Warped Penguins

Yuhsin Tsai

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In collaboration with Csaba Csáki, Yuval Grossman, Philip Tanedo

Cornell University

LEPP Particle Theory Seminar

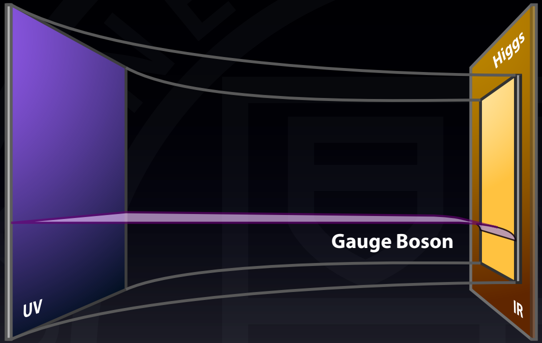
Why are Penguins important in RS ?

Randall-Sundrum in one slide

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99);

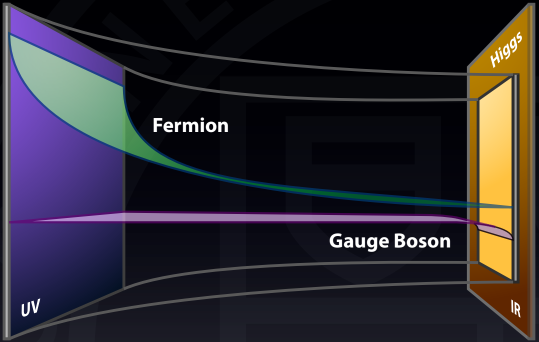
Randall-Sundrum in one slide



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99);

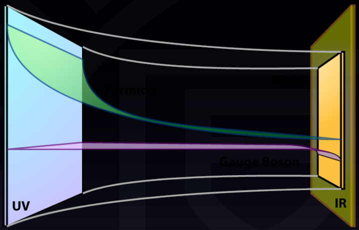
Randall-Sundrum in one slide



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99); Grossman, Neubert (00); Gherghetta, Pomarol (00); **Bulk Higgs:** Agashe, Contino, Pomarol (04); Davoudiasl, Lille, Rizzo (05)

Flavor changing & the wavefunction overlap



4D Yukawa Coupling: $Y_*^{ij}\bar{L}_i H E_j \times f_{L_i}(R')\, f_{E_j}(R')$

4D Gauge Coupling: $g_{ii}\bar{L}_i \not{Z} L_i \times \int_R^{R'} dz \left(\frac{R}{z}\right)^4 f_{L_i}(z) f_Z(z) f_{L_i}(z)$

Before we proceed,

Two assumptions for an interesting model:

- M_{kk} is not too heavy. Give me a number! $M_{kk} \sim 3\mathrm{TeV}$.

  $\Rightarrow$ Can be seen in LHC!

- Y_*^{ij} are ancharic, $\mathcal{O}(1)$ numbers.

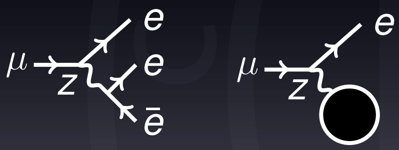
  $\Rightarrow$ No tuning is required!

FCNC from the loop & tree

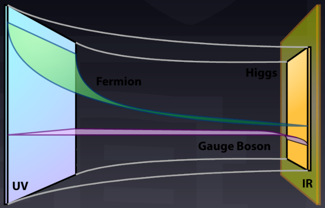
$\mu \to e\gamma$:

$\propto Y_*^3$

$\mu \to 3e$ & $\mu \to e$:

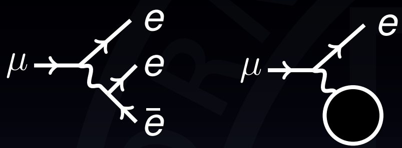


$\propto Y_*^{-1}$



The opposite Y_* bounds

For Y_* ,



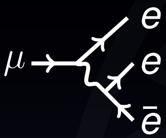
gives a lower bound.

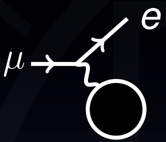
gives an upper bound.

