Stability in and of de Sitter space

arXiv : 0911.3142 (hep-th)

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Outline

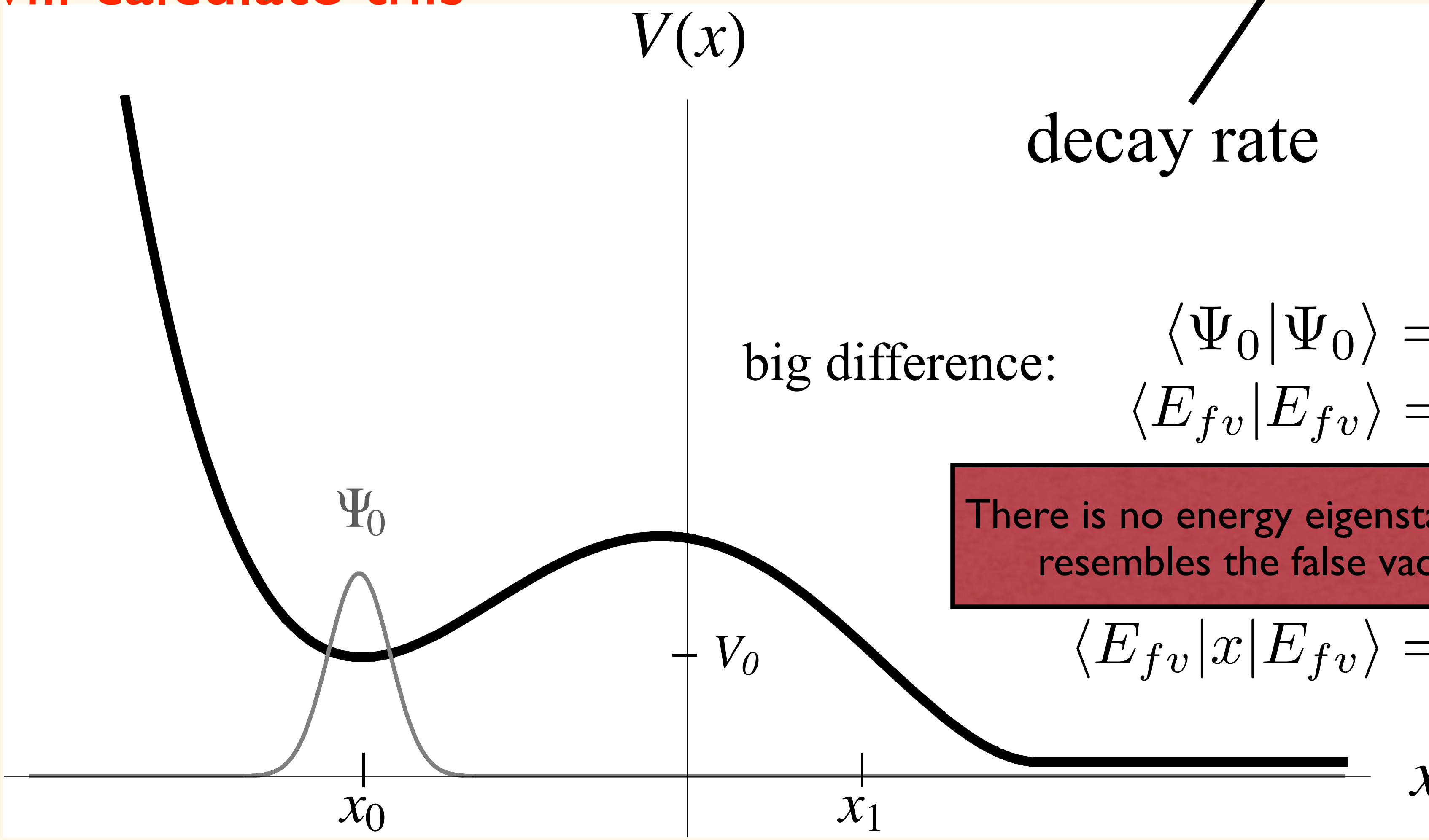
- Review of instantons and exponential decay
- The asymmetric double well
- Field theory in flat space & finite volume
- Coleman - De Luccia
- Canonical quantization methods and the failure of the probe approximation
- Outlook

exponential decay

$$|\langle\Psi_0|\exp(-iHt)|\Psi_0\rangle|^2 \approx \exp(-\Gamma|t|)$$

We will calculate this

decay rate



big difference:

$$\langle\Psi_0|\Psi_0\rangle = 1$$

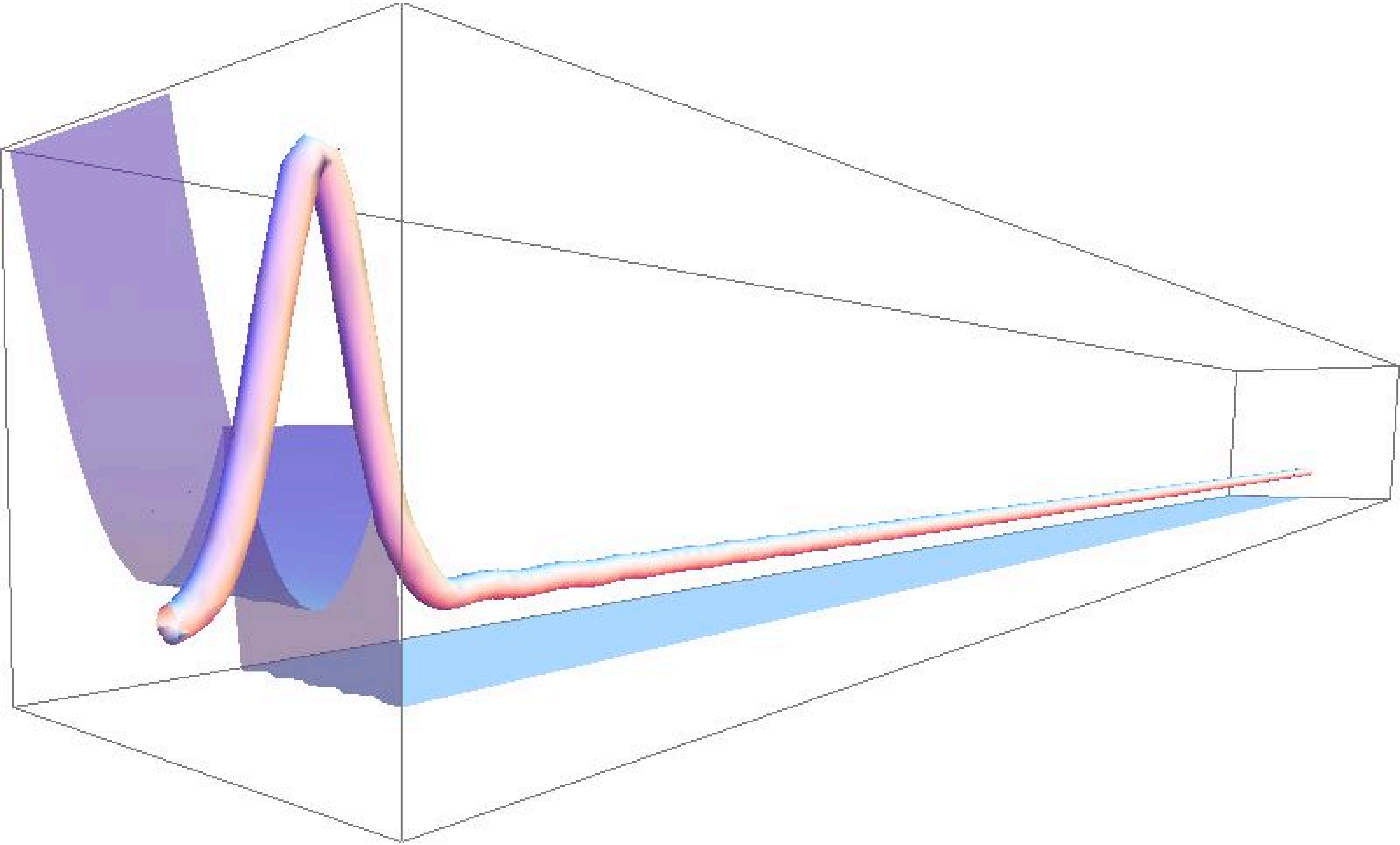
$$\langle E_{fv}|E_{fv}\rangle = \infty$$

There is no energy eigenstate which resembles the false vacuum.

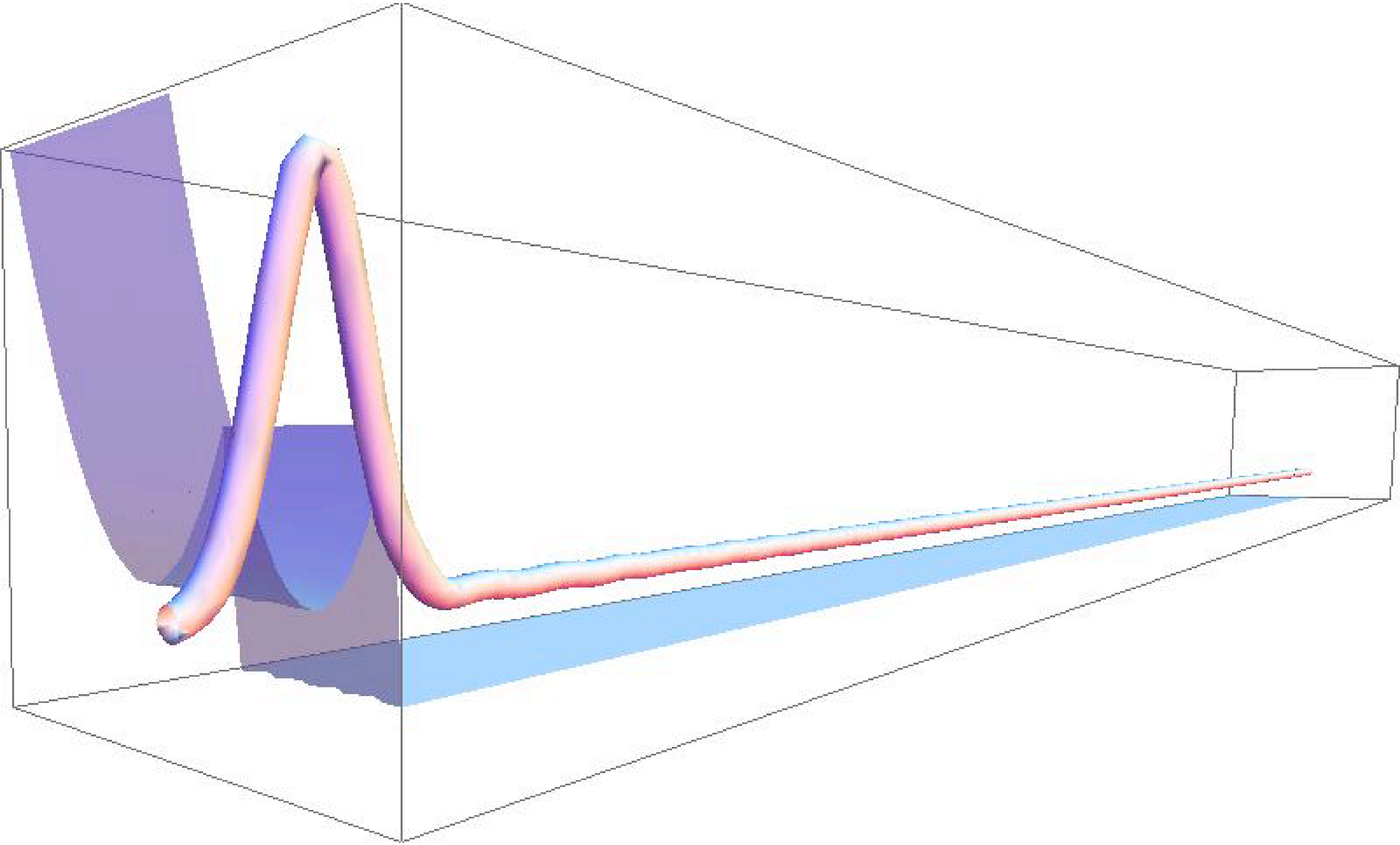
$$\langle E_{fv}|x|E_{fv}\rangle = \infty$$

"false vacuum" $|\Psi_0\rangle$ = ground state of *perturbative* Hamiltonian

Early time behavior: Probability current flows outward.



Early time behavior: Probability current flows outward.



Late time behavior: Survival probability is zero. (not pictured)

instanton method

S. Coleman
"The uses of instantons"

$$|\langle\Psi_0|\exp(-iHt)|\Psi_0\rangle|^2 \approx \exp(-\Gamma|t|)$$

$$\langle x_f|\mathrm{e}^{-iHt}|x_i\rangle = \int [dx]\mathrm{e}^{iS[x]}$$

$$t \to -it = -T$$

This is the *definition* of the path integral (consistent with Feynman's pole prescription)

$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau$$

$$V(x) \to -V(x)$$

$$\langle x_0|\mathrm{e}^{-HT}|x_0\rangle = \int [dx]\,\mathrm{e}^{-S_E[x]}$$

On the left: Large T picks out the lowest energy states

On the right: We will use method of steepest descent to calculate the late time behavior of the low energy states.

$$H = H_{pert.} + \Delta$$

Assume no degeneracies in $H_{pert.}$ (no resonances)

$$\langle x_0 | \mathrm{e}^{-HT} | x_0 \rangle = \sum_n \langle x_0 | \, \mathrm{e}^{-(H_{pert.}+\Delta)T} \, | n \rangle \langle n | x_0 \rangle$$

H is unbounded below

$$\xrightarrow{\text{large } T} \mathrm{e}^{-(E_{fv}+\delta_{fv})T} \, |\langle x_0 | \Psi_0 \rangle|^2$$

Degeneracies mean $H_{pert.}$ basis has arbitrariness which may be totally destroyed by effects of Δ.

If we treat Ψ_0 like an eigenstate of H, its energy has an imaginary part.

Actually, Δ has off-diagonal terms which preserve unitarity.

$$\delta_{fv} = -\frac{i}{2}\Gamma$$

non-perturbative correction to false-vacuum "energy"

$$|\langle \Psi_0 | \exp(-iHt) | \Psi_0 \rangle|^2 \approx \exp(-\Gamma |t|)$$

Method of steepest descent

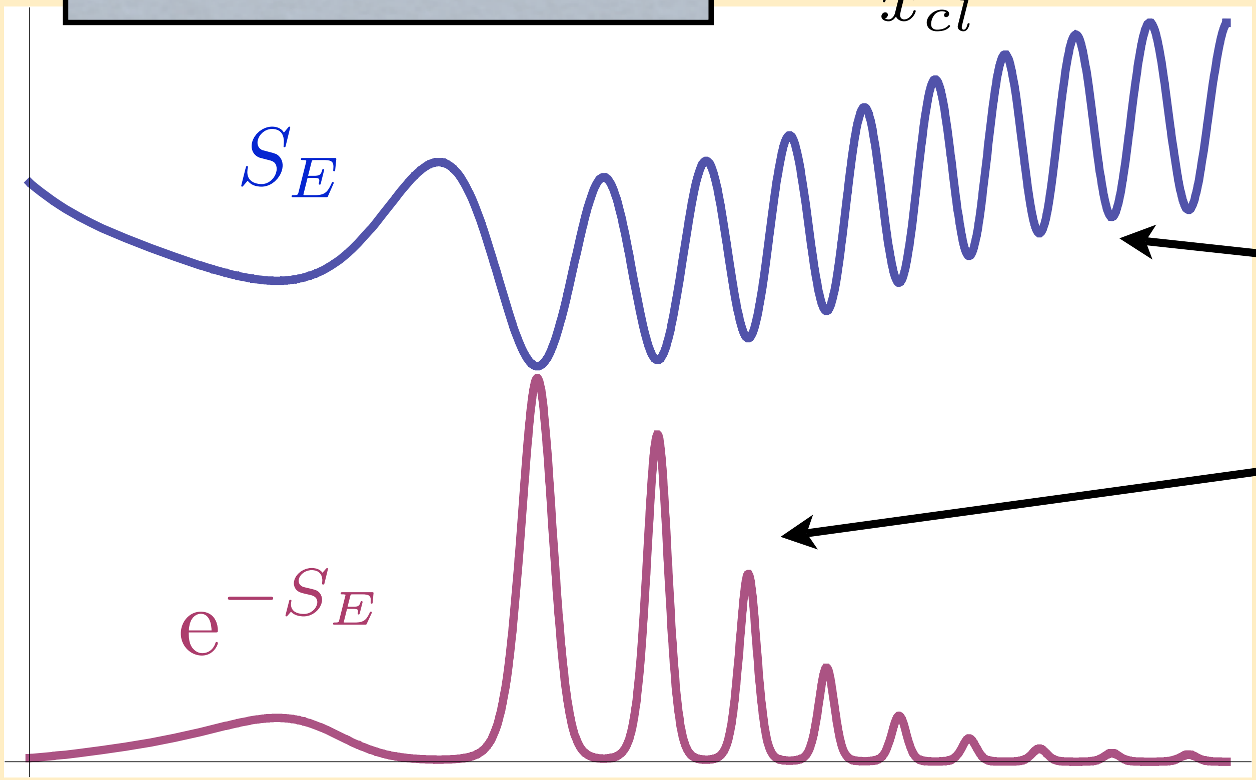
$$S_E[x] \approx \sum_{x_{cl}} S_E[x_{cl}] + \frac{1}{2}\delta x \left.\frac{\delta^2 S_E}{\delta x^2}\right|_{x_{cl}} \delta x$$

$$\left.\frac{\delta^2 S_E}{\delta x^2}\right|_{x_{cl}} = -\partial_\tau^2 + V''(x_{cl}(\tau))$$

