Supersymmetric bound states

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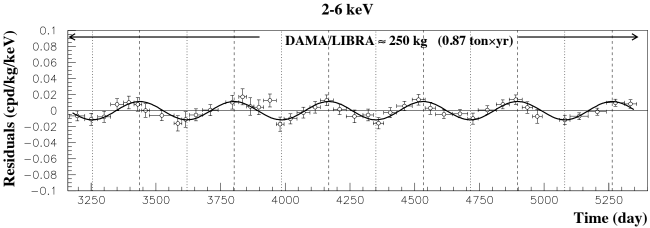
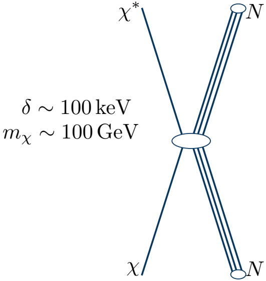
Outline

Based on
0912.2543: Jay Wacker, TR
1009.3523: Siavosh Behbahani, Martin Jankowiak, Jay Wacker, TR

Motivation I: Dark Matter



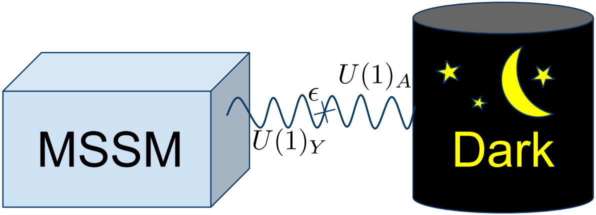


Hierarchy of scales.

CiDM/Atomic DM

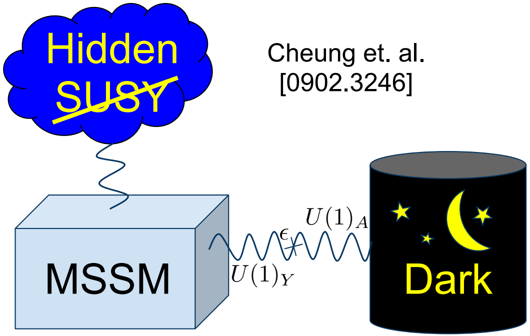
- D. Alves et. al. [0903.3945]: Composite (inelastic) DM is meson in hidden $SU(N_c)$ theory.
- D. Kaplan et. al. [0909.0753]: DM is hidden $U(1)$ atomic boundstate.

Hyperfine interaction gives $\delta \sim 100$ keV.



$$\mathcal{L} \supset \frac{\epsilon}{2} F^{(Y)}_{\mu\nu} F^{(A)\mu\nu}$$

Symmetry breaking from kinetic mixing



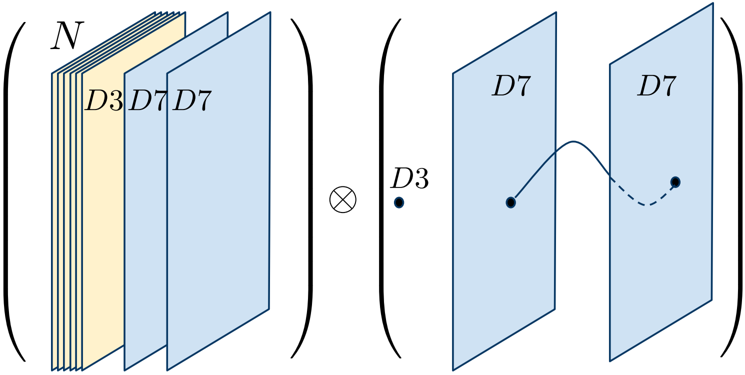
$$\mathcal{L} \supset \int d^2\theta \, \tfrac{\epsilon}{2} \mathcal{W}^{(a)} \mathcal{W}^{(b)} \supset \tfrac{\epsilon}{2} F^{(a)}_{\mu\nu} F^{(b)\mu\nu} + \epsilon D_Y D_A$$

- Induces effective F.I. $\zeta_A^2 = \epsilon D_Y \Rightarrow U(1)_A$ higgsed.
- Loop and ϵ suppressed soft masses can give tiny SUSY breaking

What is the spectrum of (approximately) SUSY bound states?