Constraint Y_* from both sides!

The opposite Y_* bounds

For Y_* ,



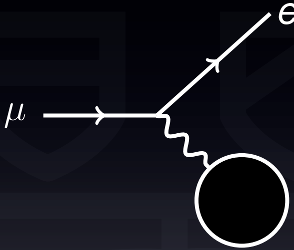


gives a lower bound.

gives an upper bound.

Want to get the precise bounds!

Tree level constraints from



K. Agashe, A. E. Blechman, and F. Petriello, hep-ph/0606021.

The origin of the flavor changing

Taking the zero mode Z as an example, the nonuniversal part of the wave function $h^{(0)}(z)$ near IR gives the flavor changing,

- Wave function:

$$h^{(0)}(z) = \frac{1}{\sqrt{R \ln R'/R}} \left[1 + \frac{M_Z^2}{4} z^2 \left(1 - 2 \ln \frac{z}{R} \right) \right]$$

- Gauge coupling:

$$g^{z f_i f_j} = g^z_{SM} \left[1 + \frac{(M_Z R')^2 \ln R'/R}{2(3-2c)} f_i f_j \right]$$

The γ' and Z' have the similar form.

The Yukawa bound

The tree level result,

- $Br(\mu \to 3e) \simeq 10^{-13} \left(\frac{3\mathrm{TeV}}{M_{KK}}\right)^4 \left(\frac{2}{Y_*}\right)^2$
- $Br(\mu \to e)_{\mathrm{Ti}} \simeq 2 \cdot 10^{-12} \left(\frac{3\mathrm{TeV}}{M_{KK}}\right)^4 \left(\frac{2}{Y_*}\right)^2$

Comparing to the experimental bounds,

- $Br(\mu \to 3e) < 10^{-12}$
- $Br(\mu \to e)_{\mathrm{Ti}} < 6.1 \cdot 10^{-13}$

The $\mu \to e$ gives the most stringent bound (for $M_{KK} = 3\mathrm{TeV}$).

$$Y_* > 3.7$$

$\mu \to e\,\gamma$ in a warped XD

Warped penguins

Warped Penguin looks weird...

- They live in 5D.
- Previous analyses suggested the brane Higgs case is log-divergent.

People thought they are divergent because of the KK sum.

Want to do a full 5D calculation to check the finiteness!

Warped penguins

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- They live in 5D.
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Can show $\mu \to e\gamma$ is finite!

$\mu \to e\,\gamma$ in 5D

The diagrams of $\mu \to e\,\gamma$:

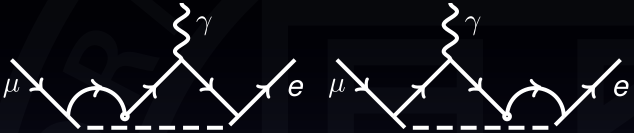
- Get the leading order result by doing mass insertions.
- Rotate all the flavor mixing to the Yukawa.
- It is an L-R operator: $a\,\bar{L}\sigma^{\mu\nu}F_{\mu\nu}E$
- From the gauge symmetry:

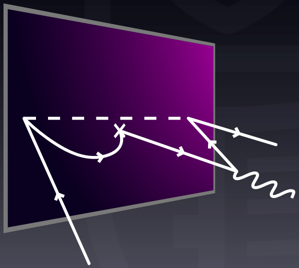
$$\epsilon_\mu \mathcal{M}^\mu \sim \epsilon_\mu \bar{u}\,(\,p^\mu + p_e^\mu - (m_\mu + m_e)\gamma^\mu\,)\,u$$

- To get a, we only need to calculate the coefficient of p^μ.