Fluctuations = measure

expand about the local minima of $S_E[x]$

$$\int [dx] e^{-S_E[x]} \approx \sum_{x_{cl}} e^{-S_E[x_{cl}]} \sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$



many minima

many Gaussians

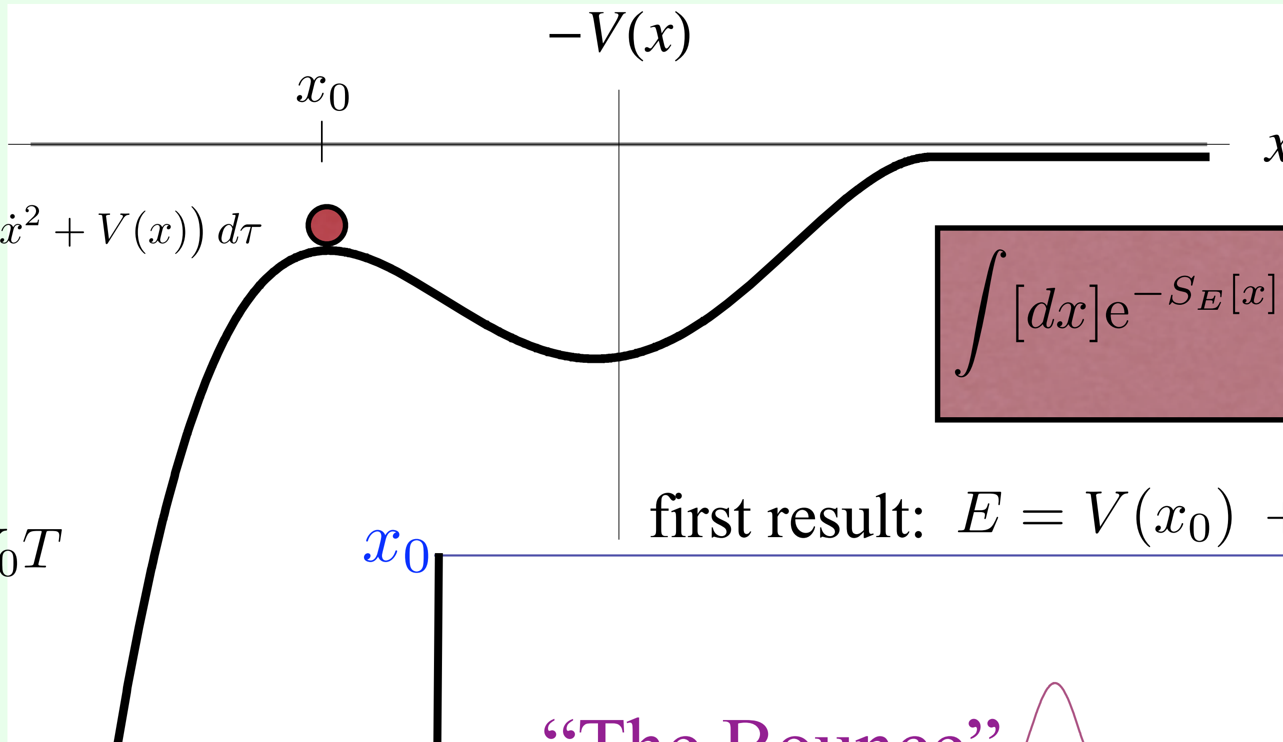
$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} e^{-\lambda x^2} = \frac{1}{\sqrt{\lambda}}$$

What are the x_{cl} ?

any number of bounces ≥ 0.

"instantons"

$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau$$



$$\int [dx] \mathrm{e}^{-S_E[x]} \approx \sum_{x_{cl}} \mathrm{e}^{-S_E[x_{cl}]} \sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$

first result: $E = V(x_0) + \frac{\hbar}{2}\sqrt{V''(x_0)} + \mathcal{O}(\hbar^2)$

$$S_E[x_0] = V_0 T$$

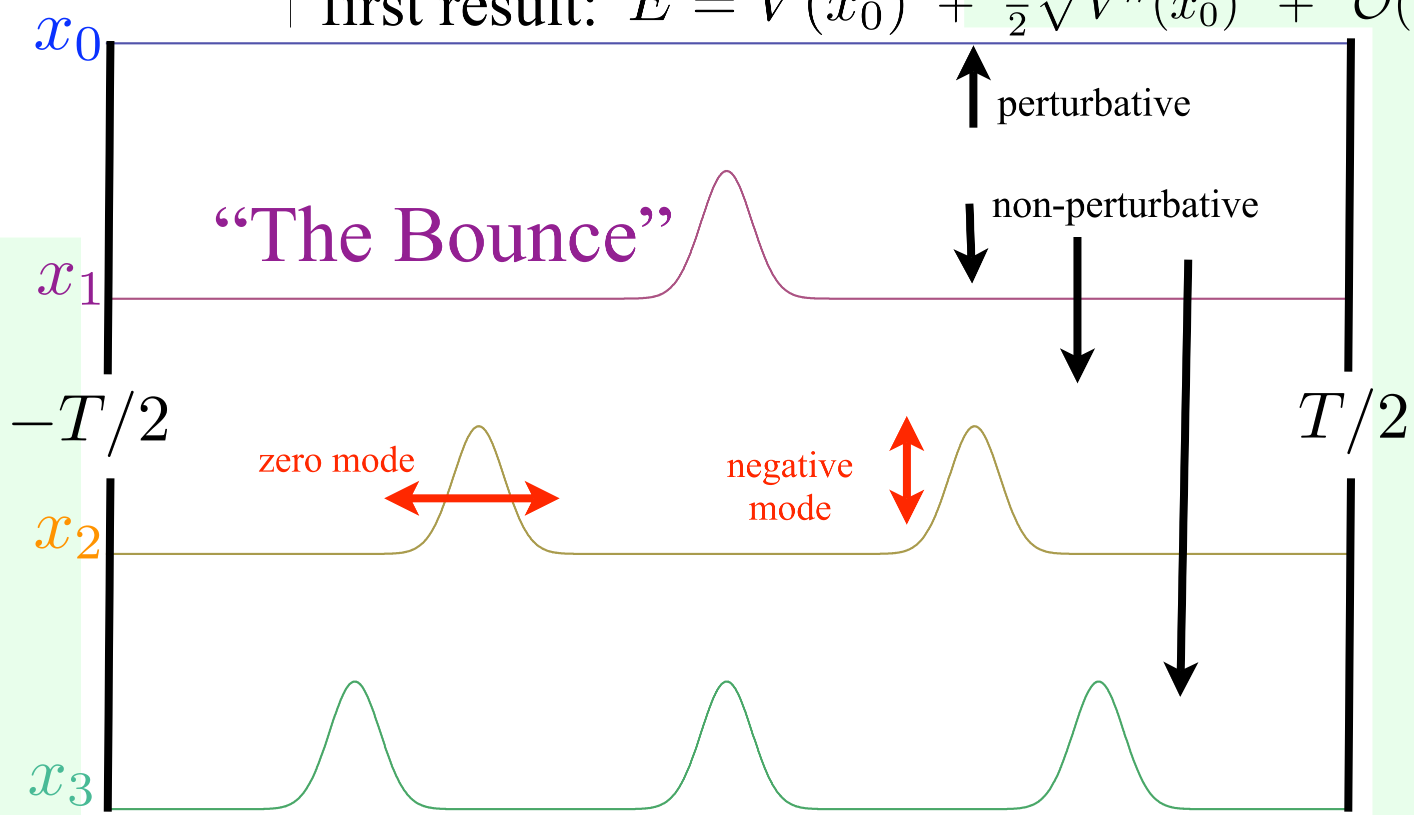
$$S_E[x_1] = V_0 T + S_B$$

$$S_E[x_n] = V_0 T + n S_B$$

n zero modes!

n negative modes!

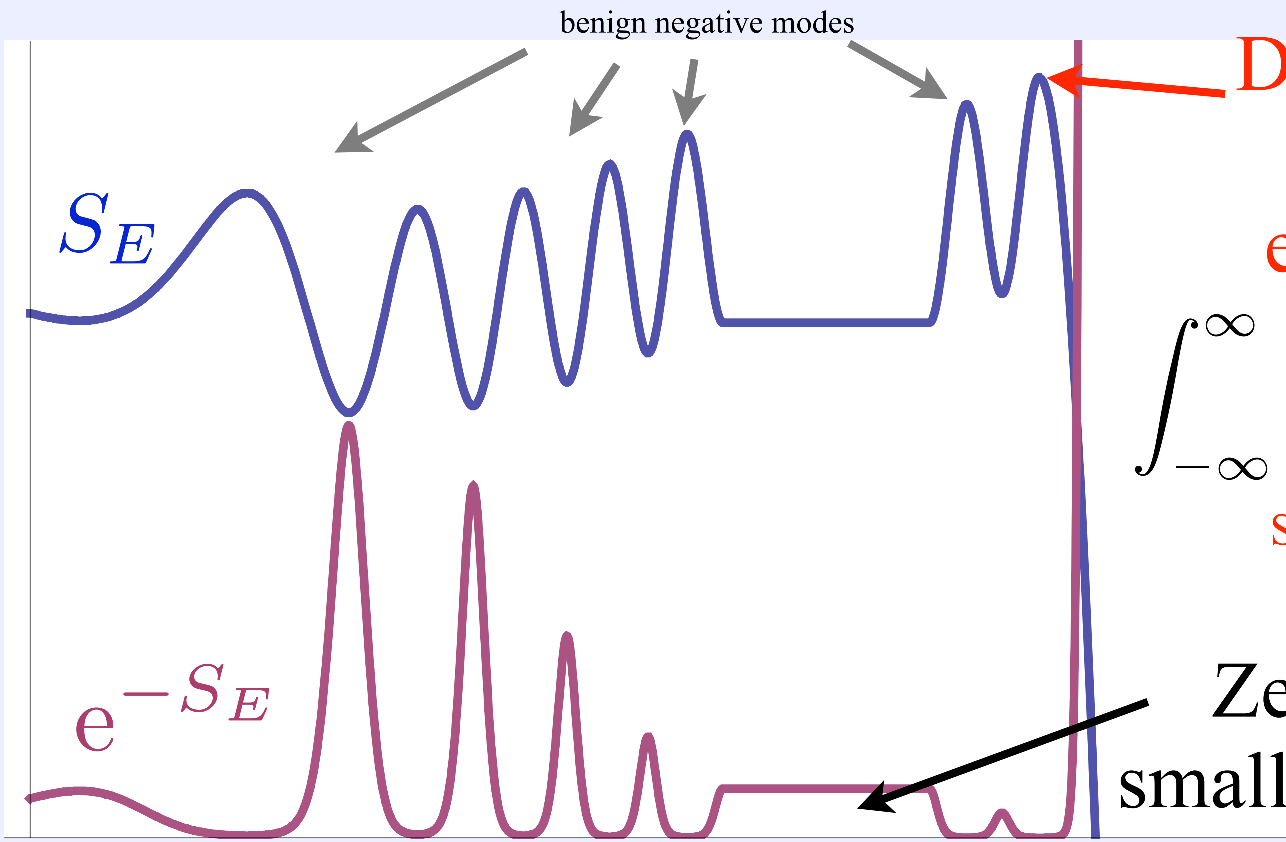
$$S_B = 2\int \sqrt{2\,(V(x) - V_0)}\,dx$$



Each zero mode contributes a large, but finite factor T .

$$\int_{-T/2}^{T/2} \mathrm{e}^{-0\tau^2} d\tau = T$$

Each "dangerous" negative mode contributes a *tiny* imaginary factor.



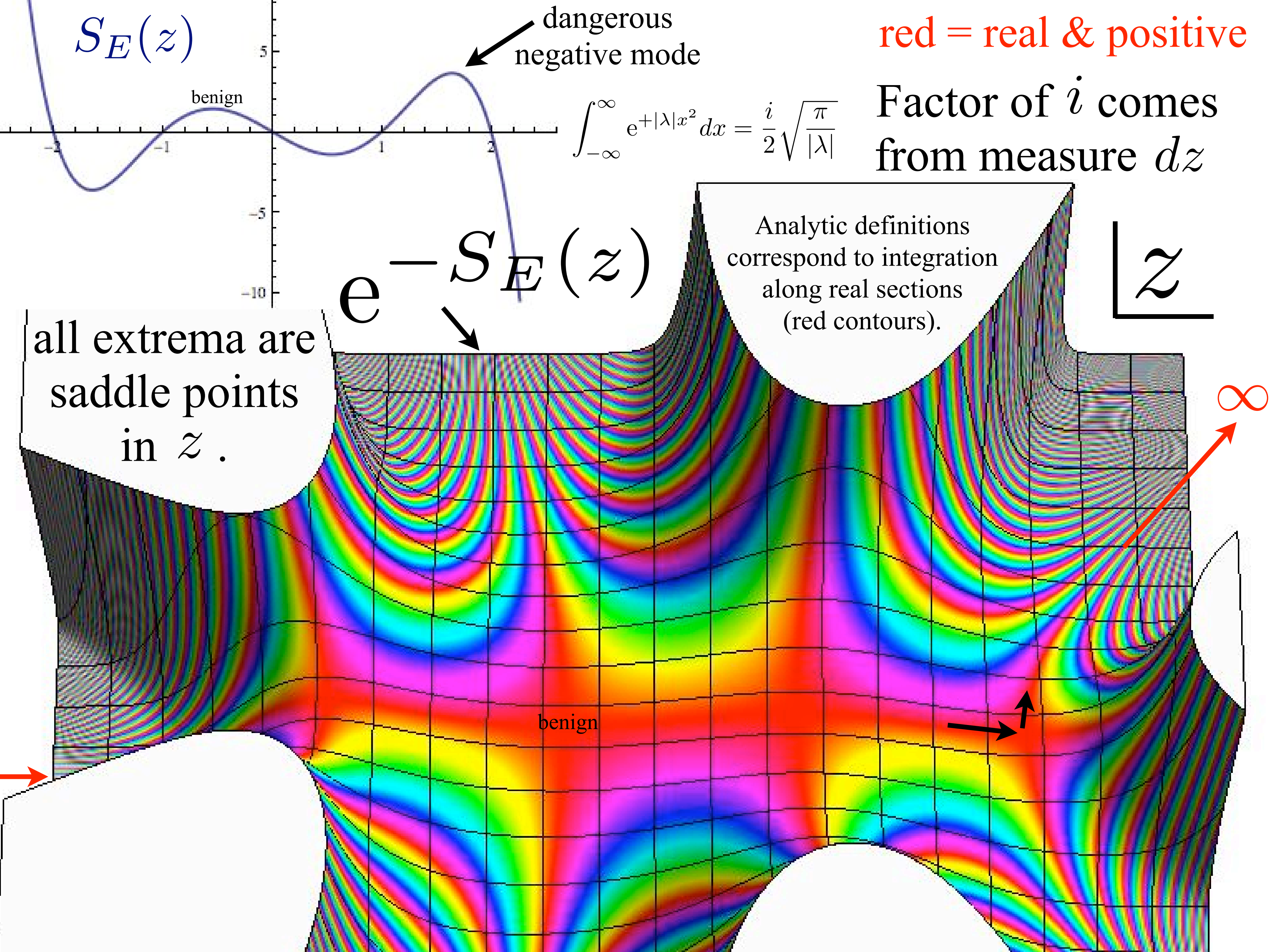
Dangerous negative modes allow for exponential decay

$$\int_{-\infty}^{\infty} \mathrm{e}^{+|\lambda|x^2} dx = \frac{i}{2}\sqrt{\frac{\pi}{|\lambda|}}$$

Sagredo forgot this factor of $1/2$.

