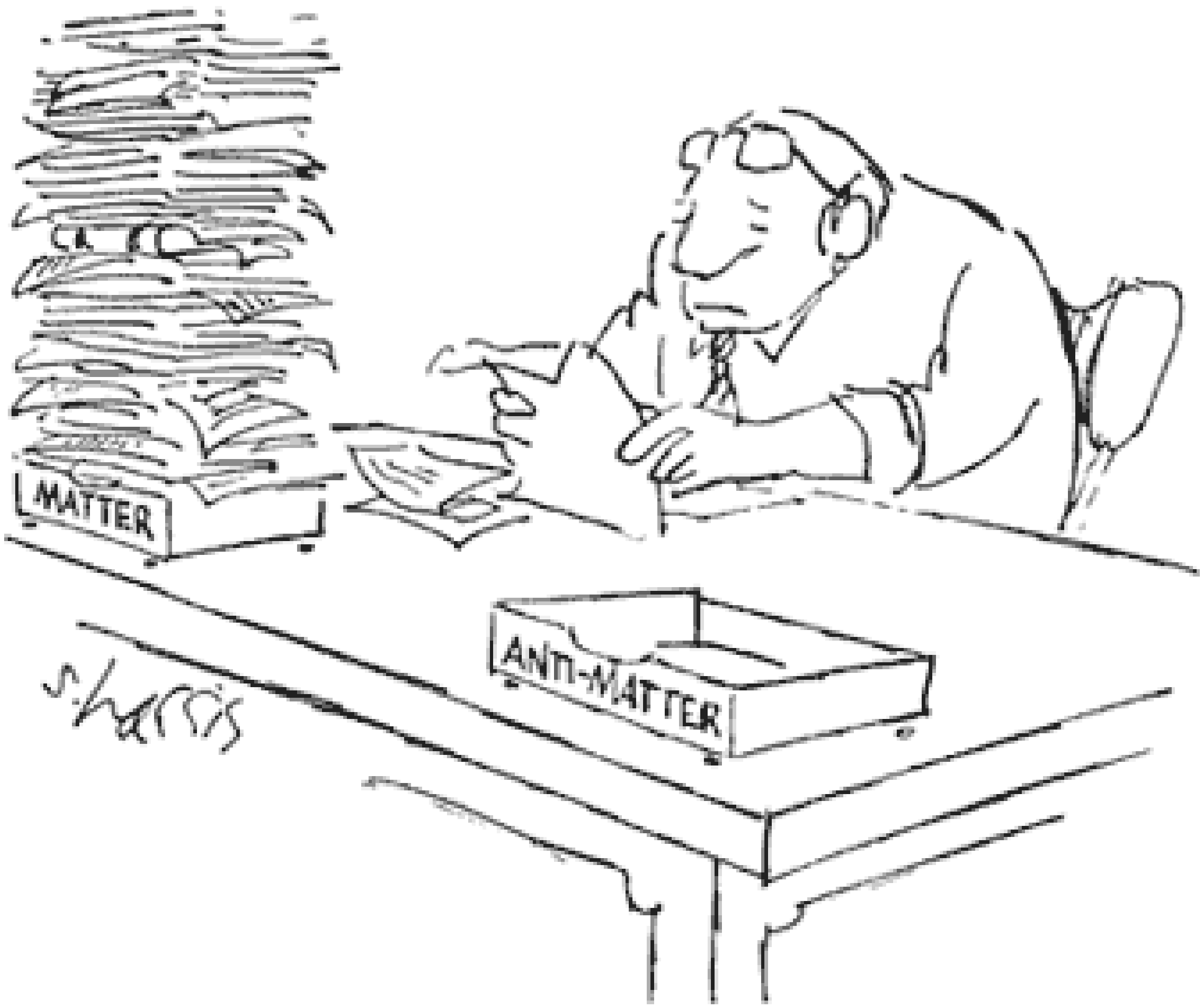
The Higgs Cubic and The Viability of Electroweak Baryogenesis