$\mu \to e\,\gamma$ in 5D

The diagrams:



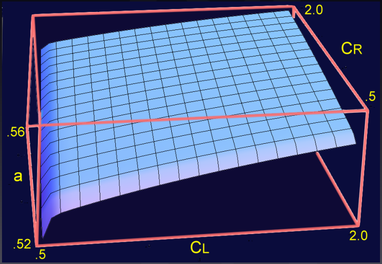


The Result

The result of the amplitude can be written as follows:

$$a_{k\ell} \times R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} \left(f_{L_i} Y_{ik} Y^{\dagger}_{k\ell} Y_{\ell j} f_{-E_j} \right) \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

The loop integral gives $a_{k\ell} \simeq 0.5$.



$$\mathrm{Br}(\mu \to e\,\gamma)_{\mathrm{thy}} > 8.2 \cdot 10^{-7} a^2 \left(\frac{3\mathrm{TeV}}{M_{KK}} \right)^4 \left(\frac{Y_*}{2} \right)^4$$

$$\mathrm{Br}(\mu \to e\,\gamma)_{\mathrm{exp}} < 1.2 \cdot 10^{-11}$$

for $M_{kk} = 3\,\mathrm{TeV}$, we have

$$Y_* < 0.2$$

The Yukawa bounds

There is a tension between the two bounds!

$$Y_*(\mu \to e) > 3.7 \qquad Y_*(\mu \to e\,\gamma) < 0.2$$

To release the tension:

- Make M_{KK} heavier. $M_{KK} \sim 10\,\mathrm{TeV}$
- Give some structure to Y_*^{ij}.
- Introduce some symmetry to suppress the FCNC.

Custodially protected model

K.Agashe, R.Contino, L. Da Rold, and A. Pomarol, hep-ph/0605341.
M.E.Albrecht, M.Blanke, A.J.Buras, B.Duling, and K.Gemmler, 0903.2415.

Introducing a custodial symmetry for leptons

$$\mathrm{SU(2)_L \times SU(2)_R \times U(1)_X \times P_{L,R}}$$

This model gives:

- γ, Z, γ', Z' & Z_H.
- The coupling of Z, Z' to the LH fermions becomes flavor universal.

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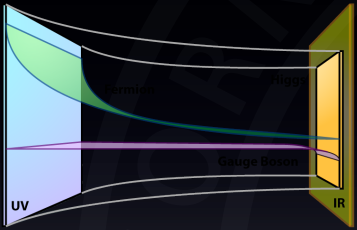
- γ, Z, γ', Z' & Z_H.
- The coupling of Z, Z' to the LH fermions becomes flavor universal.

The LH FCNC is suppressed.

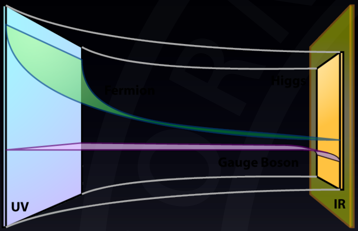
Custodially protected model



Strategy

- Make the RH leptons towards UV.
- Make the LH leptons towards IR.

Custodially protected model



This reduces the Y_* bound to

$$Y_* > 1$$

$Y_* > 3.7$ for the non-custodial case.

Strategy

- Make the RH leptons towards UV.
- Make the LH leptons towards IR.

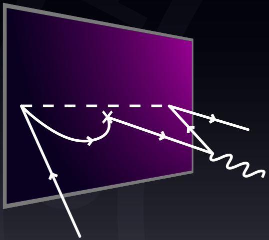
Tension between the two bounds

It's not enough...

$$Y_*(\mu \to e) > 1 \qquad Y_*(\mu \to e\,\gamma) < 0.2$$

A mild tension still exists.

Very interesting detail for the 5D loops!



The fermion propagator in 5D

$$(-\not{p} + i\gamma^5\partial_5 + m)\Delta(p, z, z') = i\delta(z - z')$$

Trick:

$$\left(p^2 - \partial_5^2 + m^2\right)F(p, z, z') = i\delta(z - z')$$

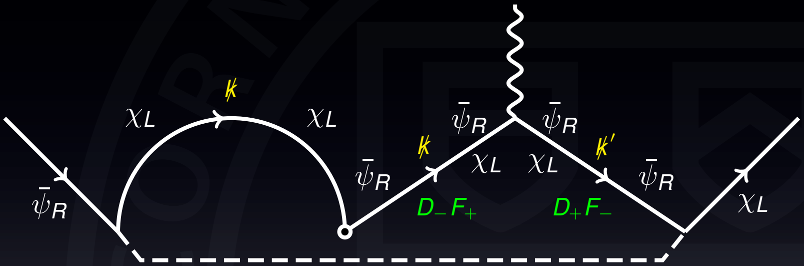
$$\Delta(p, z, z') = (\not{p} - i\gamma^5\partial_5 + m)F(p, z, z')$$