Zero modes make a small effect observable at macroscopic time (extensive).

When a physical quantity appears divergent, it is defined via analytic continuation. We must choose amongst the various branches by appealing to the physics. In the case at hand, the "unstable state" is defined as the ground state of an analytically deformed potential. In the figure above, **every** maxima marks a crossroad. Only the one leading to an actual divergence must be avoided.



$S_E(z)$
dangerous
negative mode
red = real & positive
benign
5
-5
-10
-2
-1
1
2
$\int_{-\infty}^{\infty} e^{+|\lambda|x^2} dx = \frac{i}{2}\sqrt{\frac{\pi}{|\lambda|}}$
Factor of i comes
from measure dz
$e^{-S_E(z)}$
Analytic definitions
correspond to integration
along real sections
(red contours).
z
all extrema are
saddle points
in z .
∞
benign

$$x_n \equiv n \text{ widely separated bounces}$$

$$S_E[x_n] = S_E[x_0] + nS_B \qquad S_B = 2\int\sqrt{2\left(V(x) - V_0\right)}dx$$

$$\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_n}^{-\frac{1}{2}} = \frac{T^n}{n!}\left(\frac{-i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_n}^{-\frac{1}{2}} = \frac{T^n}{n!}\left(\frac{-i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{n}{2}} \det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_0}^{-\frac{1}{2}}$$

$$\sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_0}}} = \mathrm{e}^{-\frac{1}{2}\omega T + \mathcal{O}(\hbar^2)T} \qquad \omega = \sqrt{V''(x_0)}$$

$$\int [dx]\mathrm{e}^{-S_E[x]} \approx \sum_{x_{cl}} \mathrm{e}^{-S_E[x_{cl}]}\sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$

$$= \sum_{n=0}^{\infty} \mathrm{e}^{-\left(V_0 + \frac{\omega}{2} + \mathcal{O}(\hbar^2)\right)T}\left(\frac{T^n}{n!}\left(\frac{-i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{n}{2}} \mathrm{e}^{-nS_B}\right)$$

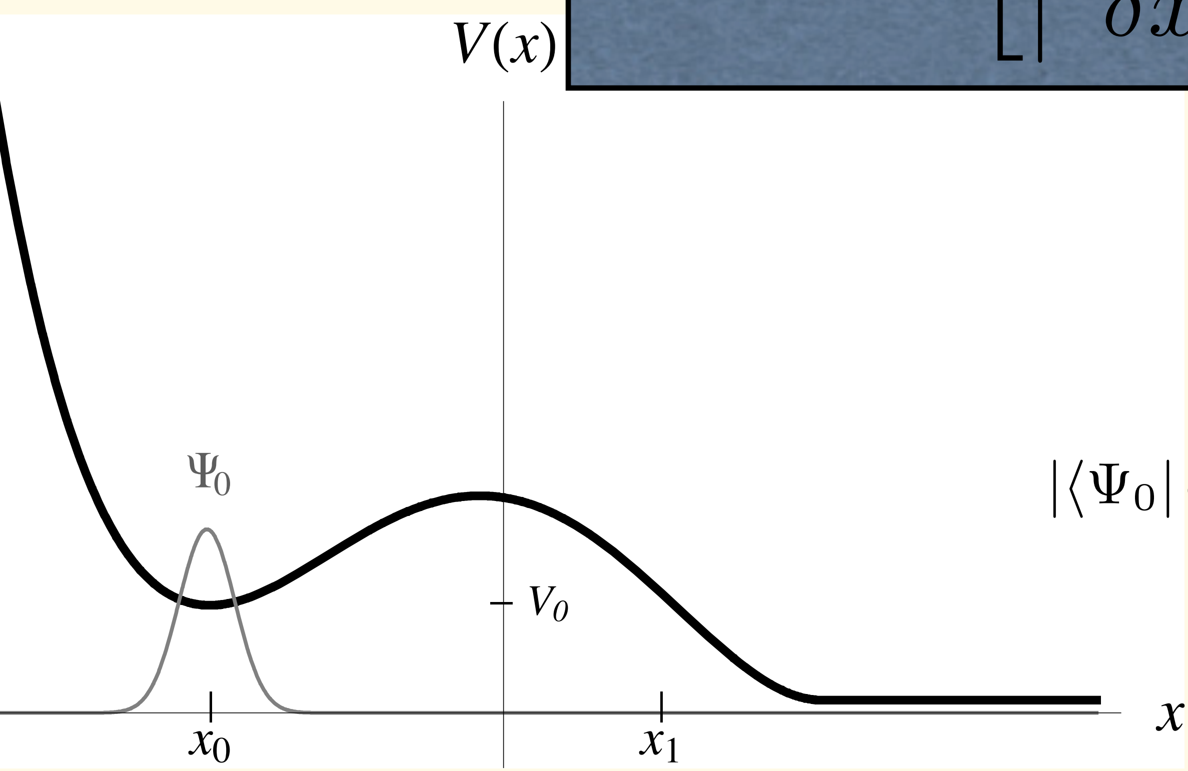
$$= \mathrm{e}^{-\left(V_0 + \frac{\omega}{2} - \frac{i}{2}\det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} \mathrm{e}^{-S_B}\right)T}$$

At large T

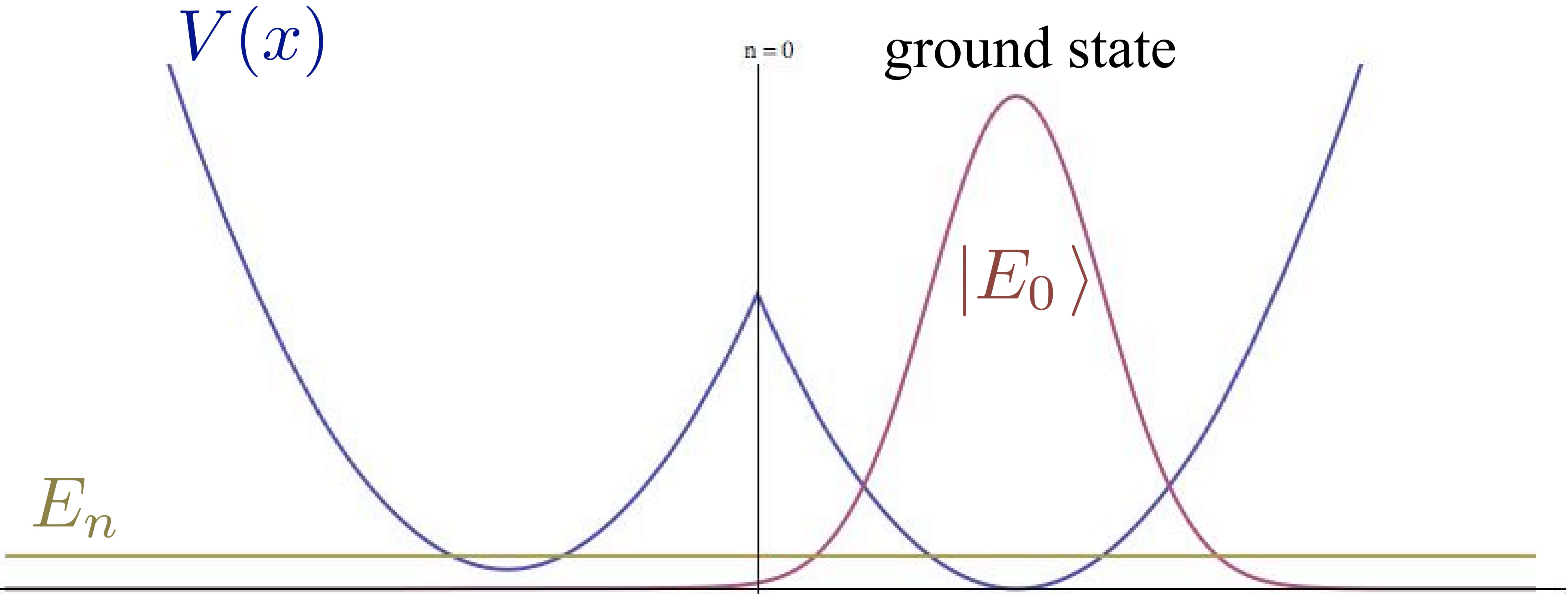
$$e^{-(E_{fv}+\delta_{fv})T} = e^{-\left(V_0 + \frac{\omega}{2} - \frac{i}{2}\det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} e^{-S_B}\right)T}$$

$$\delta_{fv} = -\frac{i}{2}\Gamma$$

$$\Gamma = \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} e^{-S_B}$$



$$|\langle\Psi_0|\exp(-iHt)|\Psi_0\rangle|^2 \approx \exp(-\Gamma|t|)$$

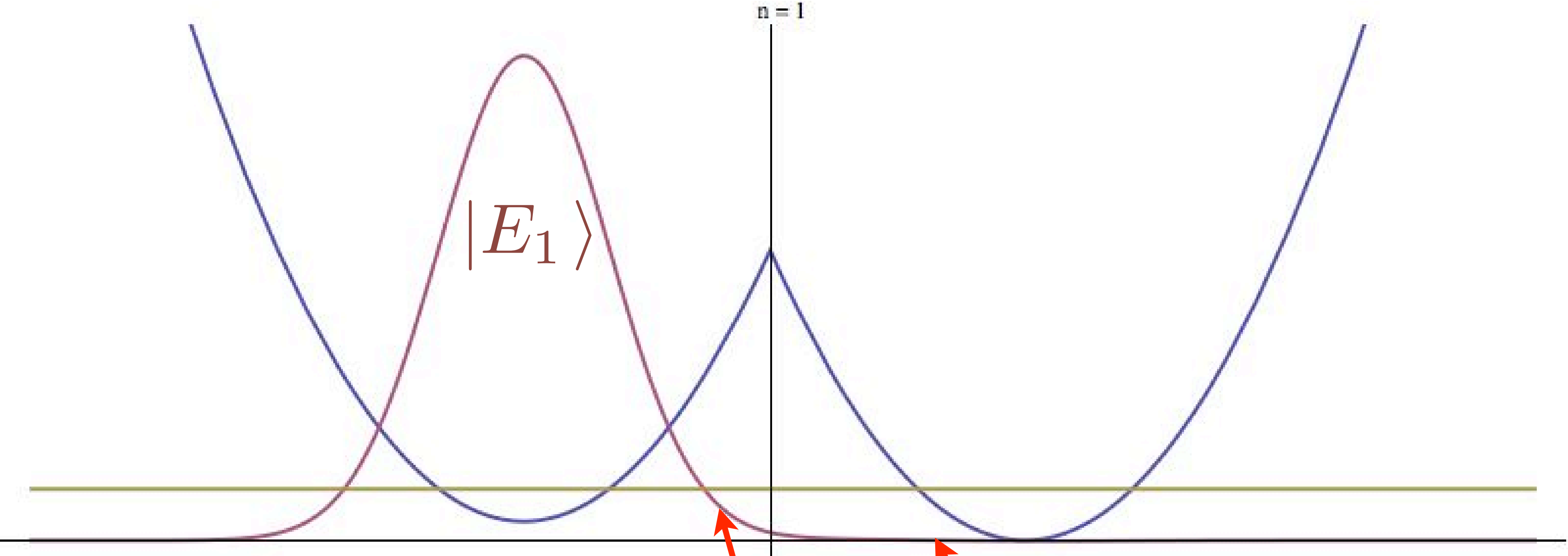


"small" lifting compared to....zero?

The asymmetric double well

not a perturbation of the symmetric double well!

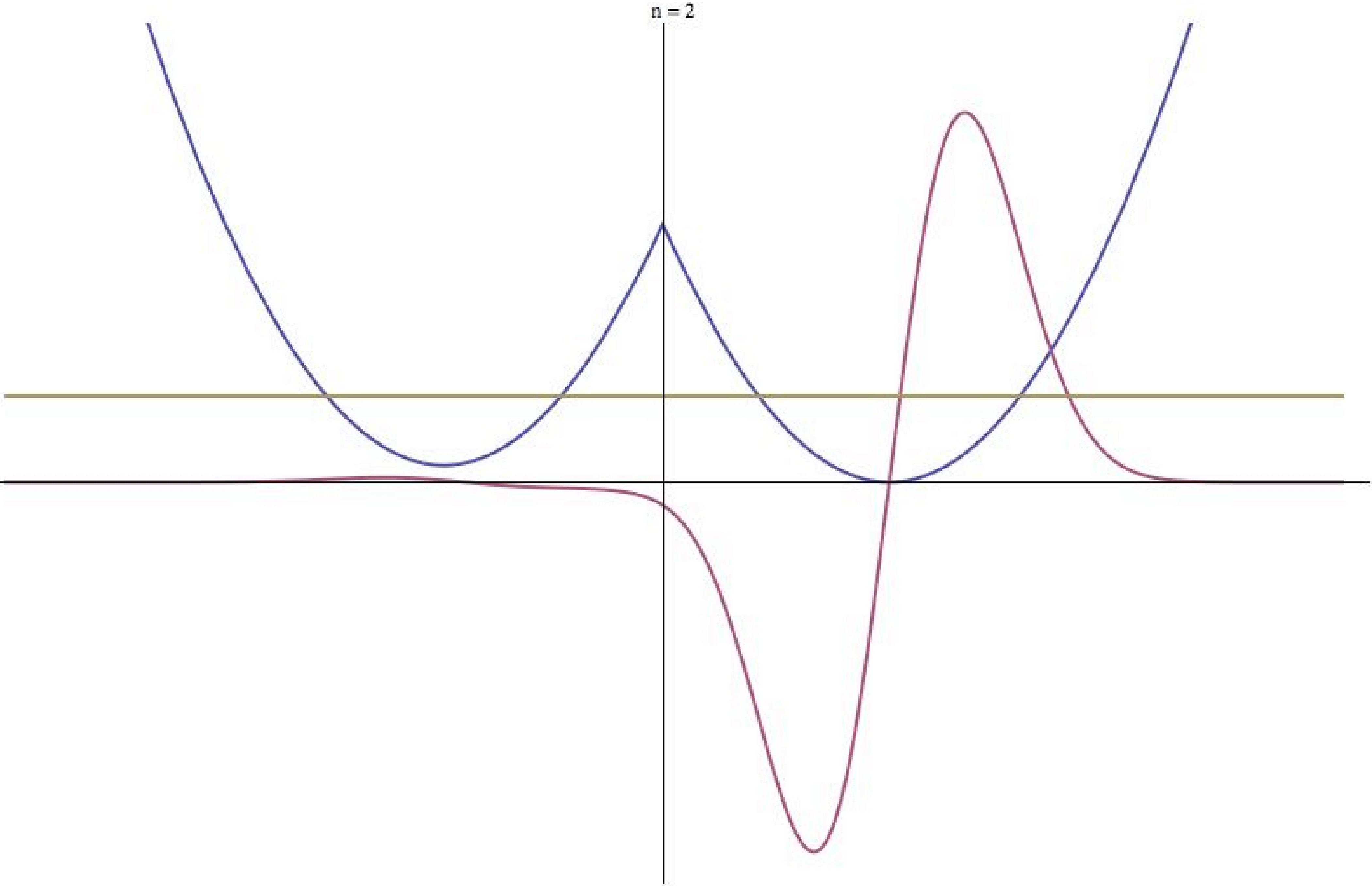
A Stable False Vacuum!