Andrew Noble
with Maxim Perelstein

hep-ph/0711.3018

After sifting through the astrophysical evidence …

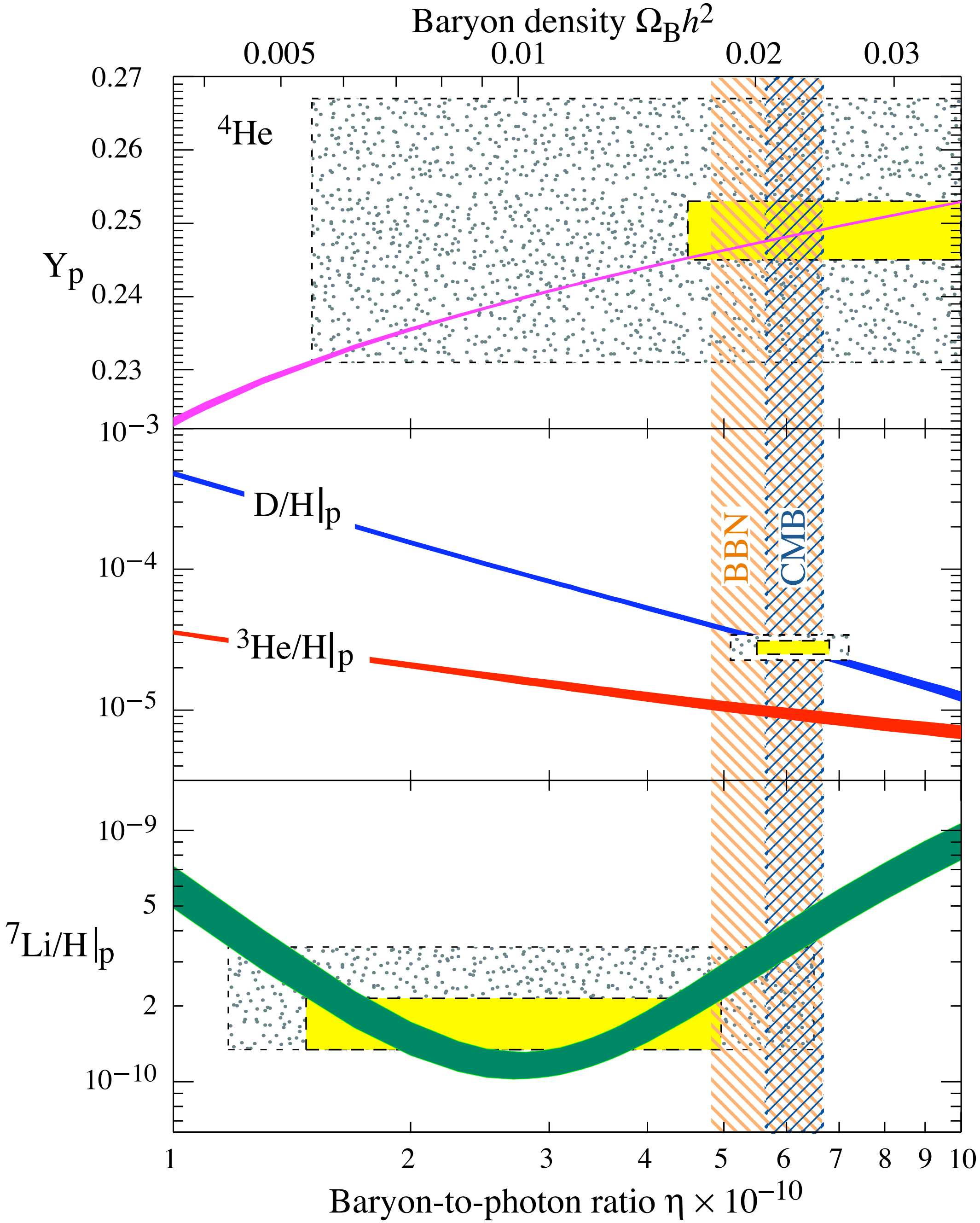


The Baryogenesis Challenge

Even though matter and anti-matter are *nearly* symmetric in the SM, the universe appears to be dominated by matter.

Is there a dynamical mechanism in the evolution of the universe that could account for this asymmetry?

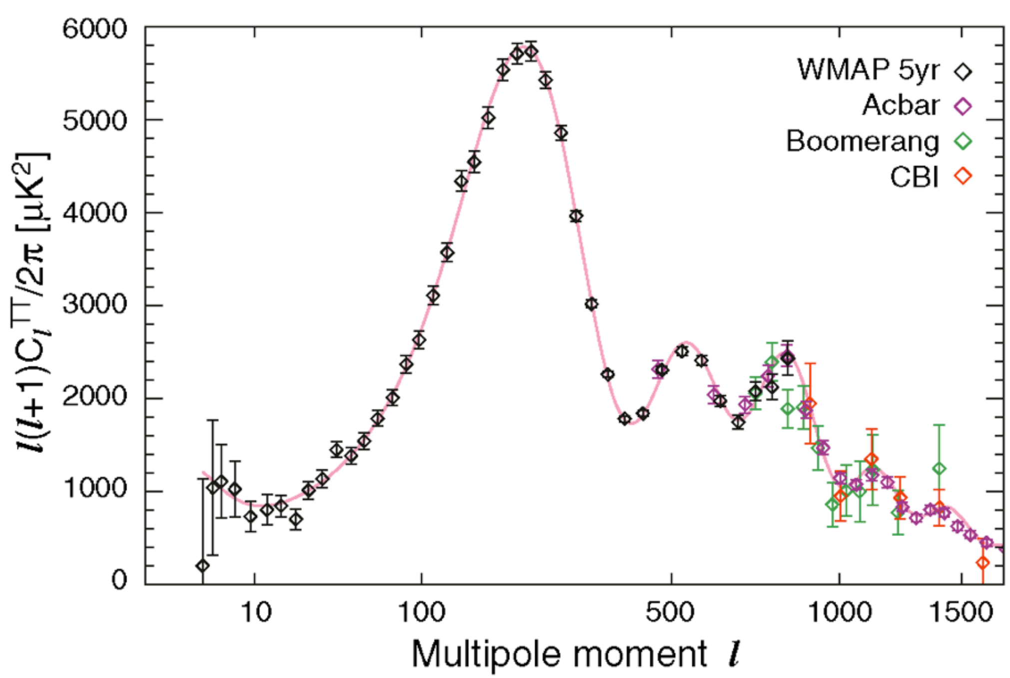
A Precise Target



$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma}$$
$$= (273.9 \times 10^{-10})\Omega_B h^2$$

$$5.9 < \eta \times 10^{10} < 6.4$$

(Simha and Steigman 2008)



Our Humble Origins

$$\text{For } t \lesssim 10^{-6}\text{s}, \quad \frac{n_q - n_{\bar{q}}}{n_q} \sim \frac{3}{100,000,000}$$

Many Creative Ideas

- Planck Scale Baryogenesis
- GUT Baryogenesis
- Electroweak Baryogenesis (EWBG)
- Leptogenesis
- Affleck-Dine Baryogenesis

Many nice reviews: Cohen, Kaplan, Nelson 1993
Trodden 1998, Riotto and Trodden 1999
Dine and Kusenko 2003

The Higgs Cubic Coupling

Our claim: The higgs cubic provides a model-independent collider probe of the viability of EWBG.

$$\lambda_3 \equiv \frac{1}{6} \frac{d^3 V_{\rm eff}}{dh^3}\Big|_{h=v}$$

$$\left(\text{e.g.} \quad \lambda_{3,SM} = \frac{m_h^2}{2v} \right)$$

- ILC measurement:
  20% precision for m_h<140GeV and 1ab^{-1}.

- Comparable precision at the SLHC/VLHC for m_h<200GeV.

Outline

- Overview of EWBG.
- The Higgs Effective Potential.
- The Higgs Cubic and EWBG.

Sakharov's Criteria

Nobel Peace Prize 1975

A successful mechanism for Baryogenesis must include:

- Violation of B.
- Violation of C and CP.
- Nonequilibrium dynamics.