Motivation II: Heavy-Light $\mathcal{N} = 2$ mesons in AdS/CFT

Christopher Herzog, Thomas Klose [0802.2956], [0912.0733]



$$D = 3 + 1$$

No hyperfine!

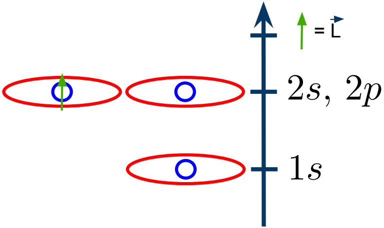
Motivation 3

It's fun!!

Undergrad quantum mechanics: Principal structure

Electron = , Proton = 

$$\mathcal{H}_\alpha = -\frac{\alpha}{r} + \ldots$$



- $E = -\frac{m_e \alpha^2}{2n^2}$
- Binding energy independent of L.

Undergrad quantum mechanics: Fine structure

Terms contributing to $\mathcal{O}(m_e\alpha^4) \sim 10^{-4}$eV:

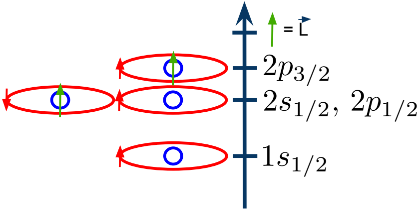
$$\mathcal{H}_\alpha = -\frac{\alpha}{r} - \frac{p^4}{8m_e^3} + \frac{\alpha\vec{L}\cdot\vec{S}_e}{2m_e^2 r^3} + \frac{\alpha\pi}{2m_e^2}\delta(\vec{r}) + \ldots$$

- Energy from two terms:

$$-\frac{m_e\alpha^2}{2n^2} - \frac{m_e\alpha^4}{2n^4}\left(\frac{n}{l+1/2} - \frac{3}{4}\right)$$

- Adding Spin-orbit coupling and Darwin term:

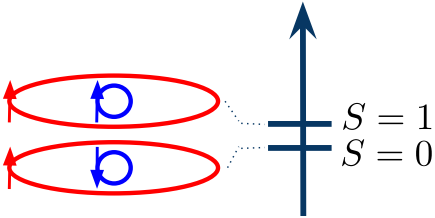
$$-\frac{m_e\alpha^2}{2n^2} - \frac{m_e\alpha^4}{2n^4}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)$$



Undergrad quantum mechanics: Hyperfine strure

At $\mathcal{O}(\alpha^4)$ there are terms suppressed in $\frac{m_e}{m_p}$, e.g. $\frac{\alpha \vec{S}_e \cdot \vec{S}_p}{m_e m_p r^3}$

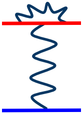
- Hyperfine splits $1s_{1/2}$. Gives 21 cm line.



- Also splits $2s_{1/2}$ and $2p_{1/2}$. Smaller than Lamb shift, but lower order in α.

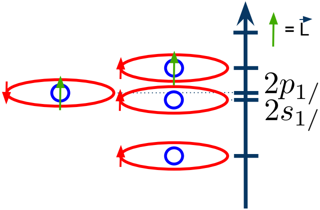
Undergrad quantum mechanics cont.

The next order in α is



$$\mathcal{H}_{\alpha^2} = \frac{4\alpha^2}{3m_e^2}\left(-2\ln\alpha + \frac{5}{6} - \ln K\right)\delta(\vec{r}) + \dots$$

Lamb shift: $\mathcal{O}\left(m_e\alpha^5\right) \sim 10^{-5}$eV



Intuitive $m_p \to \infty$ solution

SQED matter content:

	Superfield	$U(1)_V$	*Fermions*	*Bosons*
"Electrons"	$E, \bar{E}^c$	-1	e	$\tilde{e}_1, \tilde{e}_2$
"Protons"	$P, \bar{P}^c$	1	p	$\tilde{p}_1, \tilde{p}_2$
"*Photons*"	V		λ	A^μ

$$\mathcal{K} = |E|^2 + |E^c|^2 + |P|^2 + |P^c|^2 \quad \mathcal{W} = m_e E E^c + m_p P P^c$$

$$\begin{aligned} \mathcal{L} = &(\text{Charged scalar \& fermion kinetic terms}) \\ &+ (\text{Gauge and gaugino kinetic terms}) \\ &+ ie\lambda(p\tilde{p}_1^\dagger + \ldots) - e^2\left(|\tilde{p}_1|^2 - |\tilde{p}_2|^2 + \ldots\right)^2 \end{aligned}$$

SUSY interaction vanish in non-relativistic limit

Non-relativistic fields

$$\tilde{p}_{1,2} = \frac{e^{im_p t}}{\sqrt{2m_p}} \phi_{\tilde{p}_{1,2}}, \quad \Psi_p^D = e^{im_p t} \begin{pmatrix} \psi_p \\ \frac{i\vec{\sigma}\cdot\vec{\nabla}}{2m_p} \psi_p \end{pmatrix}$$

Charged scalar & fermion kinetic terms:

$$\bar{\psi}_p \left(i\partial_t - \frac{\nabla^2}{2m_p} \right) \psi_p + \bar{\phi}_p \left(i\partial_t - \frac{\nabla^2}{2m_p} \right) \phi_p + \text{(Charge terms)}$$

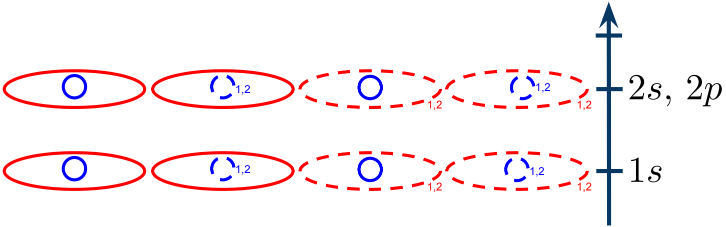
Supersymmetric interactions:

$$e\bar{\lambda} \left(\frac{1}{\sqrt{m_p}} (\psi_p \phi_p) + ... \right) + h.c. - e^2 \left(\frac{1}{m_p} \left(|\phi_{\tilde{p}_1}|^2 - |\phi_{\tilde{p}_2}|^2 \right) + ... \right)^2$$

Proton SUSY interactions suppressed by $1/\sqrt{m_p}$.

Principal splitting

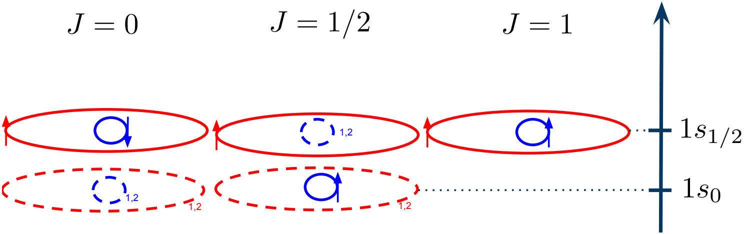
Spectrum insensitive to spin to $\mathcal{O}(m_e\alpha^2)$:



SUSY finestructure, $m_p \to \infty$ limit

- From before we know the spectrum for fermion/boson is:

$$E_{n,j_e} = -\frac{m_e\alpha^2}{2n^2} - \frac{m_e\alpha^4}{2n^4}\left(\frac{n}{j_e+\frac{1}{2}} + \frac{3}{4}\right),$$



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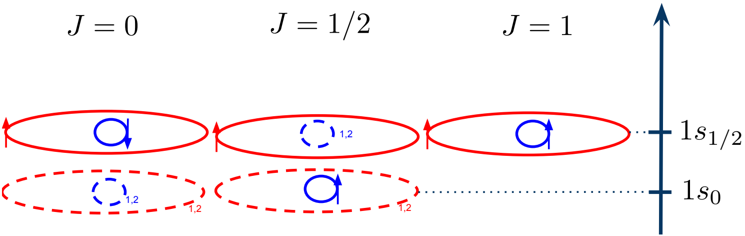
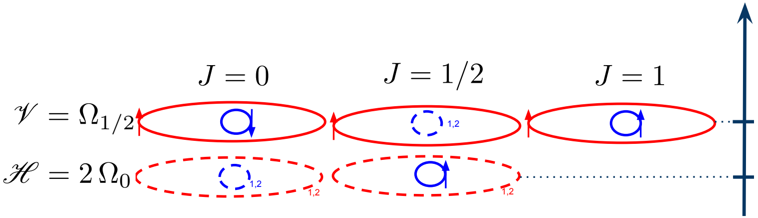