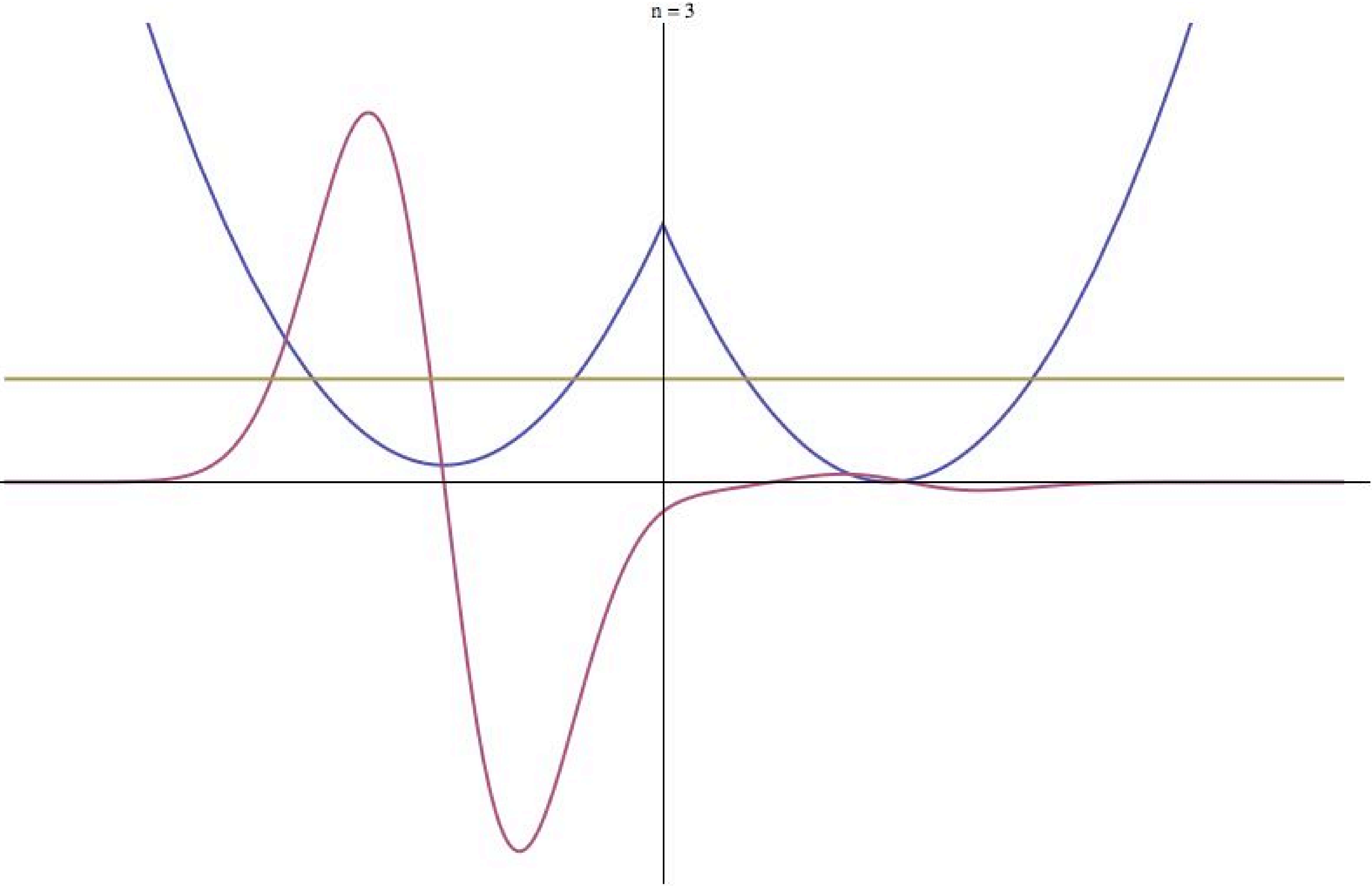


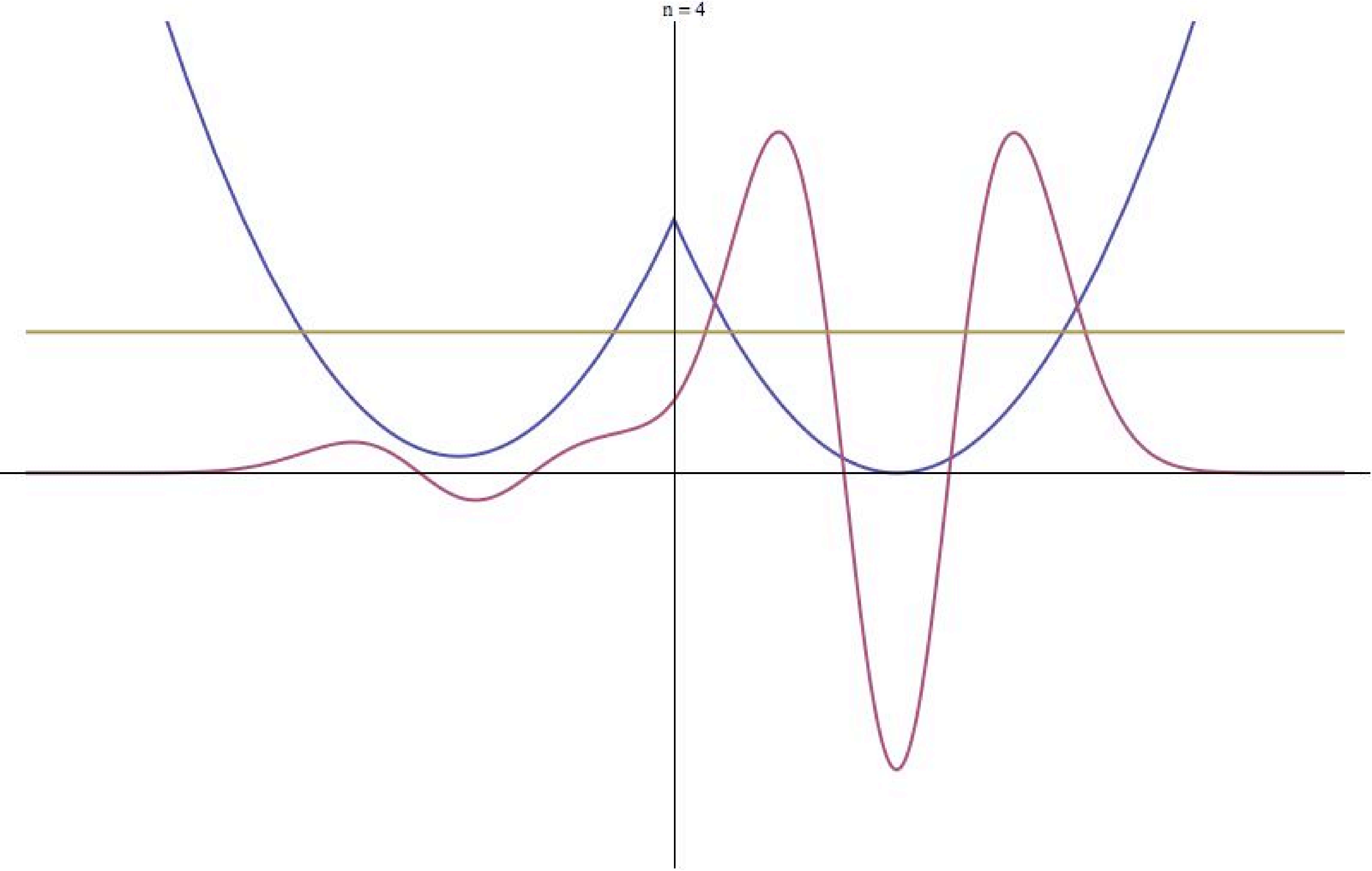
1. Fix 2 boundary conditions here

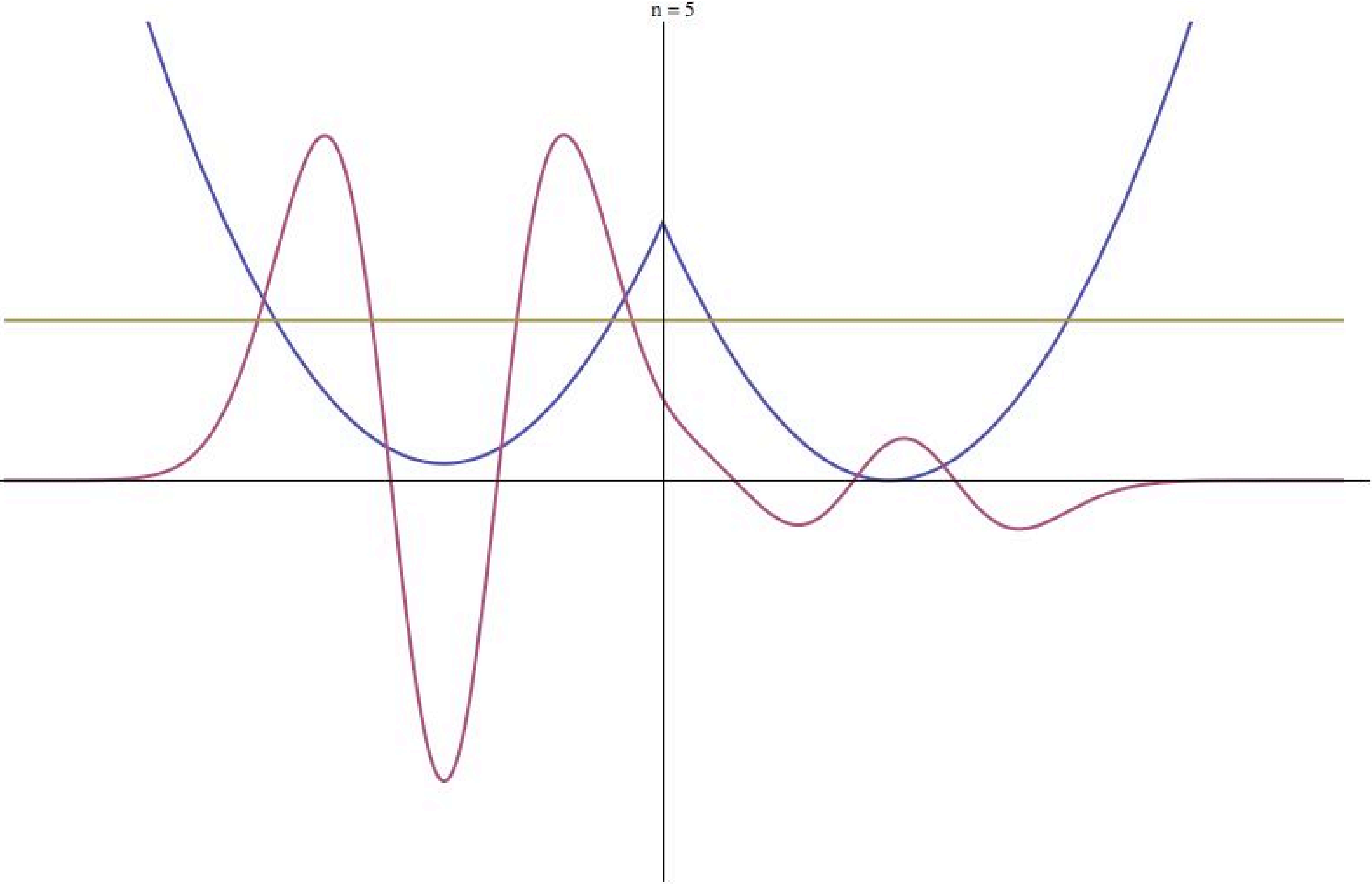
2. Choose energy for convergence here

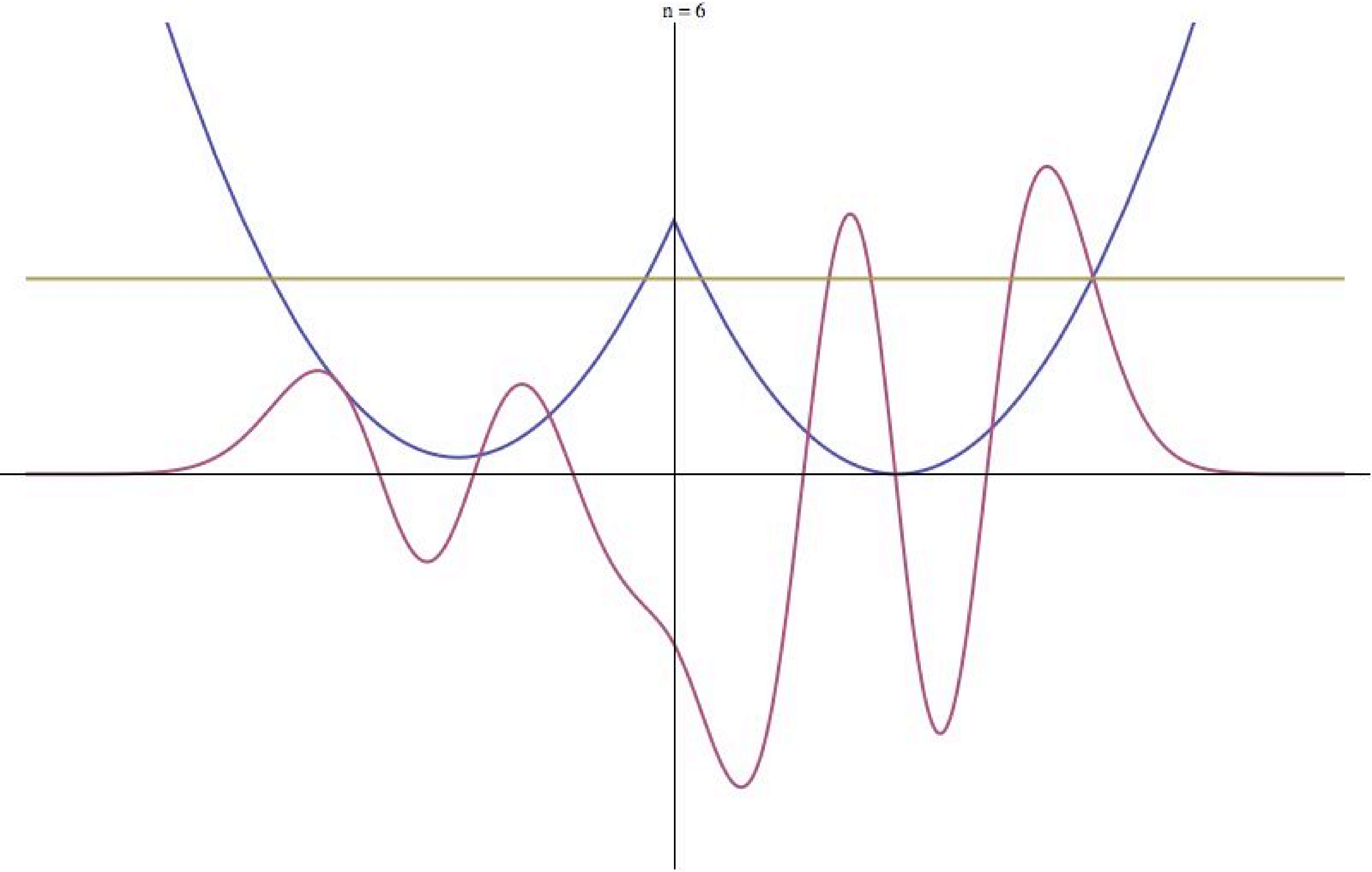
3. We are left with no free parameters. All wave functions will have a growing and a decaying mode with $\mathcal{O}(1)$ coefficients. One of them will thus always dominate, and all solutions are primarily supported on the left, or the right.