In flat XD case:

$$F(p, z, z') = A(p, z')\cos(p\,z) + B(p, z')\sin(p\,z)$$

The chiral BC

The chiral BC constraints the form of the amplitude!



$$\Psi_L = \begin{pmatrix} \chi_L \\ \bar{\psi}_L \end{pmatrix} \qquad \Psi_R = \begin{pmatrix} \chi_R \\ \bar{\psi}_R \end{pmatrix}$$

$$\Delta = \begin{pmatrix} \Delta_{\psi\chi} & \Delta_{\psi\psi} \\ \Delta_{\chi\chi} & \Delta_{\chi\psi} \end{pmatrix} = \begin{pmatrix} D_+ F_- & \sigma^\mu p_\mu F_+ \\ \bar{\sigma}^\mu p_\mu F_- & D_- F_+ \end{pmatrix}$$

How to calculate the 5D loop?

Feynman's trick with Bessel functions?? We don't know that...

However, we can

- Tayler expand the propagator into powers of the external momentum (p^μ, q^μ). $\frac{1}{(k^2+2k\cdot q)} = \frac{1}{k^2}\left(1 - \frac{2k\cdot q}{k^2} + ...\right)$.
- Isolate the p^μ terms. Get the coefficient. $\not{p}\gamma^\mu = 2p^\mu - \gamma^\mu \not{p}$.
- Solve the numerical integral, get the coefficient of $\mu \to e\,\gamma$.
  $a \times R'^2 \frac{e}{16\pi^2}\frac{v}{\sqrt{2}}\left(f_L YY^\dagger Y f_E\right)\bar{u}\not{\epsilon}u$; $a = \int dx \int dy\,(\text{scalar function})$.

Why is $\mu \to e\,\gamma$ is finite?

Lorentz invariance $+$ chiral BC gives the finiteness:

The propagators are composed of two parts

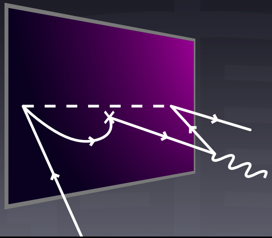
- The Dirac operator part (gives the γ^{μ} structure).
- The Bessel function part (contains the 5D profile).

In the UV limit, the photon vertex is pulled back to the IR brane.

- Can just look at the Dirac operator part.

Each loop contains two sectors:

- photon emission
- mass insertion



Why is $\mu \to e\,\gamma$ is finite?

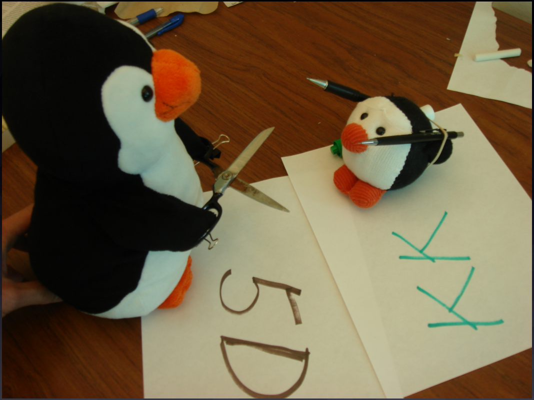
- γ emission: Lorentz inv $+$ chiral BC gives $(\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu)$
- mass insertion: The chiral BC gives $\not{k}$.
- Combining the two m-insertion amp, the leading order becomes

$$\mathcal{M}_{(a)} + \mathcal{M}_{(b)} \sim \not{k}\,(\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu) + (\not{k}\gamma^\mu\not{k} - k^2\gamma^\mu)\,\not{k} = 0$$

- From NDA, only the leading order term can be divergent.

$\mu \to e\,\gamma$ is finite!