- SUSY transformation: $Qp \sim Q\tilde{p}_{1,2} \sim \sqrt{m_p} \gg Qe$

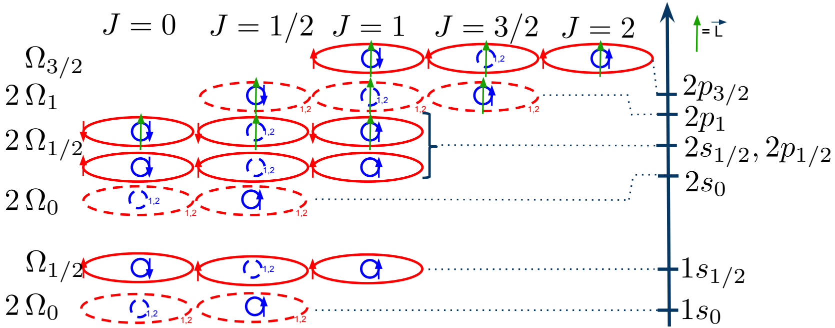
Matching to representations

The massive SUSY representations classified according to the spin of the Clifford vacuum. Counting gives

s	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
Ω_0	2	1			
$\Omega_{\frac{1}{2}}$	1	2	1		
Ω_1		1	2	1	
$\Omega_{\frac{3}{2}}$			1	2	1



SUSY finestructure, $m_p \to \infty$ limit



$$2\Omega_0 \oplus 2\Omega_{1/2} \oplus \dots \oplus \Omega_{n-1/2}$$

Spectrum of SUSY hydrogen, m_p finite

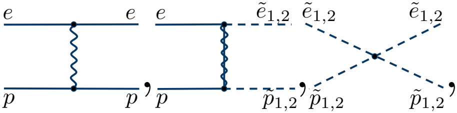
SUSY positronium

'82: Buchmller, Love & Peccei

'85: Di Vecchia & Schuchhart

Hydrogen calculation

$\hat{H}_{\text{eff}} = \Bigg\{$



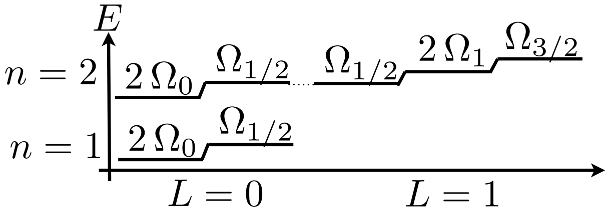
$, \ldots \Bigg\}$

$$E_{nj} = -\frac{\mu\alpha^2}{2n^2} + \frac{\mu\alpha^4}{n^3}\left[\frac{3}{8n} - \frac{\mu^2}{m_p m_e n} - \frac{1}{2j+1}\right]$$

j = spin of SUSY representation

μ = reduced mass

Spectrum of SUSY hydrogen cont.



- For $m_p \to \infty$, $\Omega_{1/2}$ contains both fermion-fermion states. Split for finite m_p in regular hydrogen, but protected in SUSY. Instead eigenstates rotate

$$\cos(2\theta)|(pe)_0\rangle + \sin(2\theta)(|\tilde{p}_2\tilde{e}_1\rangle - |\tilde{p}_1\tilde{e}_2\rangle)/\sqrt{2}, \quad \tan(\theta) \equiv \frac{m_e}{m_p}$$

- No apparent symmetry explains n=2, $\Omega_{1/2}$ degeneracy.

Q: How can we understand the wave function above?