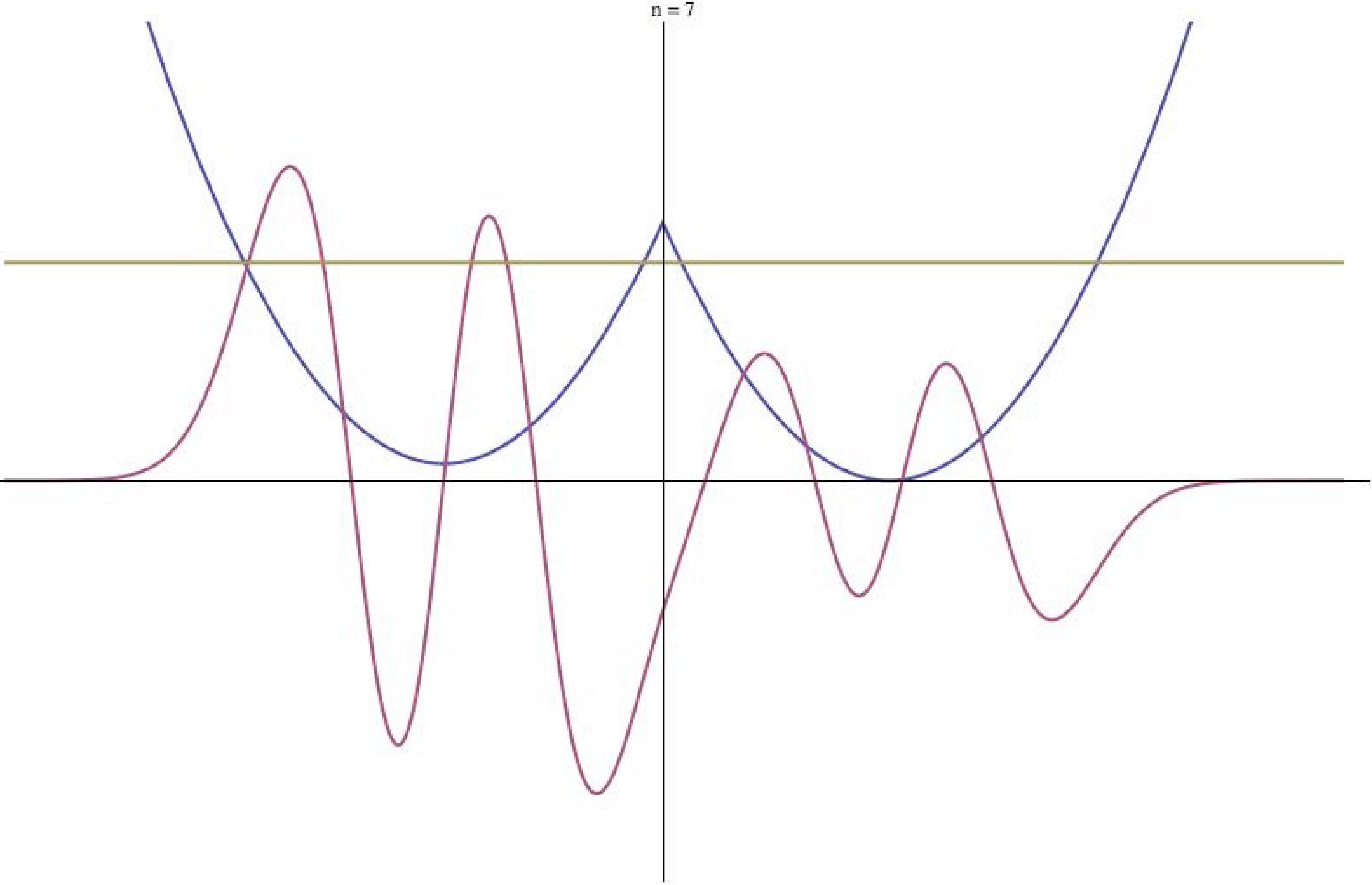

n = 2

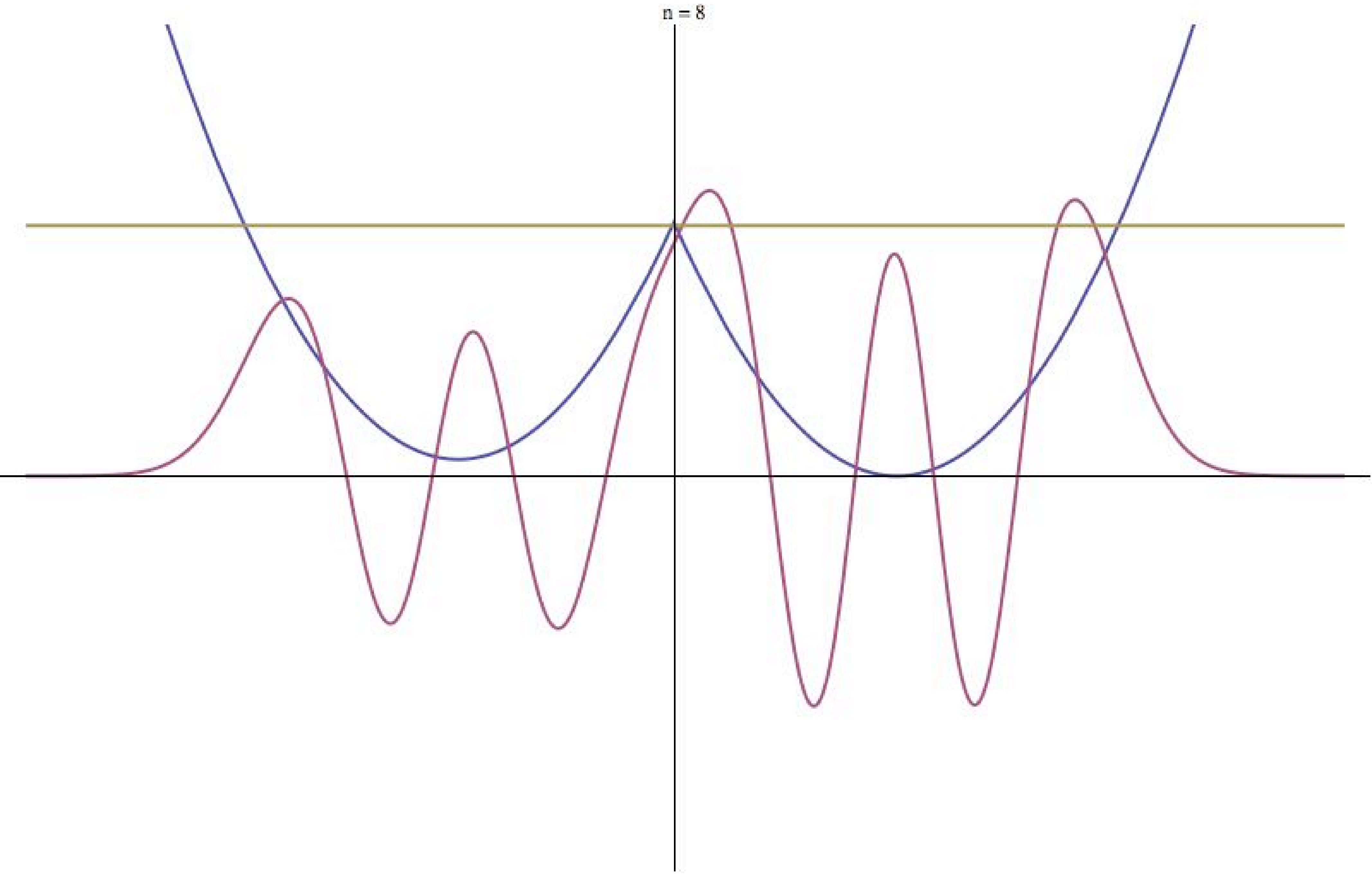

n = 3

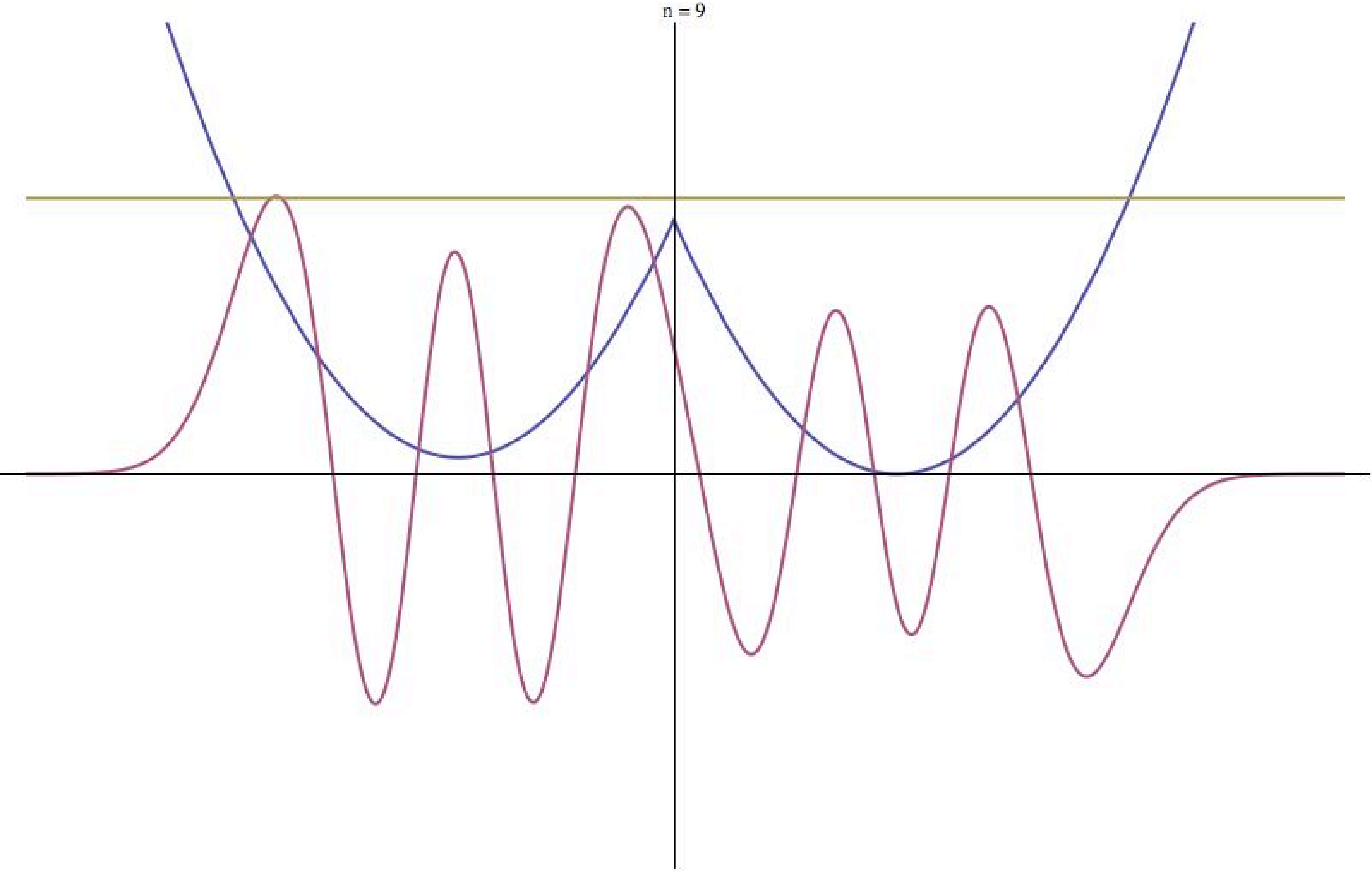

n = 4


n = 5

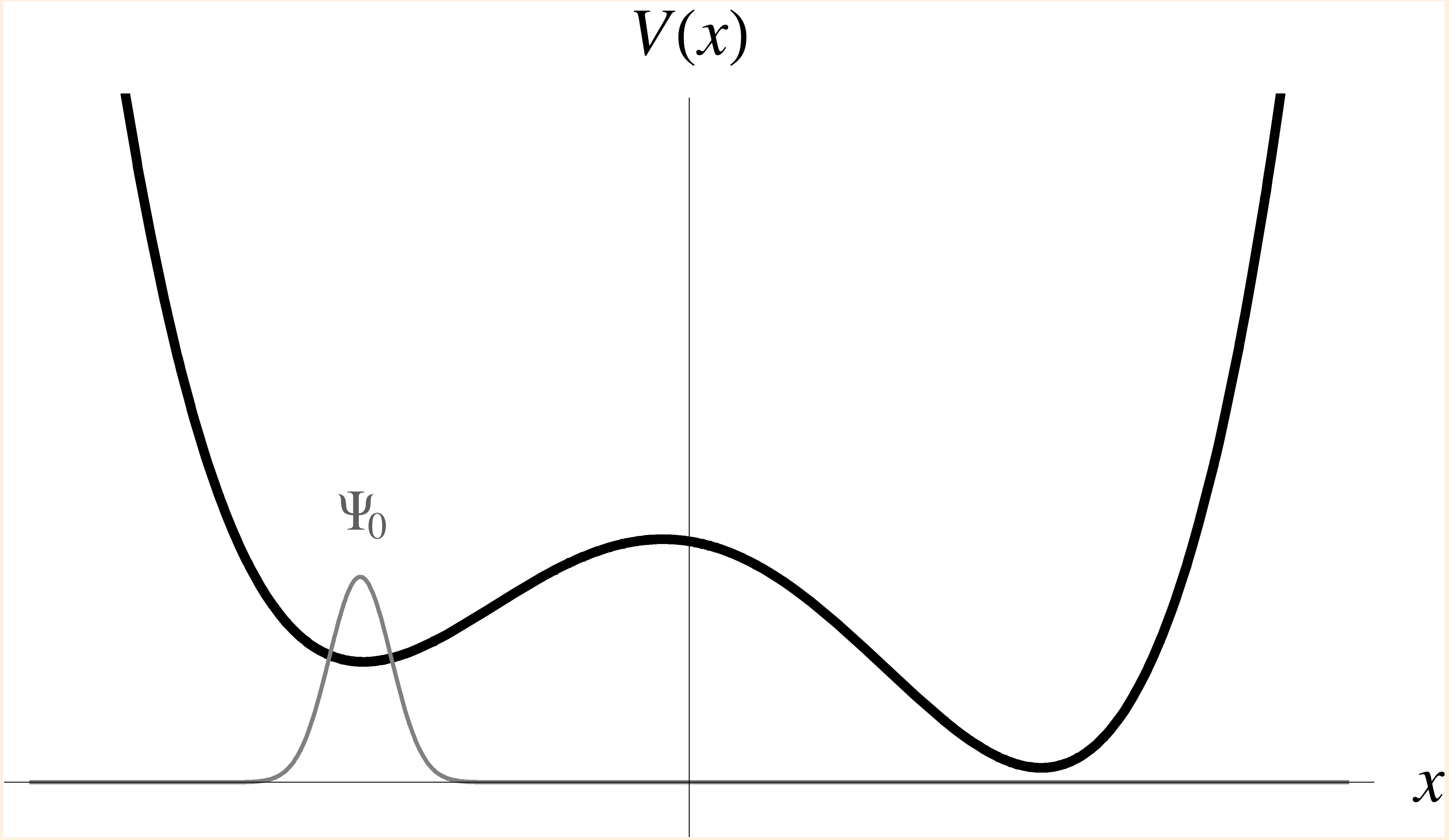

n = 6


n = 7


n = 8


n = 9

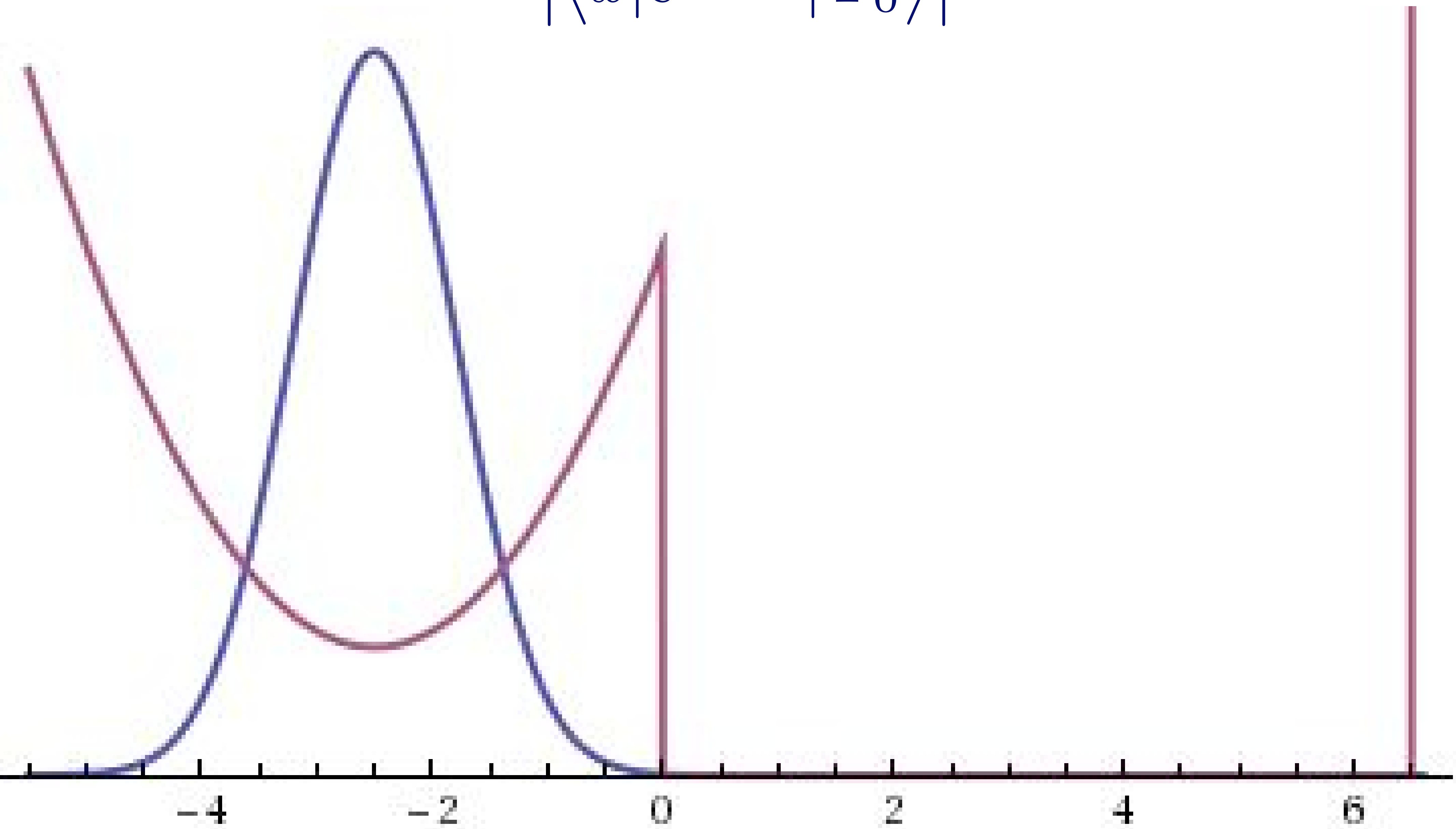
The asymmetric double well has a stable false vacuum



