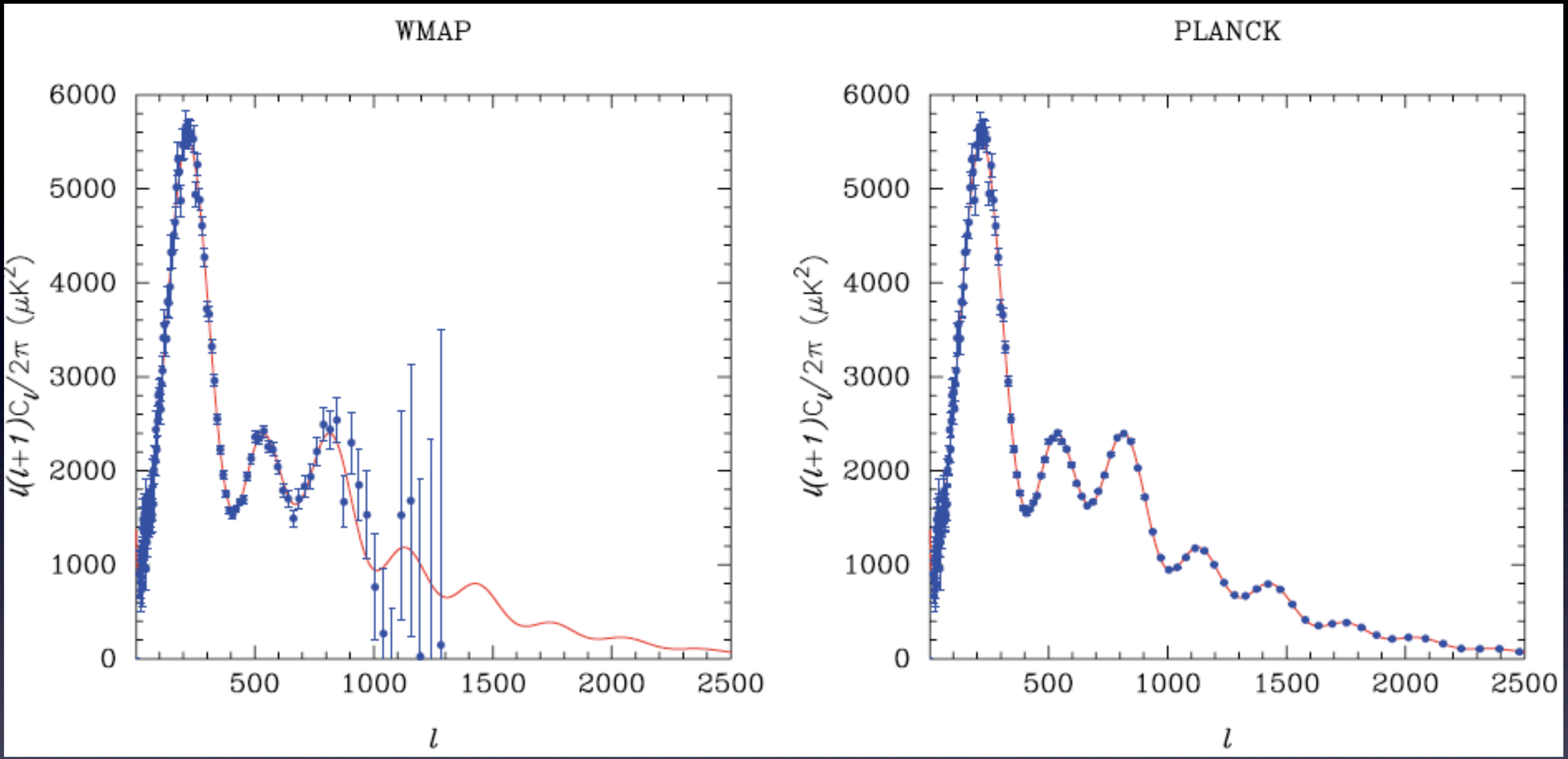
Fluctuations in the Power Spectrum



20-30 multiples/bin

3~4 efolds,
200~300 multiples/efold





On small angular scales, PLANCK has much better systematics than WMAP, and cosmic variance is negligible. There may be a better chance to see the fluctuations. Also check both TT and TE spectrums.

Small Non-Gaussianity

- Generically, order of magnitude

$$f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \qquad \zeta \sim \sqrt{\frac{N_e}{\Delta N_e}}\Theta$$

$$f_{NL} \sim \frac{D^3\Theta^3 \left(\frac{N_e}{\Delta N_e}\right)^{3/2}}{\left(D^2\Theta^2 \frac{N_e}{\Delta N_e}\right)^2} \sim \frac{1}{D\Theta\sqrt{\frac{N_e}{\Delta N_e}}}$$

Conclusions

- Multi-field Inflation with a random potential is a natural scenario motivated by the string landscape
- The random walk nature of the inflaton allows the entropic modes to feed into the adiabatic mode in a random way. Our analysis is only the first step towards fully quantifying such an effect.
- The final power spectrum has a dominant contribution from entropic modes, while the adiabatic mode gives fluctuations, might be observable by PLANCK.
- Non-Gaussianity is generically small.