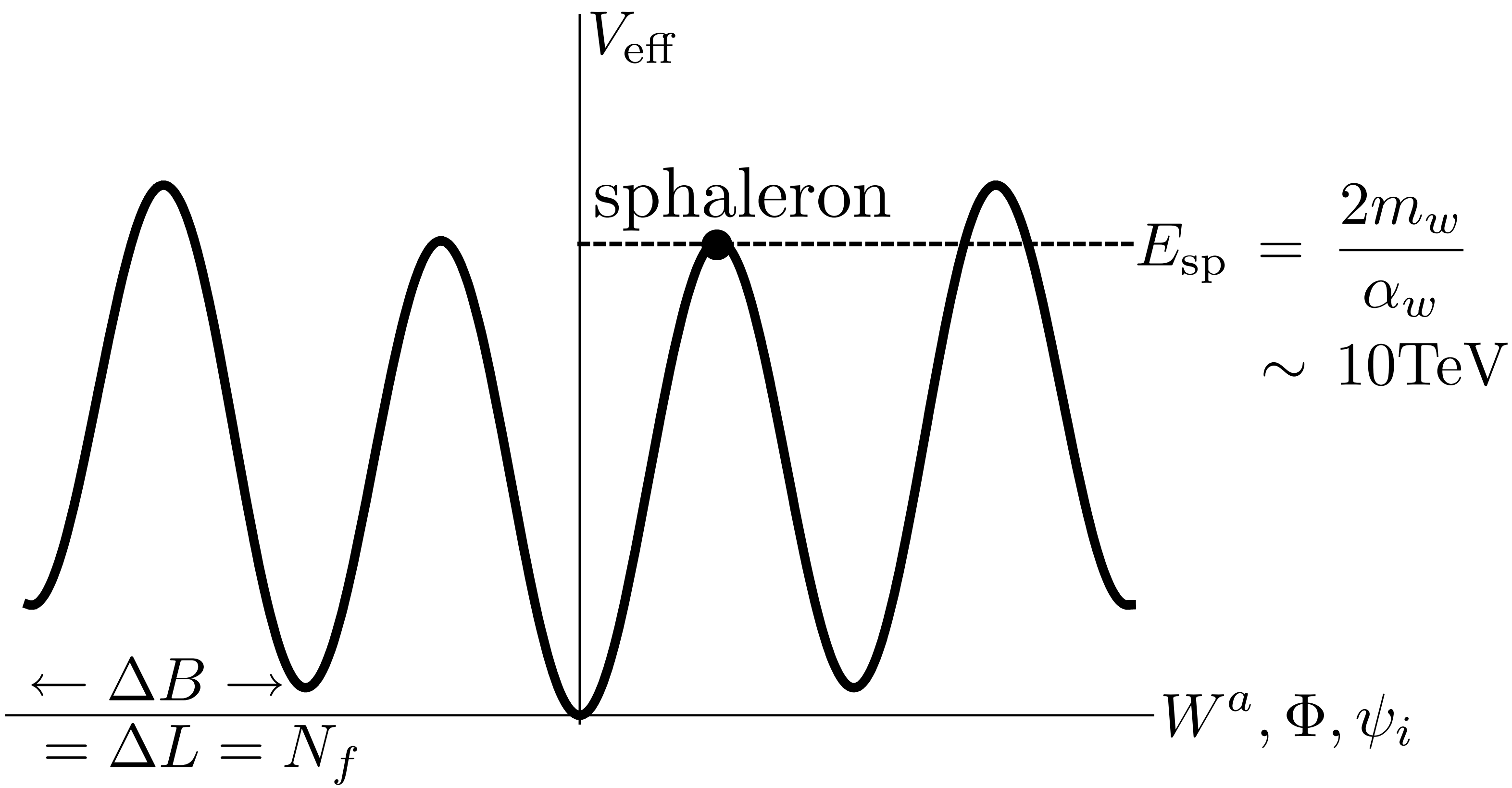
SM: Violation of B

(t'Hooft 1976)

Anomalous violation of B and L:

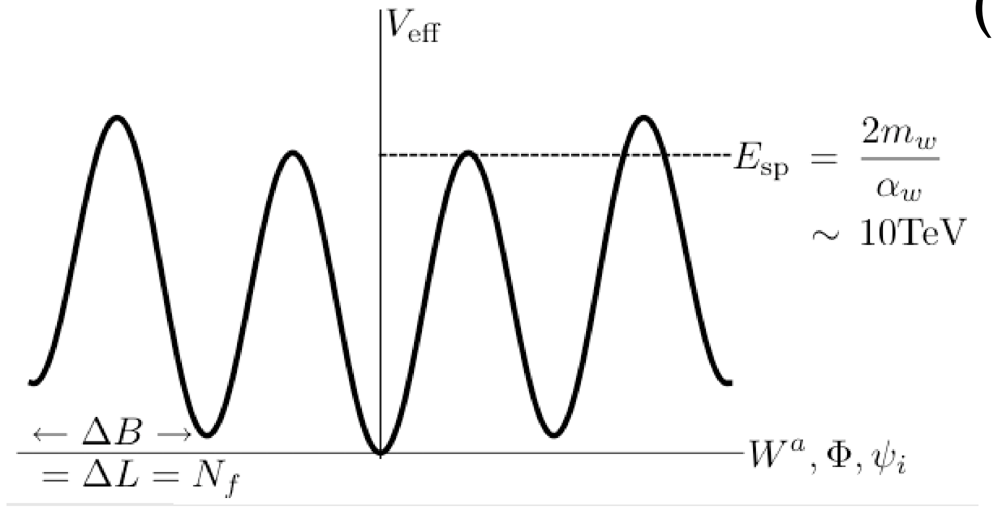
$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L \sim N_f \frac{g^2}{32\pi^2} W\tilde{W}$$

Multiple vacua related by topologically non-trivial gauge transformations.



The Instanton

(t'Hooft 1976)



Conserves $\mathrm{B - L}$

Violates $\mathrm{B + L} : \mathcal{O}_{B+L} \sim \prod_{i=1}^{N_f} u_{Li} u_{Li} d_{Li} e_{Li}$

$$T = 0 : \quad \Gamma/V \sim e^{-2\pi/\alpha_w} \sim 10^{-80}, \quad \tau \gg t_{\rm universe}$$

$$T \neq 0 : \quad \Gamma/V \sim T^4 e^{-E_{\rm sp}(T)/T} \quad \text{broken phase}$$

$$\sim T^4 (\alpha_w T)^4 \quad \text{symmetric phase}$$

SM: Violation of C and CP

- Maximal violation of C under $SU(2)_L$.