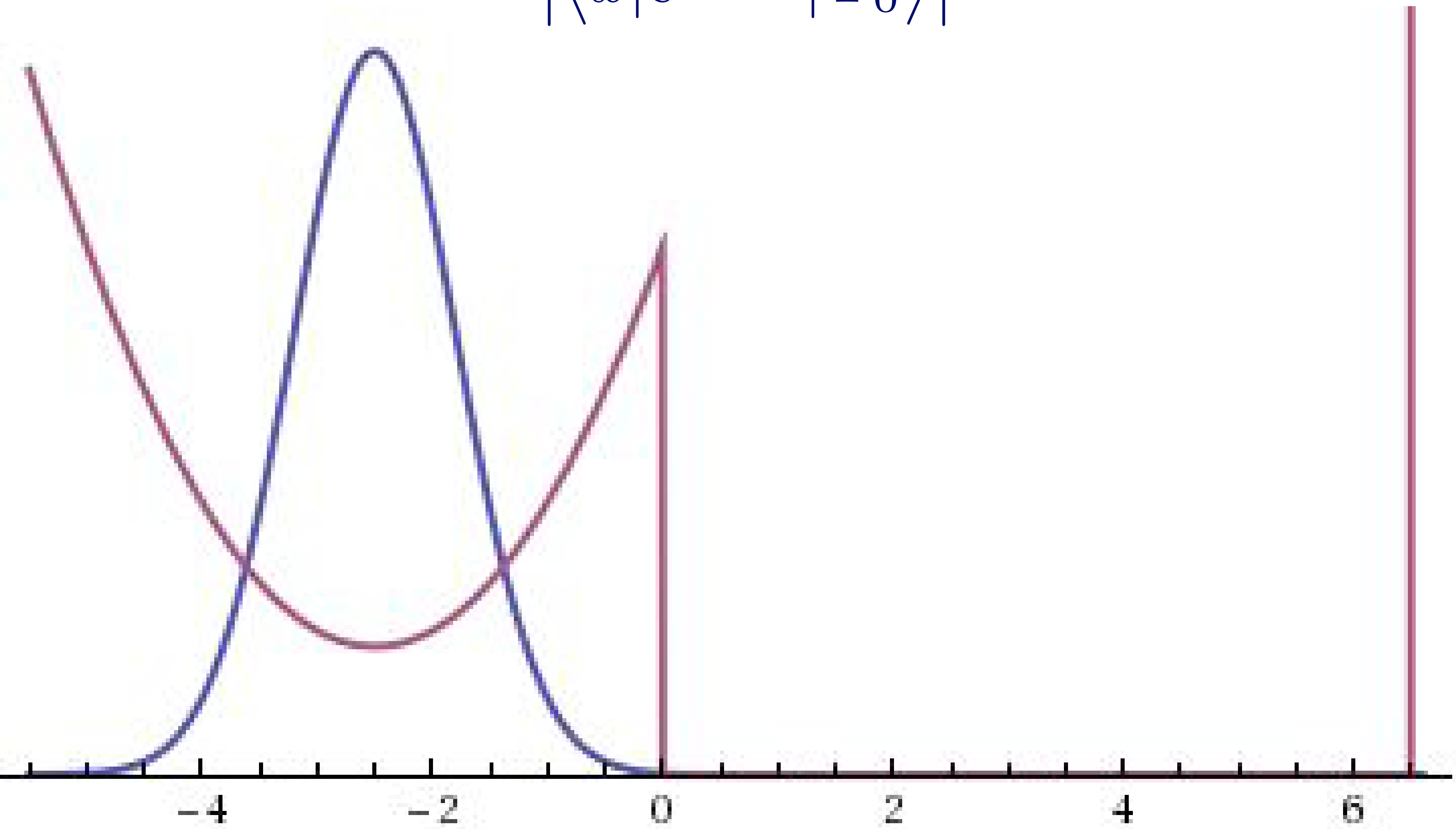
The (LHS) perturbative vacuum **is** an approximate energy eigenstate when $S_B \gg 1$.

$$|\Psi_0\rangle \approx |E_n\rangle$$

$$\left|\langle x|e^{-iHt}|\Psi_0\rangle\right|^2$$



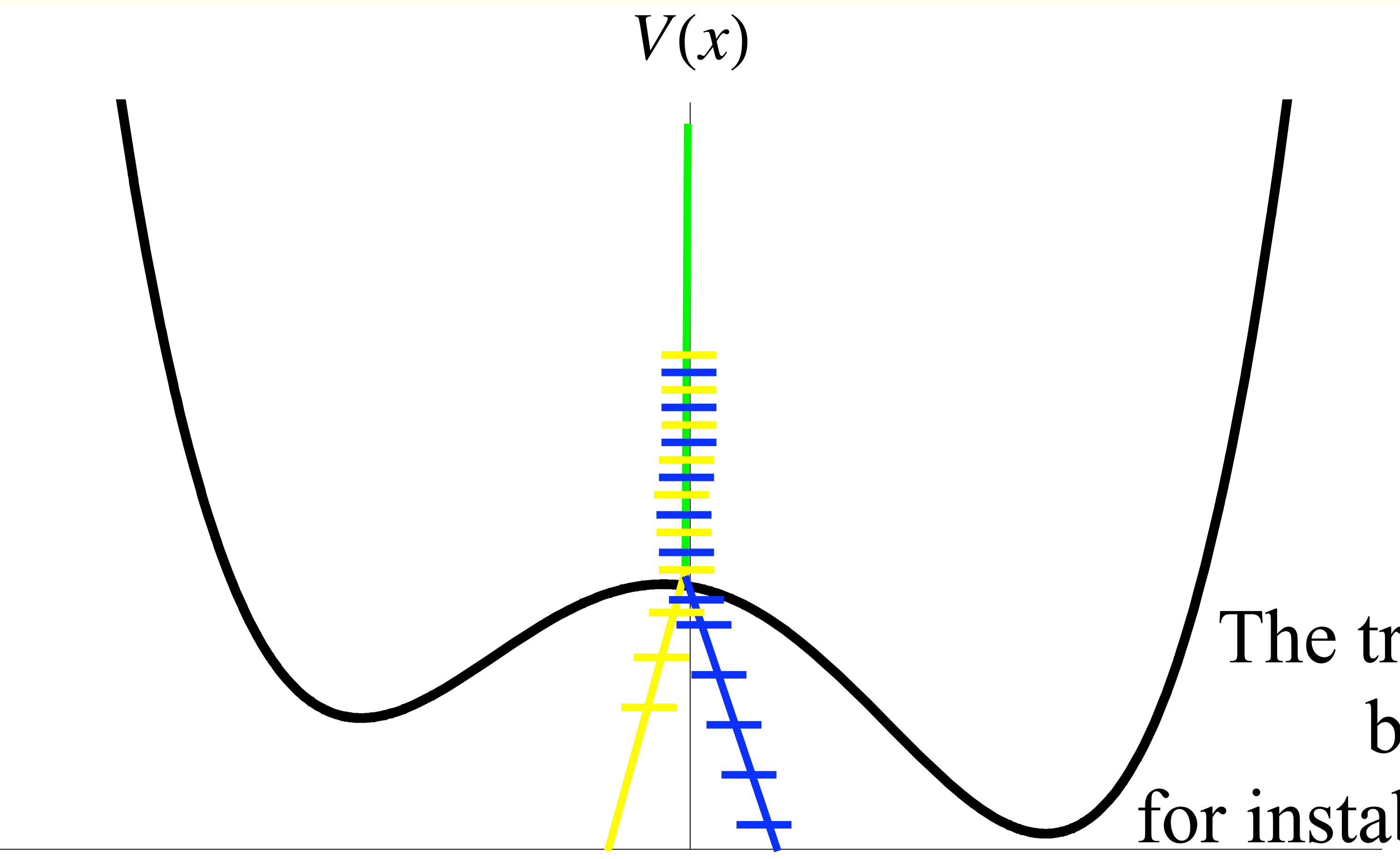
$$\left|\langle x|\mathrm{e}^{-iHt}|\Psi_0\rangle\right|^2$$



The false vacuum cannot decay because there are no excited true vacuum states with overlapping energy. This is found experimentally in cavity QED.

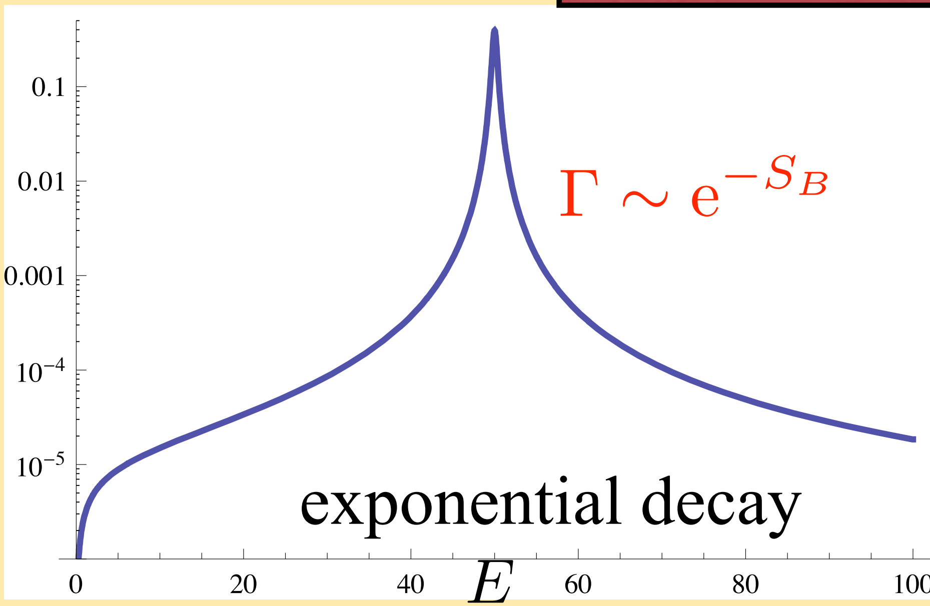
$$\Delta E_{\Psi_0} \approx \Gamma \sim e^{-S_B}$$

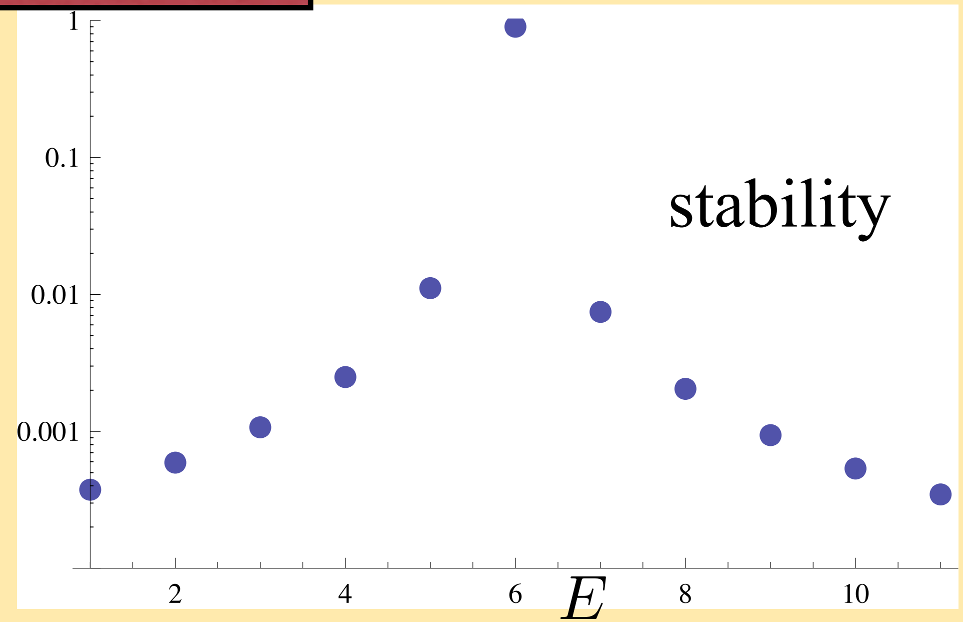
$$\Delta E_{tv} \sim 1/\ell$$

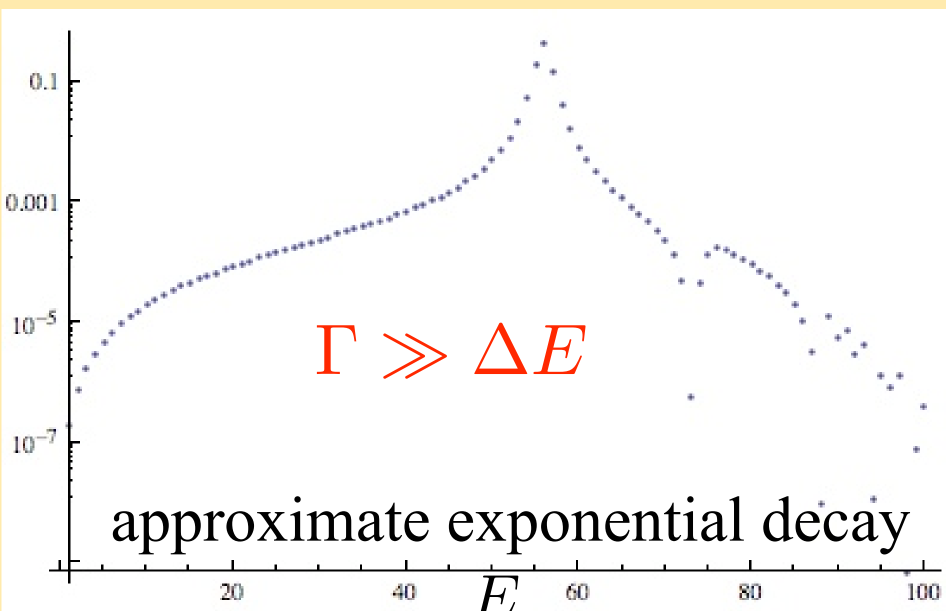


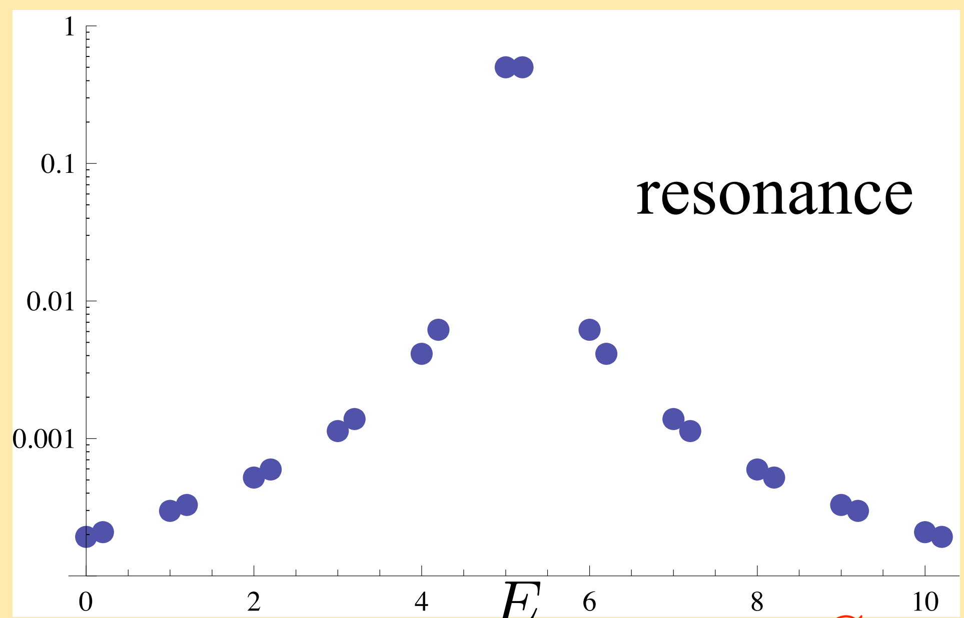
The true vacuum must be *very* large for instability to be generic.