Supersymmetric spectroscopy

Assumption:

Binding dynamics is spin independent at $\mathcal{O}(\mu\alpha^2)$

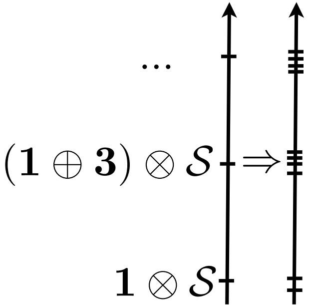
The eigenstates then factorize

$|\Psi_{nl\mathcal{S}}\rangle = |\psi_{nl}(r)\rangle \otimes |\mathcal{S}\rangle + \mathcal{O}(\alpha)$

At $\mathcal{O}(\mu\alpha^4)$ irreducible reps. generally split

Plan:

1. Decompose $\mathcal{S}$ for $L = 0$
2. Decompose $L \otimes \mathcal{S}$ for $L > 0$



Decomposition of $\mathcal{S} = \Omega_0^p \otimes \Omega_0^e \simeq 2\Omega_0 \oplus \Omega_{1/2},\ L = 0$

When hard, use superfields!!!!

- Use superfields with on-shell d.o.f.

$$E(y,\theta) = \tilde{e}_1 + \sqrt{2}\theta^\alpha e_\alpha - \theta^2 m_e\, \tilde{e}_2,\ \ldots$$

- Non-relativistic normalization: $e \to \psi_e,\ \tilde{e}_{1,2} \to \phi_{\tilde{e}_{1/2}}$
- SUSY acts on spatial wavefunction $\psi(r)$ through $\vec{\nabla} \sim v \sim \alpha\ \Rightarrow$ Ignore spatial dependence
- Write down neutral bilinears

$$PE,\ P\bar{E}^c,\ \bar{P}^c E,\ \bar{P}^c \bar{E}^c,\ P\mathcal{D}^\alpha E,\ \ldots$$

- Decompose using standard projection operators:

$$\mathcal{P}_1 = \frac{\bar{\mathcal{D}}^2\mathcal{D}^2}{16\Box},\quad \mathcal{P}_2 = \frac{\mathcal{D}^2\bar{\mathcal{D}}^2}{16\Box},\quad \mathcal{P}_T = -\frac{\mathcal{D}\bar{\mathcal{D}}^2\mathcal{D}}{8\Box}.$$

Decomposition of $\mathcal{S} = \Omega_0^p \otimes \Omega_0^e$ cont.

All states contained in three superfields:

$$\begin{aligned}
\mathcal{P}_1 PE = \quad & PE \propto \phi_{\tilde{p}_1}\phi_{\tilde{e}_1} + \sqrt{2}\Theta^a\left(c_\theta \psi_p^a \phi_{\tilde{e}_1} + s_\theta \phi_{\tilde{p}_1}\psi_e^a\right) \\
& \quad -\Theta^2\left(s_\theta^2 \phi_{\tilde{p}_1}\phi_{\tilde{e}_2} + c_\theta^2 \phi_{\tilde{p}_2}\phi_{\tilde{e}_1} - s_{2\theta}(\psi_p\psi_e)_0\right),
\end{aligned}$$

$$\mathcal{P}_2 \bar{P}^c \bar{E}^c = \quad \bar{P}^c \bar{E}^c \propto \ldots$$

$$\mathcal{P}_T P \bar{E}^c = \quad \left\{ \begin{array}{lll} D & \propto & c_{2\theta}(\psi_p\psi_e)_0 + s_{2\theta}(\phi_{\tilde{p}_2}\phi_{\tilde{e}_1} - \phi_{\tilde{p}_1}\phi_{\tilde{e}_2})/\sqrt{2} \\ \bar{\lambda}_1 & \propto & s_\theta \psi_p \phi_{\tilde{e}_2} - c_\theta \phi_{\tilde{p}_2}\psi_e \\ \lambda_2 & \propto & s_\theta \psi_p \phi_e - c_\theta \phi_p \psi_e \\ v^\mu & \propto & \psi_p \vec{\sigma} \psi_e \end{array} \right.$$

$$\tan^2\theta = \frac{m_e}{m_p}, \;\; \bar{\Theta}^\alpha = \sqrt{m_p + m_e}\,\theta^\alpha.$$

$$m_p \to \infty \text{ OK.}$$

R-symmetry and Parity

Two symmetries besides SUSY:

- $U(1)_R: \; R[P] = R[E] = 1, \; R[\bar{P}^c] = R[\bar{E}^c] = -1$
- Parity $\mathcal{P}$: $\mathcal{P}P = \bar{P}^c$, $\mathcal{P}E = \bar{E}^c$.

$$\text{In particular, } \mathcal{P}\tilde{e}_1 = \tilde{e}_2 \text{ but } R[\tilde{e}_1] = -R[\tilde{e}_2]$$
$$\Rightarrow [U(1)_R, \mathcal{P}] \neq 0$$

$$U(1)_R \rtimes P \cong O(2)_R$$

Conclusions:

- $\Omega_0^p \otimes \Omega_0^e$ decomposes into one hyper multiplet $\mathscr{H} = \{PE, \bar{P}^c\bar{E}^c\}$ and one vector $\mathscr{V} = \mathcal{P}_T P\bar{E}^c$.
- Same method can be used for baryons, positronium, super Yukawa bound states etc..
- $L > 0$ analogous

What is spectrum of weakly broken SUSY bound states?

SUSY breaking effects:

- Binding dynamics, e.g. gaugino interactions etc. Velocity suppressed.
- Soft masses
  - $O(2)_R$ preserving masses, e.g. $\Delta^2(|\tilde{e}_1| + |\tilde{e}_2|^2)$
  - $O(2)_R$ violating masses, e.g. $Bm_e \tilde{e}_1 \tilde{e}_2^* + c.c$

  Effect of soft masses captured in

$$\delta H_{\text{soft}} = \delta m_{e\pm} |\phi_{e\pm}\rangle\langle\phi_{e\pm}| + \delta m_{p\pm} |\phi_{p\pm}\rangle\langle\phi_{p\pm}|$$

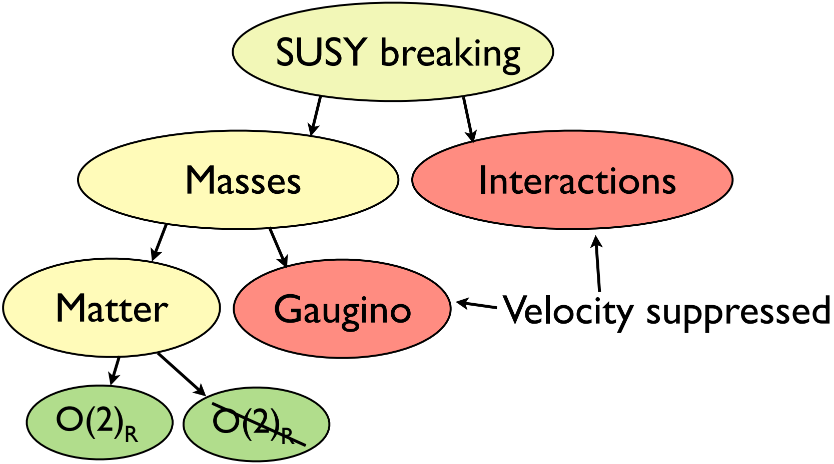
Assumption:

Soft masses smaller than principal splitting $\mu\alpha^2$

$\Rightarrow$ Spectrum given by diagonalizing by finite dimensional matrix

What is spectrum of weakly broken SUSY bound states?