5D V.S. KK



The KK result should be the same as 5D

The mass matrix:

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{KK,1} & m^{23} \\ 0 & 0 & M_{KK,2} \end{pmatrix}$$

In the mass basis, the Yukawa takes the form ($\epsilon = m/M_{KK}$)

$$\hat{y} \sim \begin{pmatrix} 1 & 1+\epsilon & -1+\epsilon \\ 1+\epsilon & \cdots & \cdots \\ 1-\epsilon & \cdots & \cdots \end{pmatrix}$$

$$\mathcal{M} \propto (1+\epsilon)(1+\epsilon) + (-1+\epsilon)(1-\epsilon) \sim \epsilon$$

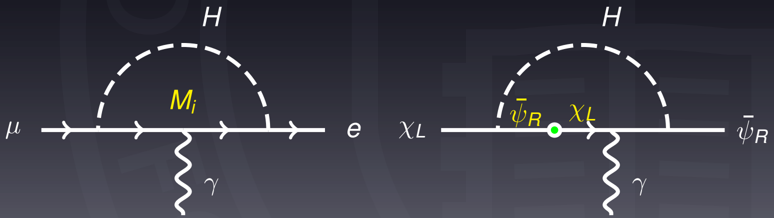
The KK result should be the same as 5D

For the finiteness:

- In the mass basis, naive NDA gives $\mathcal{M} \propto M_{KK}^{-1}$. The Yukawa gives additional $\epsilon = m/M_{KK}$ and ensures finiteness.

$$\Rightarrow \mathcal{M} \propto \sum_{KK} \frac{1}{(M_{KK})^2}$$

- In the M_{KK} basis, treat m as the mass insertion. The chiral BC effect becomes obvious.



The KK result should be the same as 5D

For the a value:
Do the momentum integral to infinity and sum the KKs.

$$\mathcal{M} = \sum_{n=1}^{N} \int_0^\infty d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

The leading order result is

$$\mathcal{M} \propto \frac{1}{(M_{KK})^4} \quad !?$$

which is different from the 5D result $\mathcal{M} \propto R'^2$!

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5D $\neq$ KK !?

The KK result should be the same as 5D

The reason is,

Wrong UV limit!

To match the 5D & KK, the momentum cutoff $\lesssim$ KK cutoff.

The correct way of doing the sum and the integral is

$$\lim_{N\to\infty} \sum_{n=1}^{N} \int_0^{N\,M_{KK}} d^4k \;\; \hat{\mathcal{M}}^{(n)}(k)$$

This gives the same leading order contribution $\propto M_{KK}^{-2}$ as in 5D.

The 5D & KK results do match!

Outlook & Conclusion

One loop $\mu \rightarrow e\gamma$ for brane Higgs case is finite.

- The bulk higgs is also finite.
- How about the finiteness for higher loops?

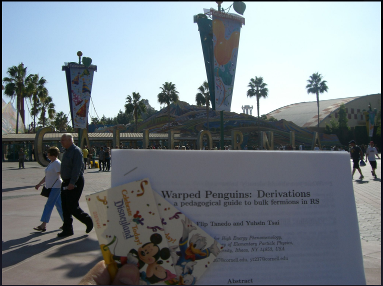
A tension between the tree-level & loop-induced Yukawa bounds.

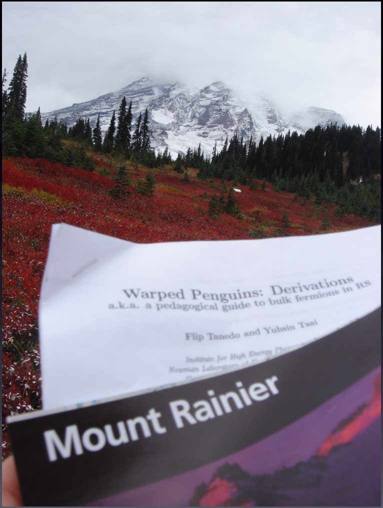
- Only having the custodial symmetry is not enough.
- Need some structure on Y_*.
- How about $b \rightarrow s\gamma$?

The nontrivial UV limit for KK is important for mathcing the 5D.

- Does this happen in other loop-induced processes?

A long journey for Warped Penguins...





Yes, we worked very hard!