$|\langle E|\Psi_0(t)\rangle|^2$ ← time independent





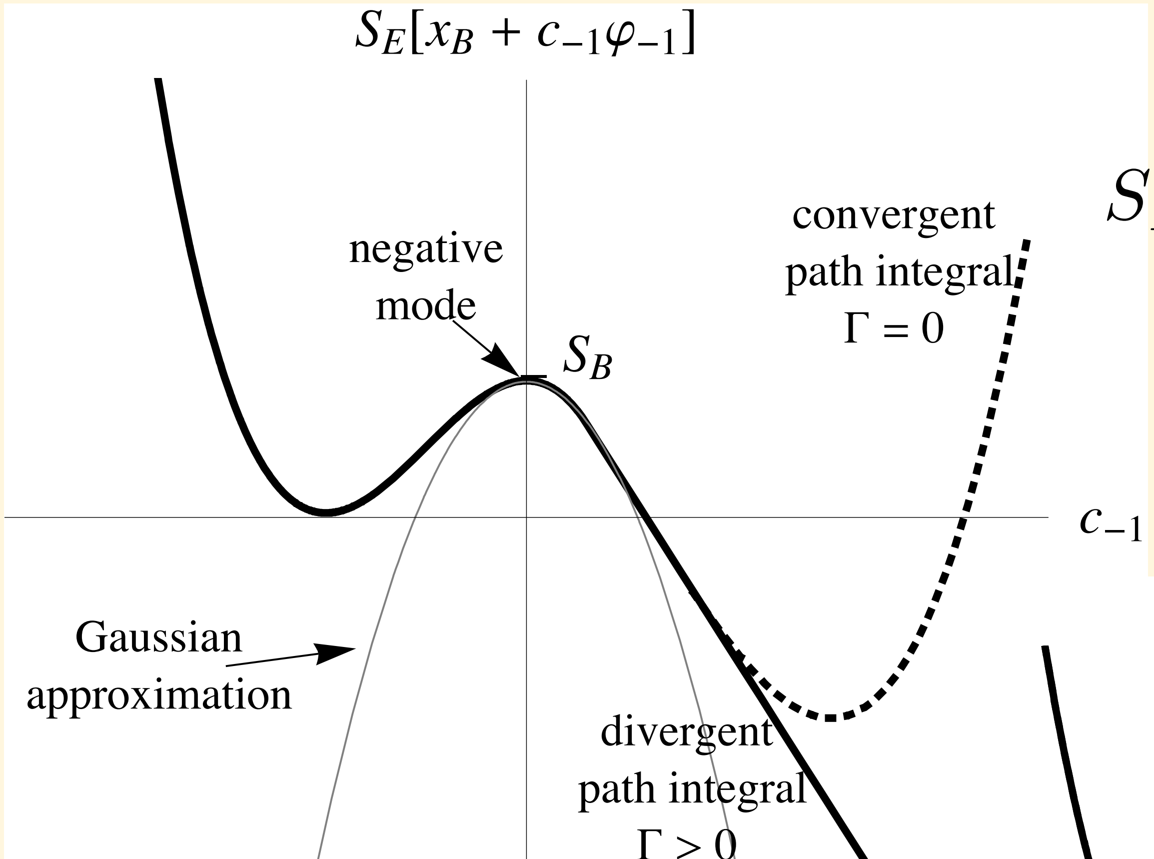




density of states set by volume

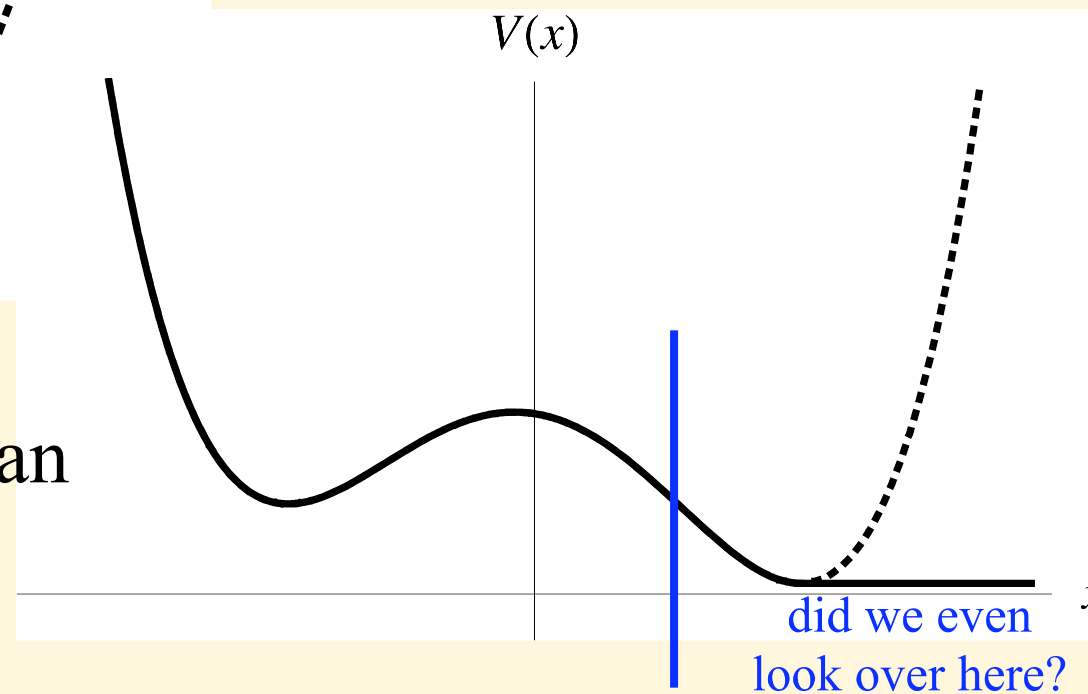
peak width Γ set by e^{-S_B}

The instanton formalism appears to predict exponential decay. The resolution is that the single negative mode is rendered benign by "compactifying" the true vacuum.

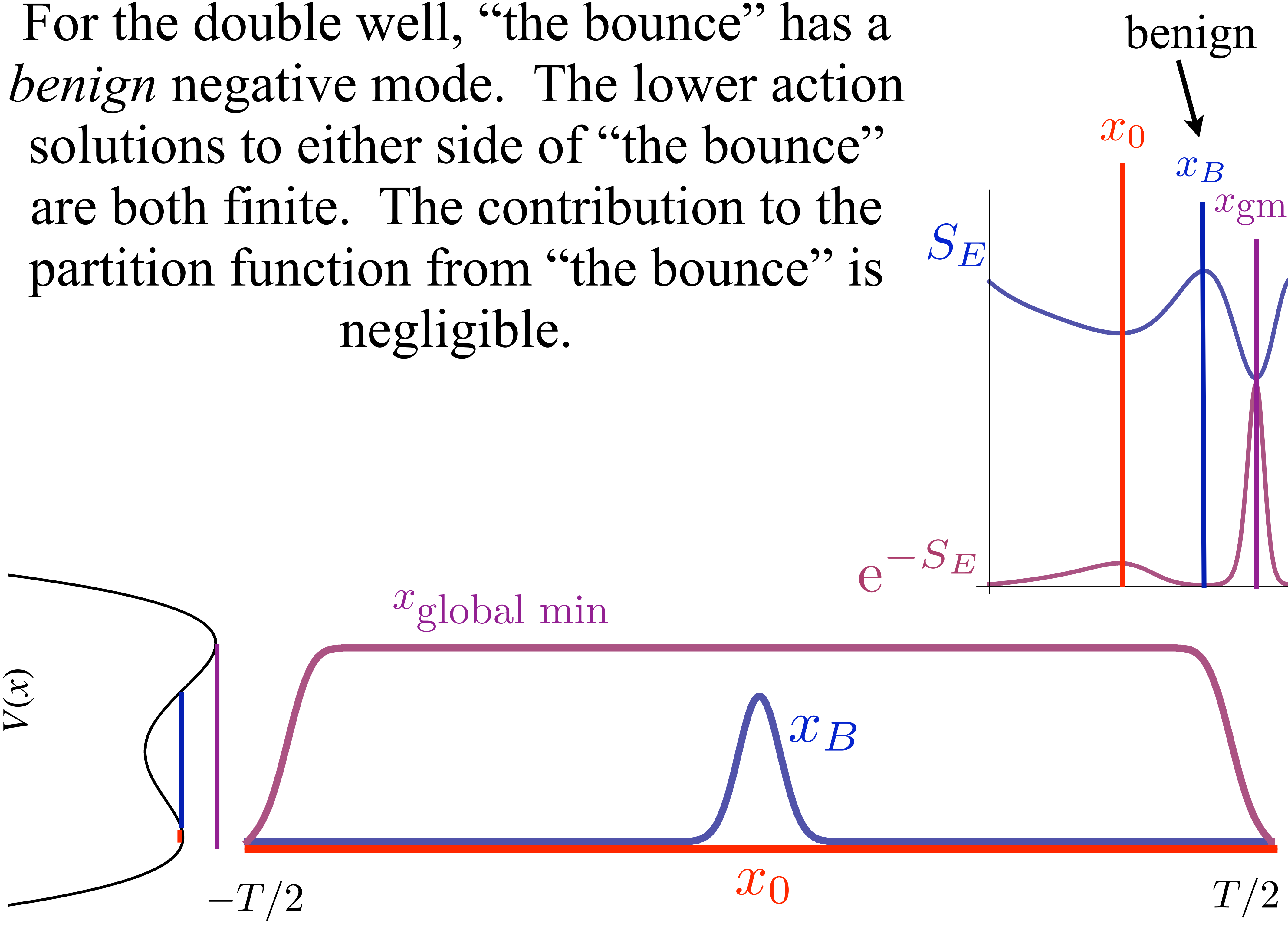


$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau$$

The double well has Euclidean action bounded below!



For the double well, "the bounce" has a *benign* negative mode. The lower action solutions to either side of "the bounce" are both finite. The contribution to the partition function from "the bounce" is negligible.

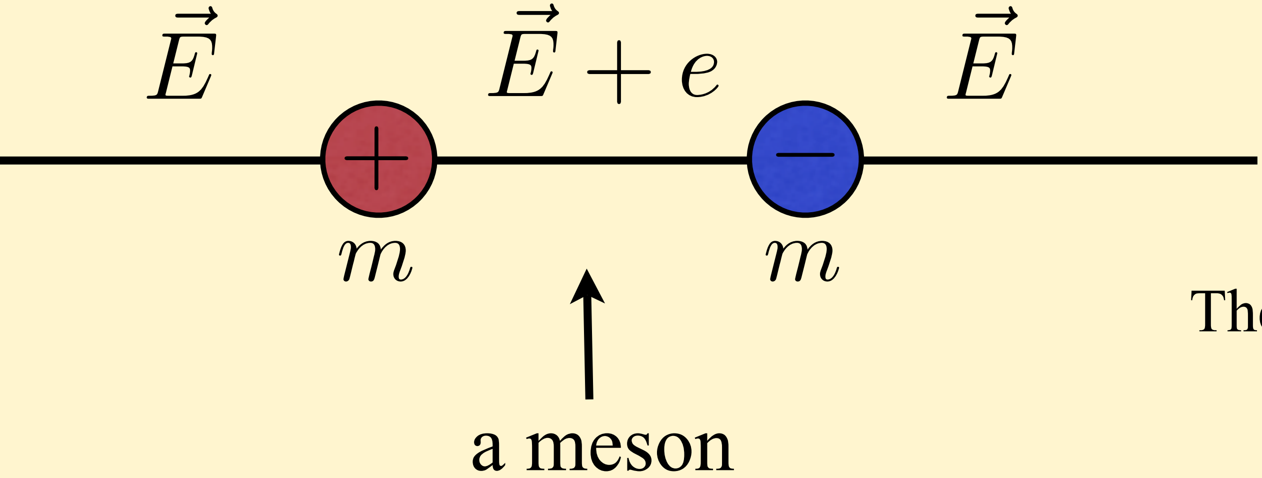


The Schwinger model

vacuum electric field $= \dfrac{\theta e}{2\pi}$.

$$S = -\int \left(\tfrac{1}{4} F^2 + \tfrac{1}{2} \overline{D_\mu \Phi} D^\mu \Phi + \frac{m^2}{2} \bar{\Phi}\Phi \right) d^2x + \frac{e}{2\pi}\theta \int F$$

$$\vec{E} = *F \quad \text{(a scalar)}$$



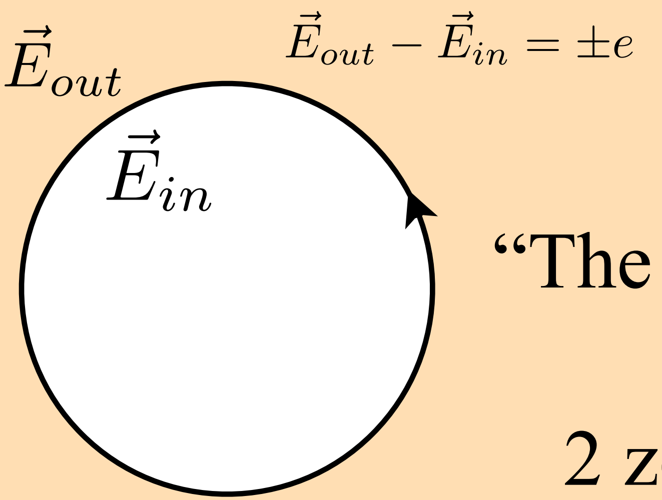
There is no photon: low energy mesons are stable

Confinement: All quarks undergo piecewise uniform acceleration

Pair production (vacuum instability)

The Schwinger model $S_E = \oint_{\partial\Sigma} m - \int_{\Sigma} \epsilon$

Exponential decay of the false vacuum is calculated precisely as it was in quantum mechanics:



$$\epsilon = \frac{1}{2}\left(\vec{E}^2_{out} - \vec{E}^2_{in}\right)$$

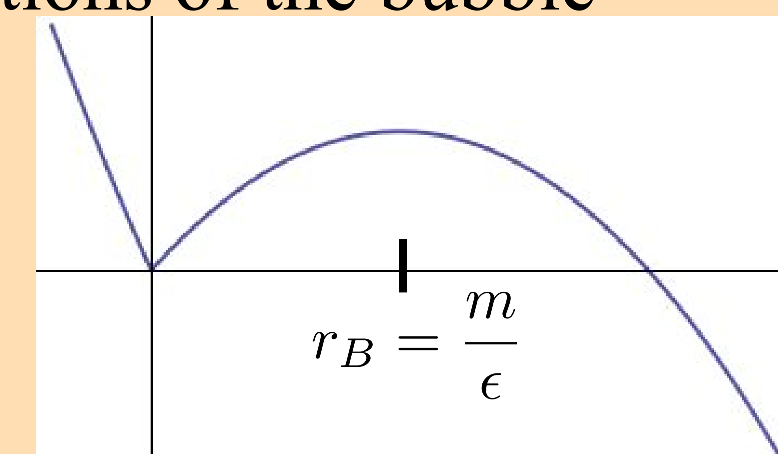
$$\mathbb{R}^{1,1} \longrightarrow \mathbb{R}^2$$

"The Bounce" = S^1 domain wall in $\mathbb{R}^2$

2 zero modes (translations in $\mathbb{R}^2$)

Negative mode corresponds to dilatations of the bubble

$$S_E = 2\pi r m - \pi r^2 \epsilon$$



Nucleation rate per unit length

$$\left\langle \vec{E} \left| \mathrm{e}^{-\mathcal{H}XT} \right| \vec{E} \right\rangle = \int [d\Sigma] \mathrm{e}^{-S_E[\Sigma]}$$

$$S_E = m \oint_{\partial\Sigma} ds + \frac{\vec{E}^2}{2} \int_{\Sigma} d^2x$$

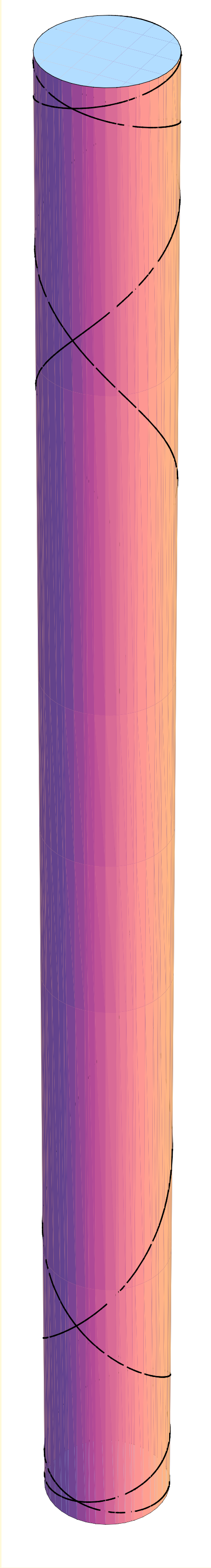
$$\vec{E}_{in} - \vec{E}_{out} = \pm e$$

$$\langle \mathcal{H} \rangle = \frac{1}{2}\vec{E}^2 - i\frac{e\vec{E}}{4\pi} \exp\left(-\pi m^2/e\vec{E}\right)$$

decay rate per unit volume

$$\Gamma = \frac{1}{2} \det{}' \left[\left| \frac{\delta^2 S_E}{\delta \partial\Sigma^2} \right| \right]^{-\frac{1}{2}} \mathrm{e}^{-S_B} = \frac{e\vec{E}}{2\pi} \exp\left(-\pi m^2/e\vec{E}\right)$$

Sagredo's missing factor



What happens if we compactify the Schwinger model on a circle?