SUSY breaking effects:



Effect of soft masses

Soft mass terms:

- $O(2)_R$ preserving masses, e.g. $\Delta^2(|\tilde{e}_1| + |\tilde{e}_2|^2 + |p_1|^2 + |p_2|^2)$
- $O(2)_R$ violating masses, e.g. $B(m_e \tilde{e}_1 \tilde{e}_2^* + m_p \tilde{p}_1 \tilde{p}_2^*) + c.c.$

$$\delta m = \sqrt{m^2 + \delta m^2} \simeq m + \frac{\delta m^2}{m}$$

Effect of soft masses captured in

$$\delta H_{\text{soft}} \,=\, \delta m_{e\pm} |\phi_{e\pm}\rangle\langle\phi_{e\pm}| \,+\, \delta m_{p\pm} |\phi_{p\pm}\rangle\langle\phi_{p\pm}|$$

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Soft masses smaller than principal splitting $\mu\alpha^2$

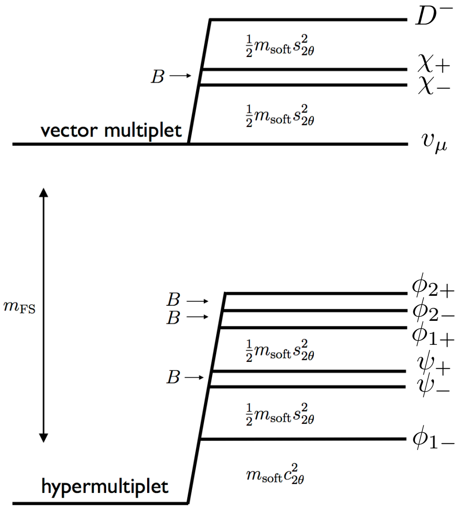
$\Rightarrow$ Spectrum given by diagonalizing by finite dimensional matrix

Example: Ground state

In ground state, splitting between $\mathscr{H}$ and $\mathscr{V}$, denoted m_{FS}, fixed by dynamics. Spin/particle eigenstates determined by SUSY.

Wealth of scales:

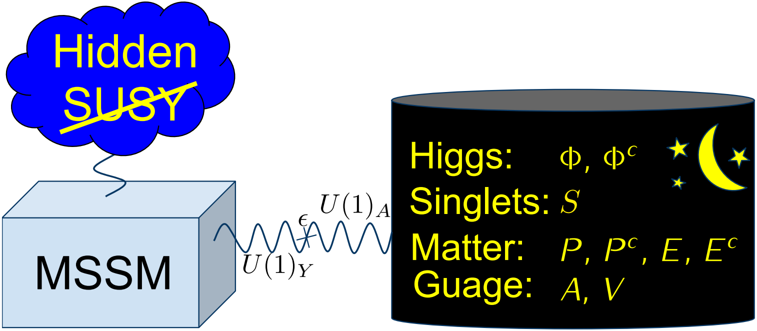
- Principal splitting m_{prin}.
- Fine structure m_{FS}
- $O(2)_R$ preserving $m_{\mathrm{soft}} \sim \Delta^2/m_e$.
- States protected by $O(2)_R$ split by B



Lowest state $\mathcal{P} = -1$ scalar.

Splittings insensitive to details of binding dynamics.

Toy model



$$\mathcal{W} \; = \; S(\Phi\Phi^c - \mu^2) + y_p \Phi P P^c + y_e \Phi^c E E^c$$

Kinnetic mixing gives effective Fayet-Iliopoulos term:

$$D_A = \epsilon D_Y - 2g_A(|\Phi|^2 - |\Phi^c|^2 + ...)$$

At SUSY minimum:

$$|\Phi_c| \sim |\Phi| \sim \mathcal{O}(100\text{GeV}), \; m_p \sim m_e \sim m_A \sim \mathcal{O}(\text{GeV})$$

SUSY breaking

$U(1)_R$-preserving mass: $\Delta_e^2 \simeq \Delta_p^2 \simeq \epsilon^2 \frac{\alpha_A}{\alpha_Y} M^2_{\tilde{e},SM}$

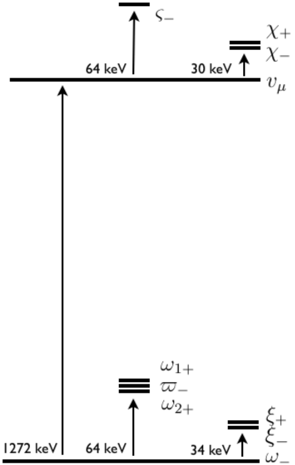
$U(1)_R$-breaking operator: $\lambda_A \frac{\epsilon^2 M_1 \Box}{\Box + M_1^2} \lambda_A \Rightarrow B \simeq \frac{\epsilon^2 \alpha_A M_1}{\pi} \log \frac{\Lambda_{UV}}{M_1}$