As we dilatate the instanton on the cylinder, it will overlap itself.

In the probe approximation, this is irrelevant:

$$S_E = 2\pi r m - \pi r^2 \epsilon$$

$$\Rightarrow \Gamma > 0$$

The negative mode is still "dangerous"

This can be confirmed by the method of Bogolyubov coefficients.

But if we include the back-reaction on the electric field, the action is bounded below.

The negative mode is actually "benign"

$$\Rightarrow \Gamma = 0$$

This can be confirmed by explicit computation of the discrete spectrum when $\Gamma\ell \ll 1/\ell$

3+1 d QFT in flat space

$$S_E = \int_{\mathbb{R}^4} \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + V(\phi) d^4x$$

Integrate out the UV:
only domain walls remain

a.k.a. Brown - Teitelboim

$$S_E \;=\; \mu \int_{S^3} d^3x \;-\; \epsilon \int_{B^4} d^4x$$

Exponential decay of the false vacuum is calculated precisely as it was in the Schwinger model:

$$\mathbb{R}^{1,3} \longrightarrow \mathbb{R}^4 \qquad ds^2 \to d\tau^2 + dx^2 + dy^2 + dz^2$$

"The Bounce" = S^3 domain wall in $\mathbb{R}^4$

4 zero modes (translations in $\mathbb{R}^4$)

Negative mode corresponds to dilatations of the bubble
It is "dangerous."

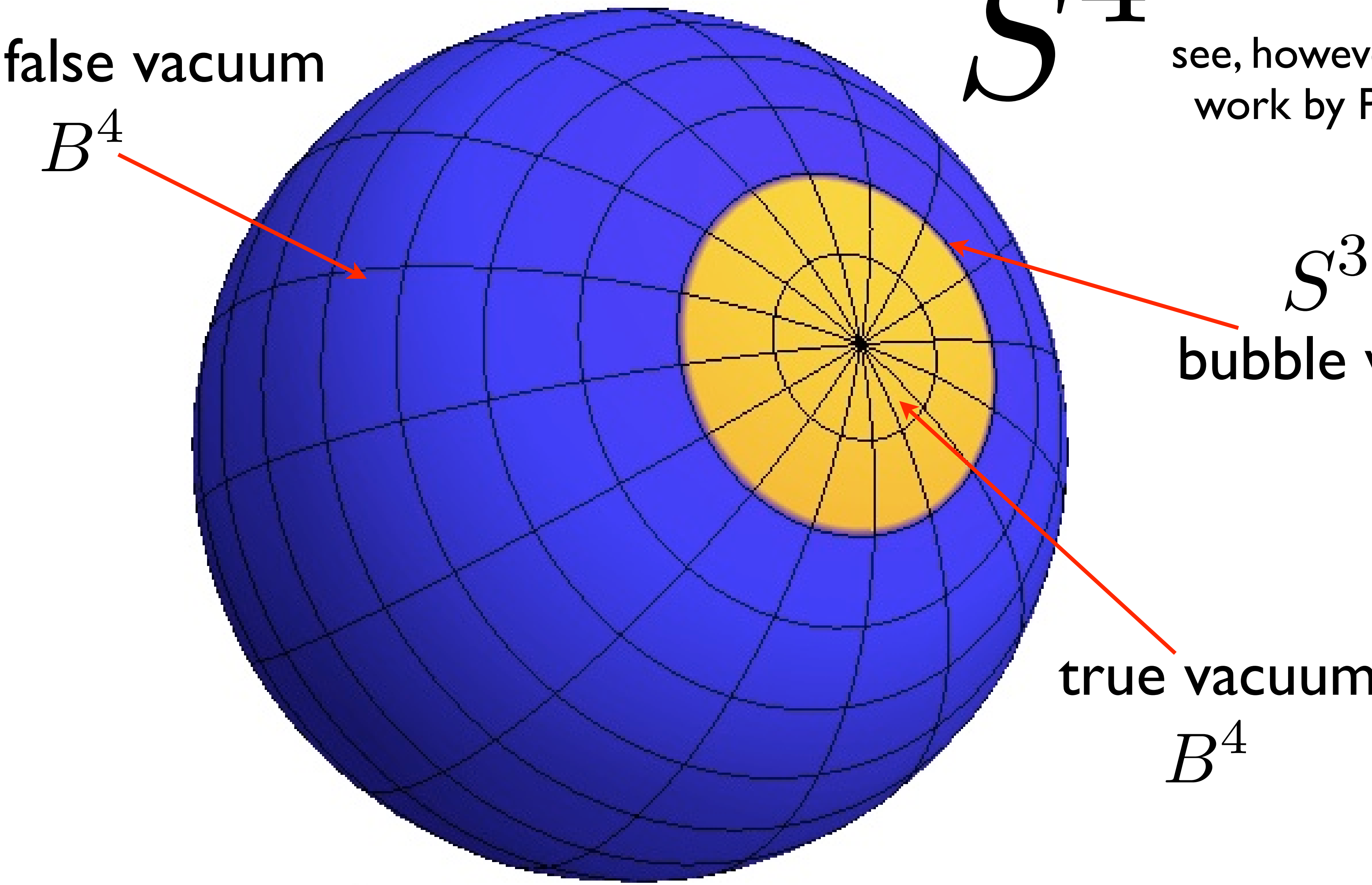
$$\Gamma \sim \mathrm{e}^{-S_B} \qquad S_B = \frac{27\pi^2\mu^4}{2\epsilon^3}$$

QFT in de Sitter space

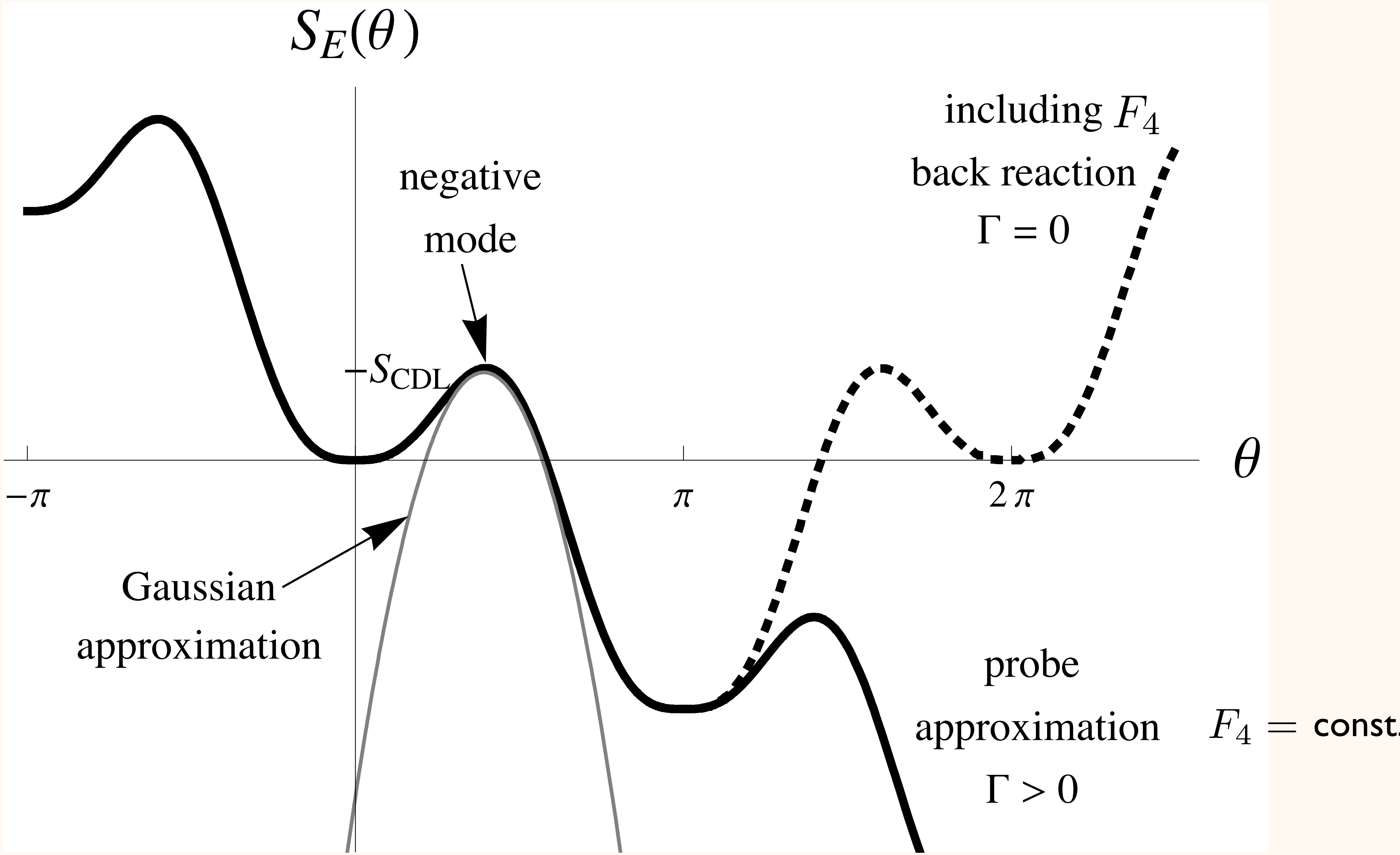
$$ds^2 = (1 - \frac{r^2}{l^2})d\tau^2 + (1 - \frac{r^2}{l^2})^{-1}\left(dr^2 + r^2 d\Omega^2\right)$$

S^4

see, however, recent work by Polyakov



Coleman - De Luccia action as a function of bubble radius



The Coleman - De Luccia negative mode is *benign* in de Sitter space.

Caveats

$M_{Pl} \to \infty$

De Sitter space is not a box: there is certainly a continuous spectrum, so our intuition about finite volumes is not useful.

Just because there is *a* stable de Sitter false vacuum doesn't mean it is the *relevant* one (for, say, eternal inflation). I have argued in favor of the existence of a de Sitter invariant false vacuum. (There is no Poincaré invariant false vacuum in flat space.)

Formalism predicts no exponential decay in dS for any parameter range. We can be sure this breaks down at low curvature just like finite volume QFT, but explicit verification is difficult.

What about the Gibbons Hawking temperature? Is that an *external* heat bath, making QFT in de Sitter space (static patch) non-unitary? Throughout my analysis I assume GH entropy is entanglement entropy, not an external heat bath.

A toy model...

An example where back-reaction can be ignored: semi-bounded 1+1d Rindler space with a uniform electric field.

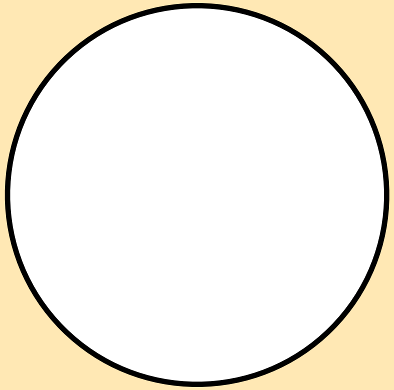
no firm boundary between:

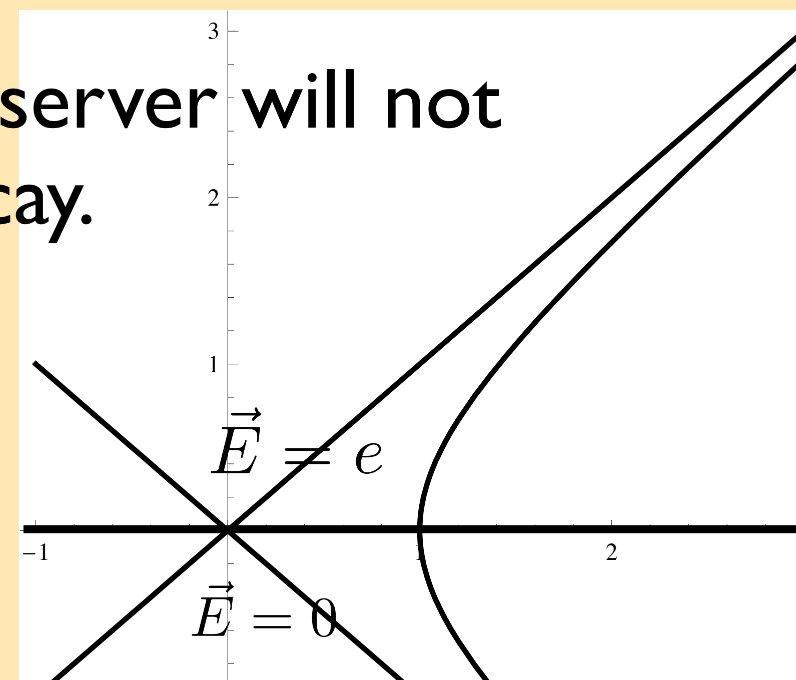
Virtual pair production ⟷ thermal activation ⟷ quantum tunneling

by an argument similar to Brown & Weinberg

Schwinger pairs are produced *at rest* w.r.t. the initial conditions (instantaneously generated electric field).

Even a low acceleration Rindler observer will not experience vacuum decay.





Vacuum instability
vs.
Black hole instability
an analogy

Flat	Anti- de Sitter	De Sitter
False vacuum is always unstable (neglecting gravitational back-reaction) Coleman	False vacuum unstable if energy difference is large enough Coleman - De Luccia	False vacuum is stable.
Black holes eat up space at any nonzero temperature. Gross - Perry - Yaffe	Black hole instability only if temperature is large enough Hawking - Page	Black holes are insignificant fluctuations Ginsparg - Perry
unstable	depends	stable

Conclusion

- At semi-classical level, de Sitter space is stable*
  - CDL instanton does not mediate decay in dS
  - Method of Bogolyubov coefficients does not predict decay unless backreaction is ignored.
  - de Sitter space is known to be stable against black hole nucleation
- This is not a question of quantum gravity, which we ignore
- The global picture of the Landscape is complex, but it is far from clear that the Coleman - De Luccia instanton is applicable.

*I assume QFT in de Sitter space is unitary, i.e., there is no <u>external</u> heat bath.