$U(1)_V$ gaugino receives a tiny soft mass $m_{\lambda V} \sim \frac{\alpha_V}{4\pi^2} B$

$$m_{\text{principal}} \gg \Delta^2 / m_e \gg B \gg m_{\lambda V}$$

Benchmark point

- $m_p \sim 50$ GeV
- $m_e \sim 3$ GeV
- $\alpha_V \sim 0.15$
- $m_{\text{Prin}} \sim 50$ MeV
- $m_{\text{FS}} \sim 1.5$ MeV
- $m_{\text{soft}} \sim 270$ keV
- $B \sim 5$ eV



Conclusion:

- SUSY hydrogen spectrum almost identical to that of Dirac/Klein-Gordon equation, to $\mathcal{O}(\mu\alpha^4)$.
- $2s_{1/2}$ and $2p_{1/2}$ degenerate, even for finite proton mass.
- Superfield methods useful to calculate states and explain degeneracies.
- Supersymmetry breaking gives a wealth of scales.

Backup slides

Outlook

SUSY molecules:

Can supersymmetric matter form molecules and complex structure?

Problem: Boson binding energy scales as $E \sim N^{7/5}$ (c.f. fermions: $E \sim N$). $D \sim N^{-1/5}$.

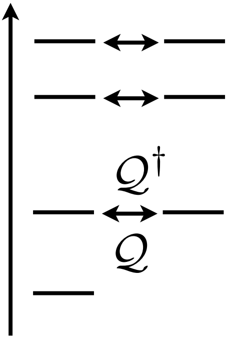
Structure Collapses. Black hole formations?

$(SUSY)^2$ hydrogen?

$-\frac{\alpha}{r} \Rightarrow$ Runge-Lenz Vector $\Rightarrow$ QM SUSY

Dirac eq. - $\frac{\alpha}{r} \Rightarrow$ Johnson-Lippmann operator $\Rightarrow$ QM SUSY

SUSY QM: Singled groundstate + doublet excited states



Decomposing $L \otimes \mathcal{S}$ into irreducible reps.

- Remember: Massive **particle multiplet** SUSY multiplet spanned by

$$|\Omega_j\rangle,\ Q^\dagger|\Omega_j\rangle,\ Q^{\dagger 2}|\Omega_j\rangle,$$

  $|\Omega_j\rangle$ irrep. with **total** angular momentum j.

- Previous slides: On shell SUSY **field multiplets** spanned by

$$\left\{ nl\rangle \otimes |\Omega_s\rangle, \quad |nl\rangle \otimes \left(Q^\dagger \otimes |\Omega_s\rangle\right), \quad |nl\rangle \otimes \left(Q^{\dagger 2}|\Omega_s\rangle\right)\right\},$$

  to lowest order in α. Reducible rep. of rotation group for $L > 0$.

- Interpret non-relativistic fields as one particle wave functions:

$$\left\{|nl\rangle \otimes |\Omega_s\rangle, \quad Q^\dagger \otimes (|nl\rangle \otimes |\Omega_s\rangle), \quad Q^{\dagger 2} (|nl\rangle \otimes |\Omega_s\rangle)\right\}$$

- $|nl\rangle \otimes |\Omega_s\rangle$ decomposes into $|\Omega_{|l-s|}\rangle, \ldots, |\Omega_{l+s}\rangle$

Decomposing $L \otimes \mathcal{S}$ into irreducible reps. cont.

This gives the decomposition

$$l \otimes \Omega_s = \Omega_{|l-s|} \otimes \ldots \otimes \Omega_{|l+s|}$$

In our case

$$l > 0 : \quad l \otimes \Omega_0^p \otimes \Omega_0^e = l \otimes \left(2\,\Omega_0 \oplus \Omega_{1/2}\right) = \Omega_{l-1/2} \oplus 2\,\Omega_l \oplus \Omega_{l+1/2}.$$