- Insufficient CP violation to achieve $\eta \sim 10^{-10}$.

$$\delta \lesssim 10^{-20} \quad \text{from CKM}$$

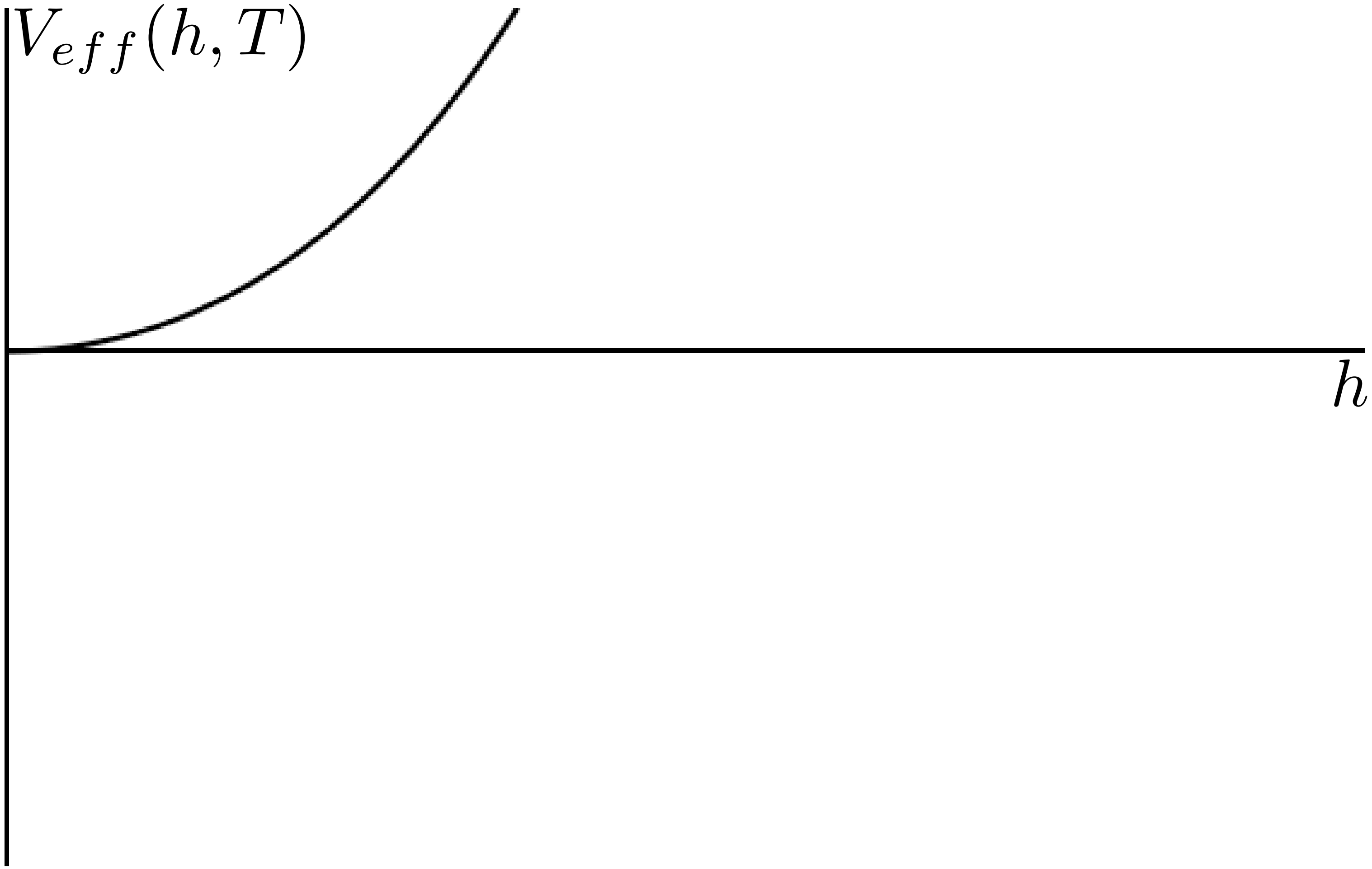
$$\theta \lesssim 10^{-9} \quad \text{from QCD instantons}$$

SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

Second Order Transition:

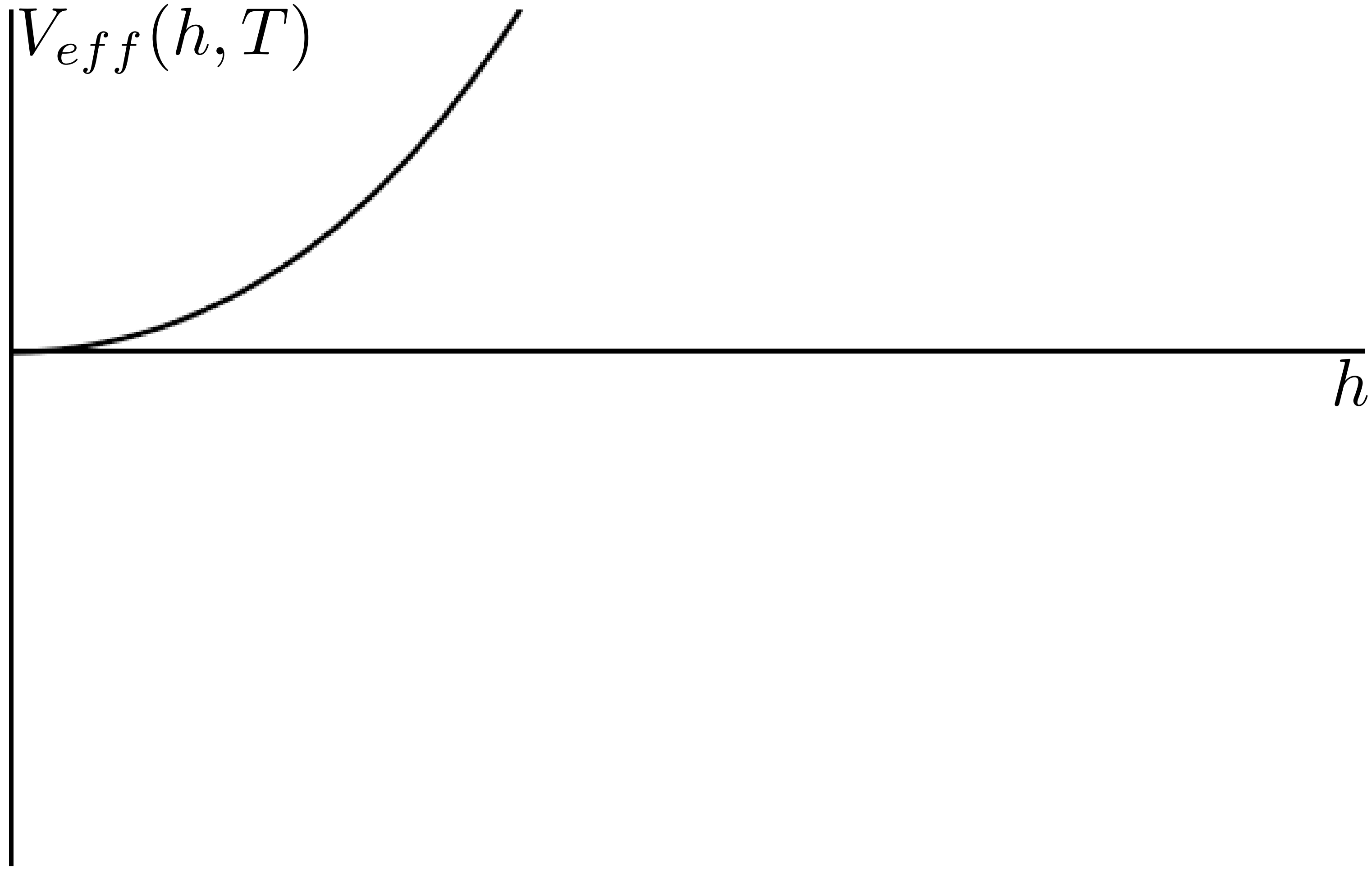


SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

Second Order Transition:

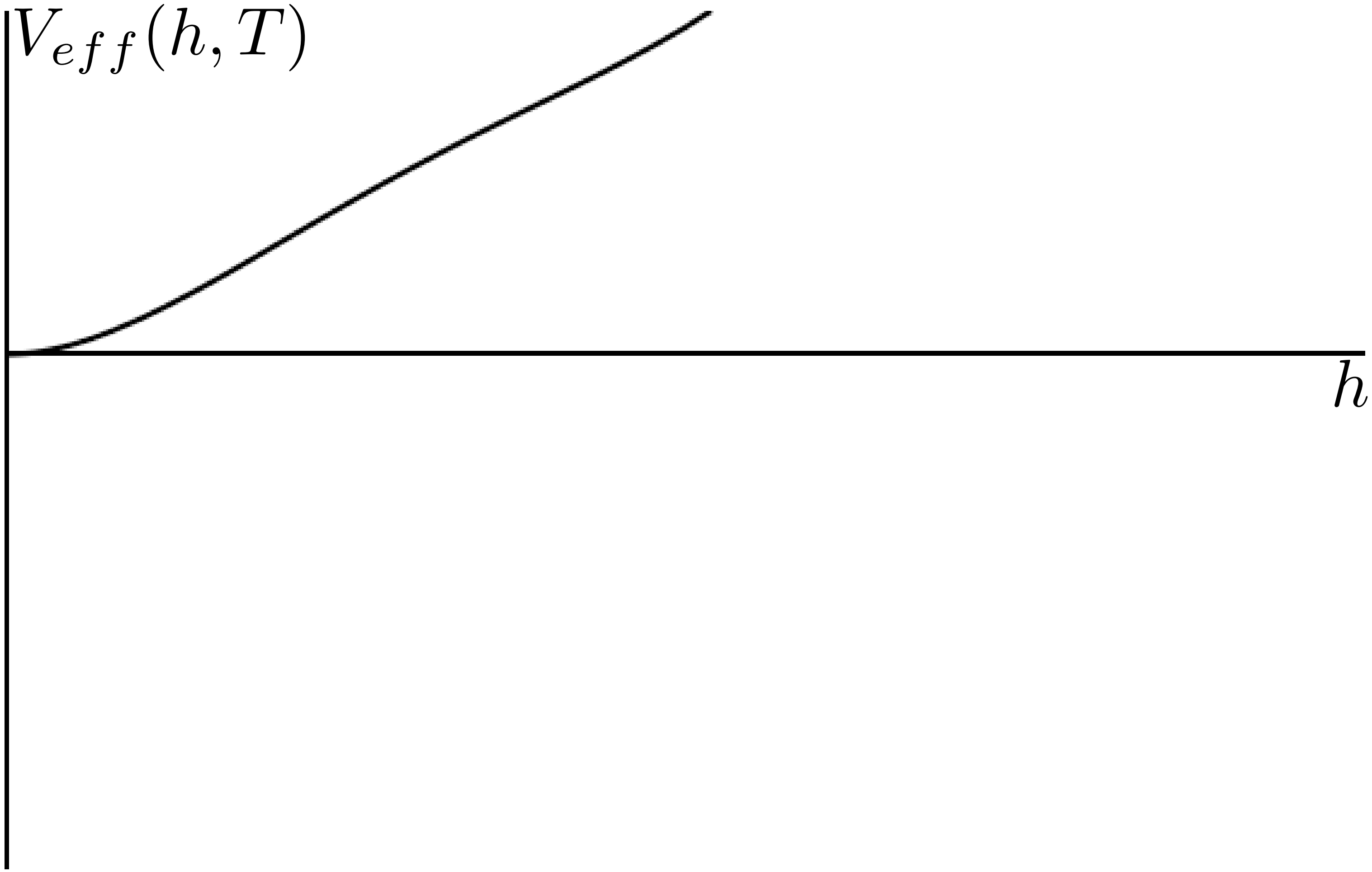


SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

First Order Transition:

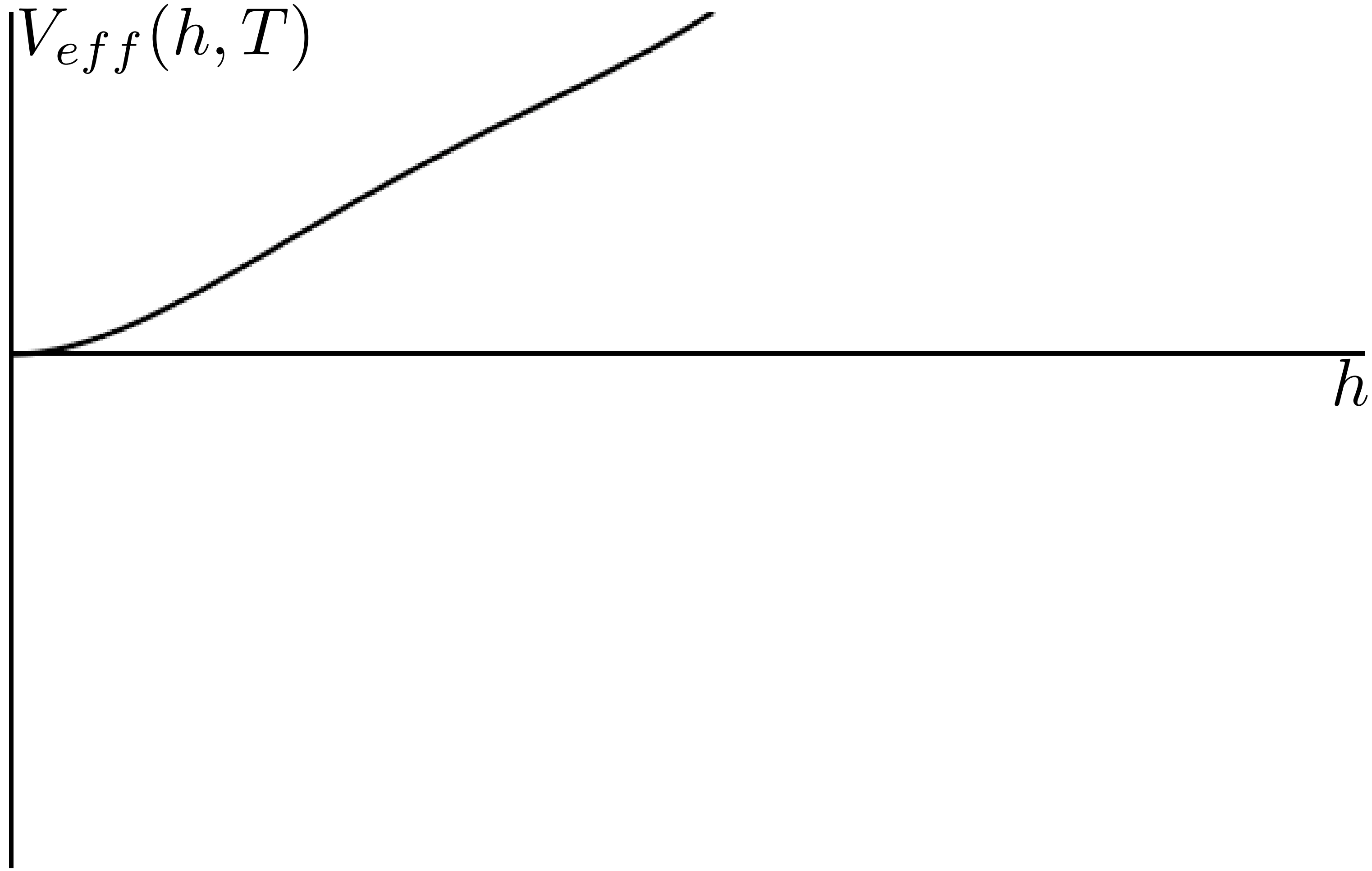


SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

First Order Transition:

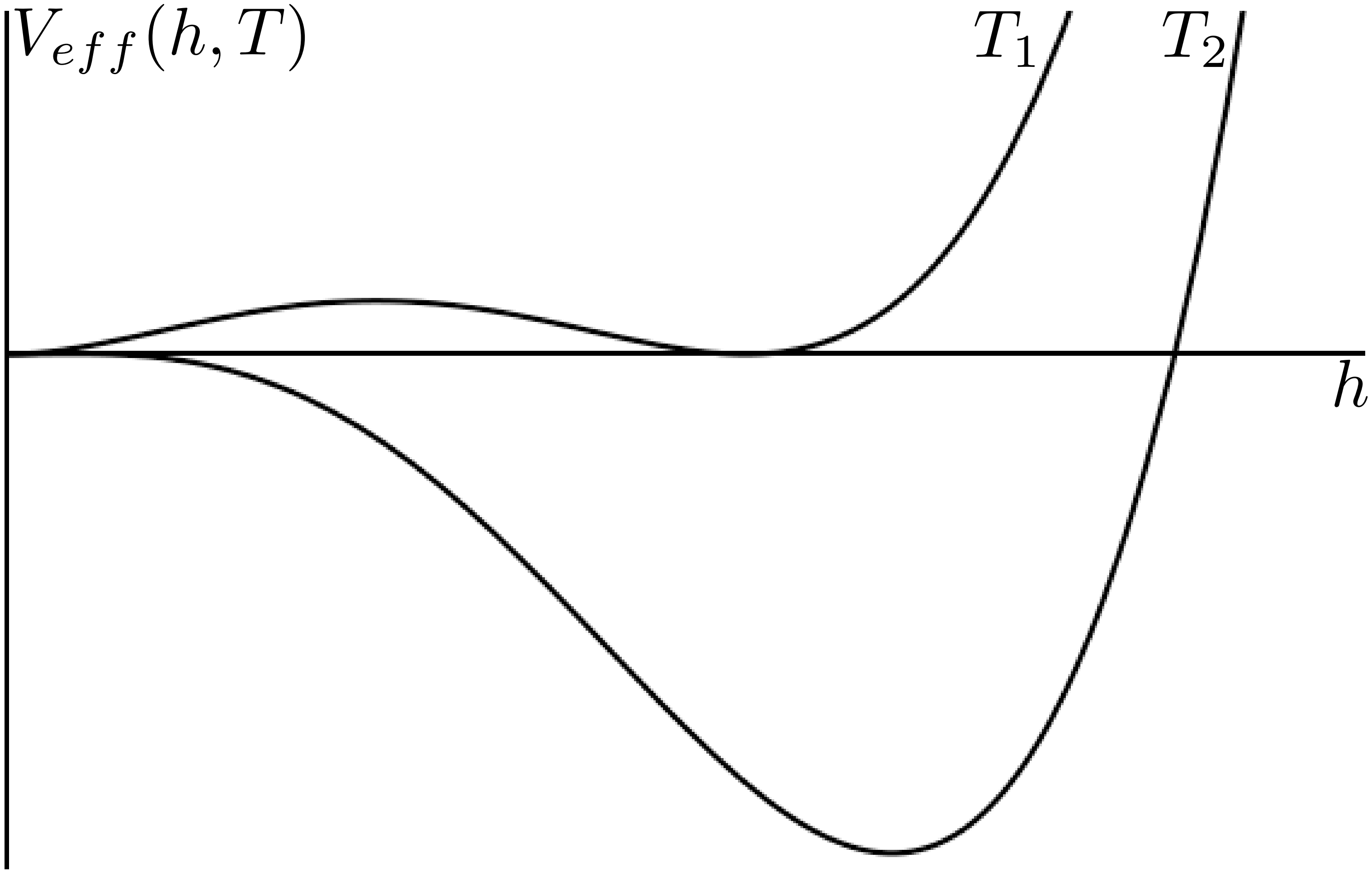


SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

First Order Transition:

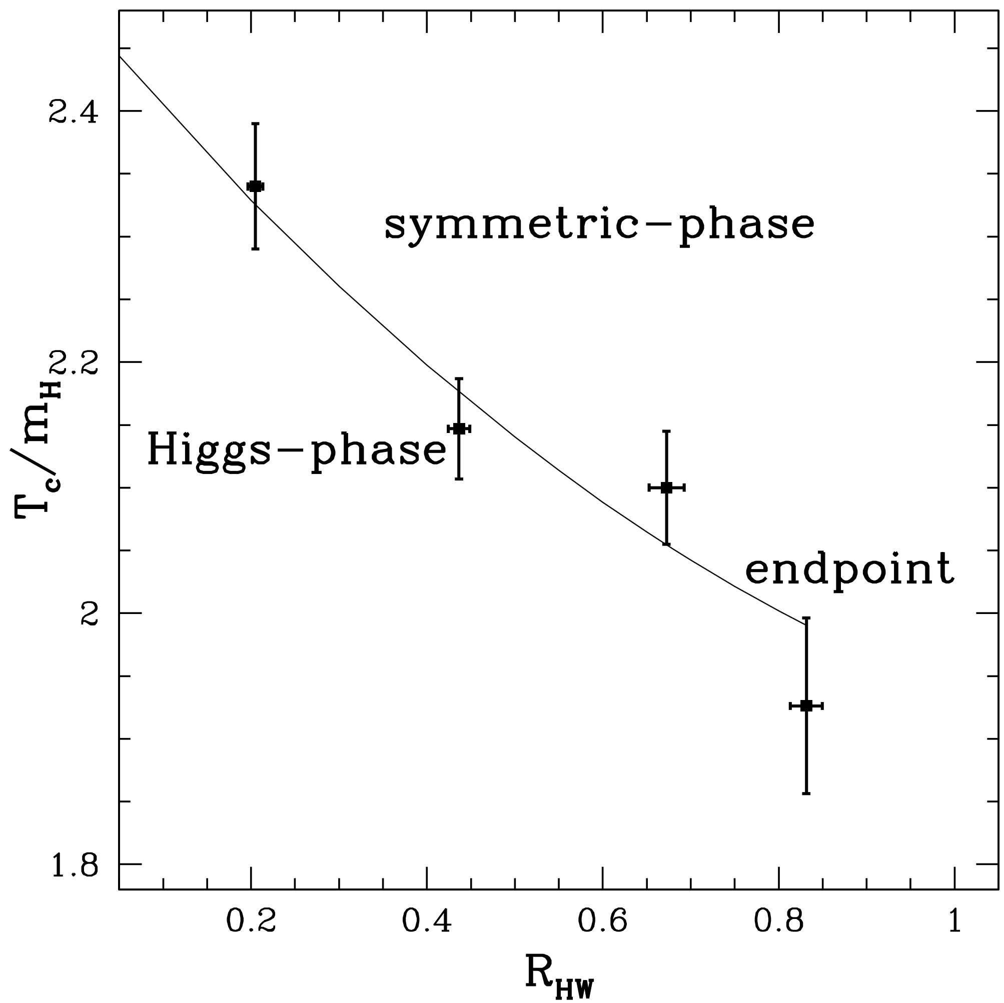


SM: Nonequilibrium Dynamics

One possibility: A First Order Phase Transition (FOPT) in the breaking of the electroweak symmetry,

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

Higgs Phase Diagram:

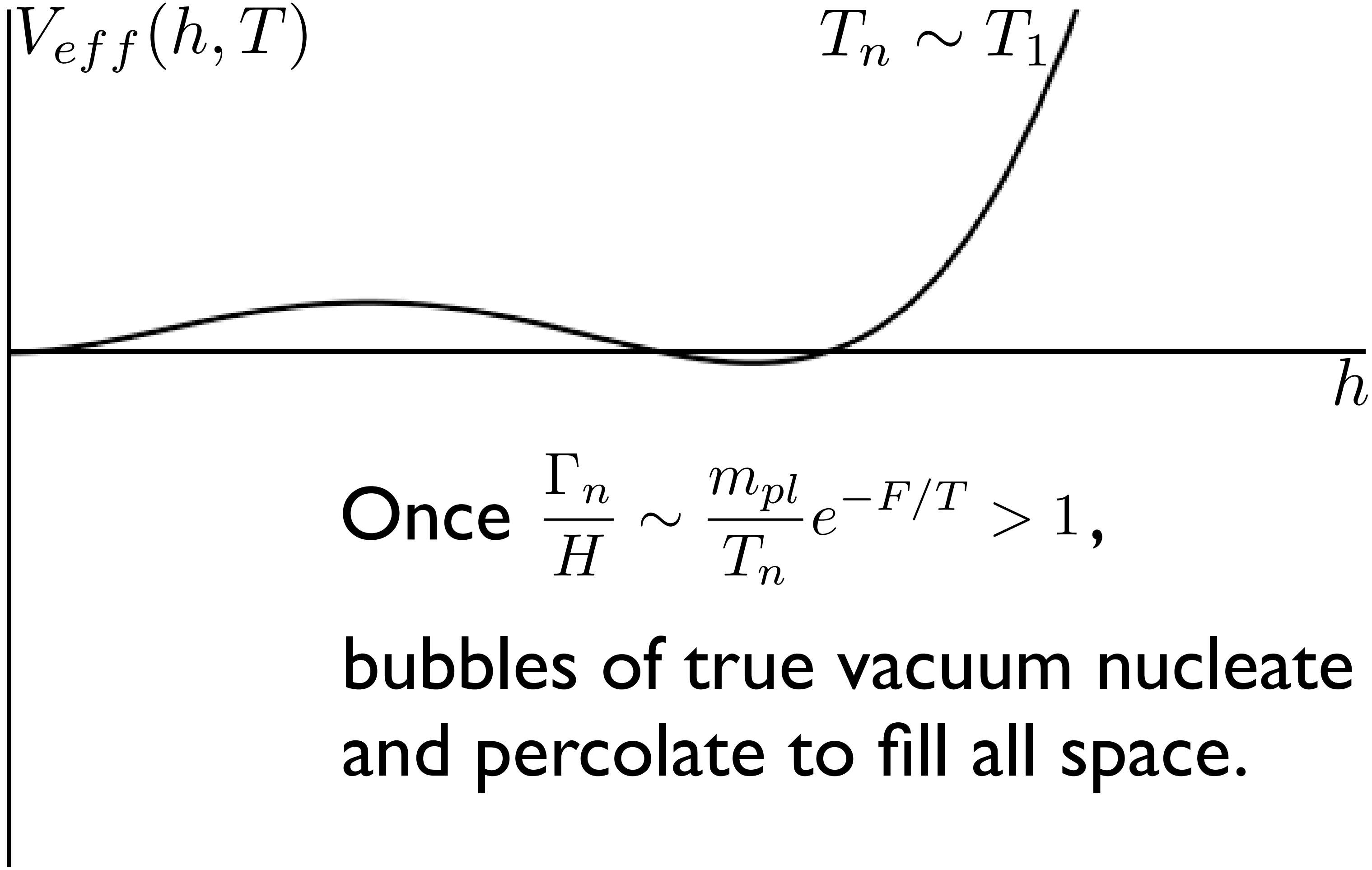


Transition is second order for $m_h > 114\mathrm{GeV}.$

(Csikor, Fodor, Heitker 1999)

Non-local, Thin-Wall EWBG

(Cohen, Kaplan, Nelson 1992)



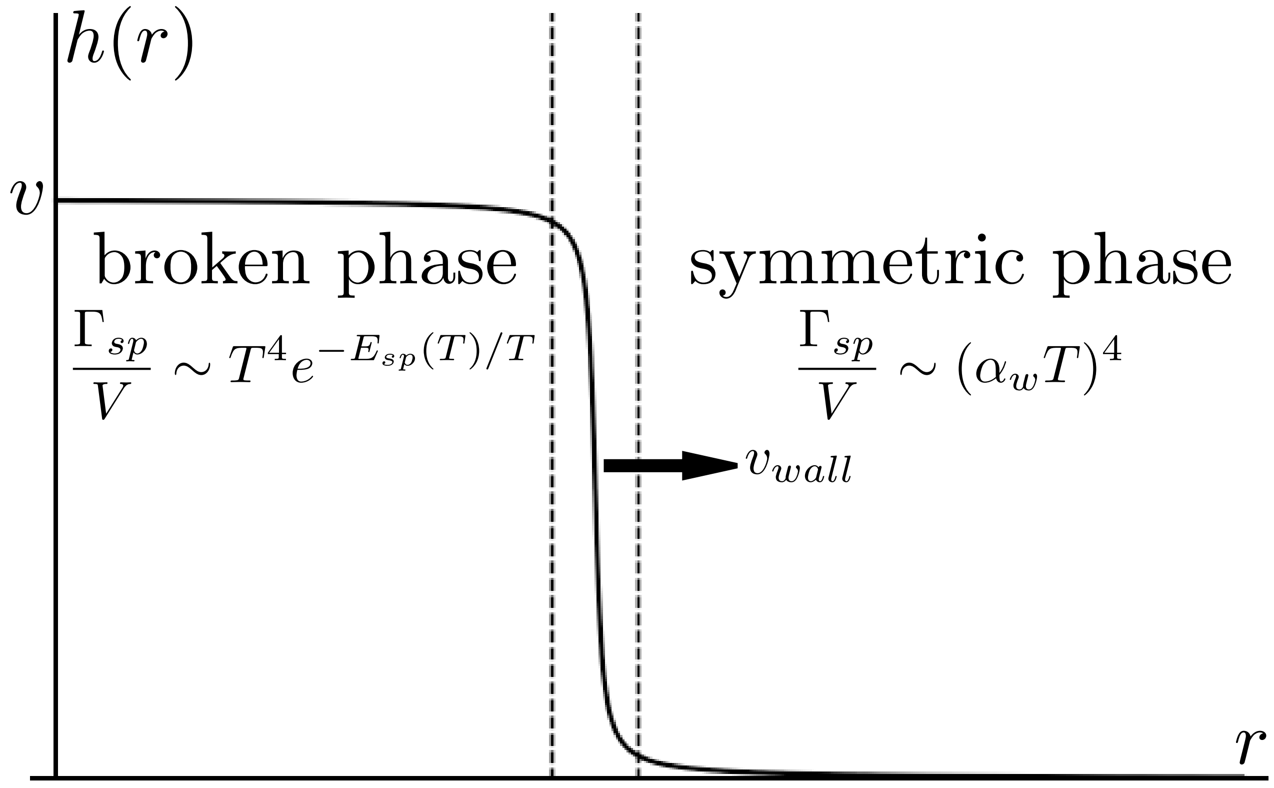
Once $\frac{\Gamma_n}{H} \sim \frac{m_{pl}}{T_n} e^{-F/T} > 1$,

bubbles of true vacuum nucleate and percolate to fill all space.

Non-local, Thin-Wall EWBG

(Cohen, Kaplan, Nelson 1992)

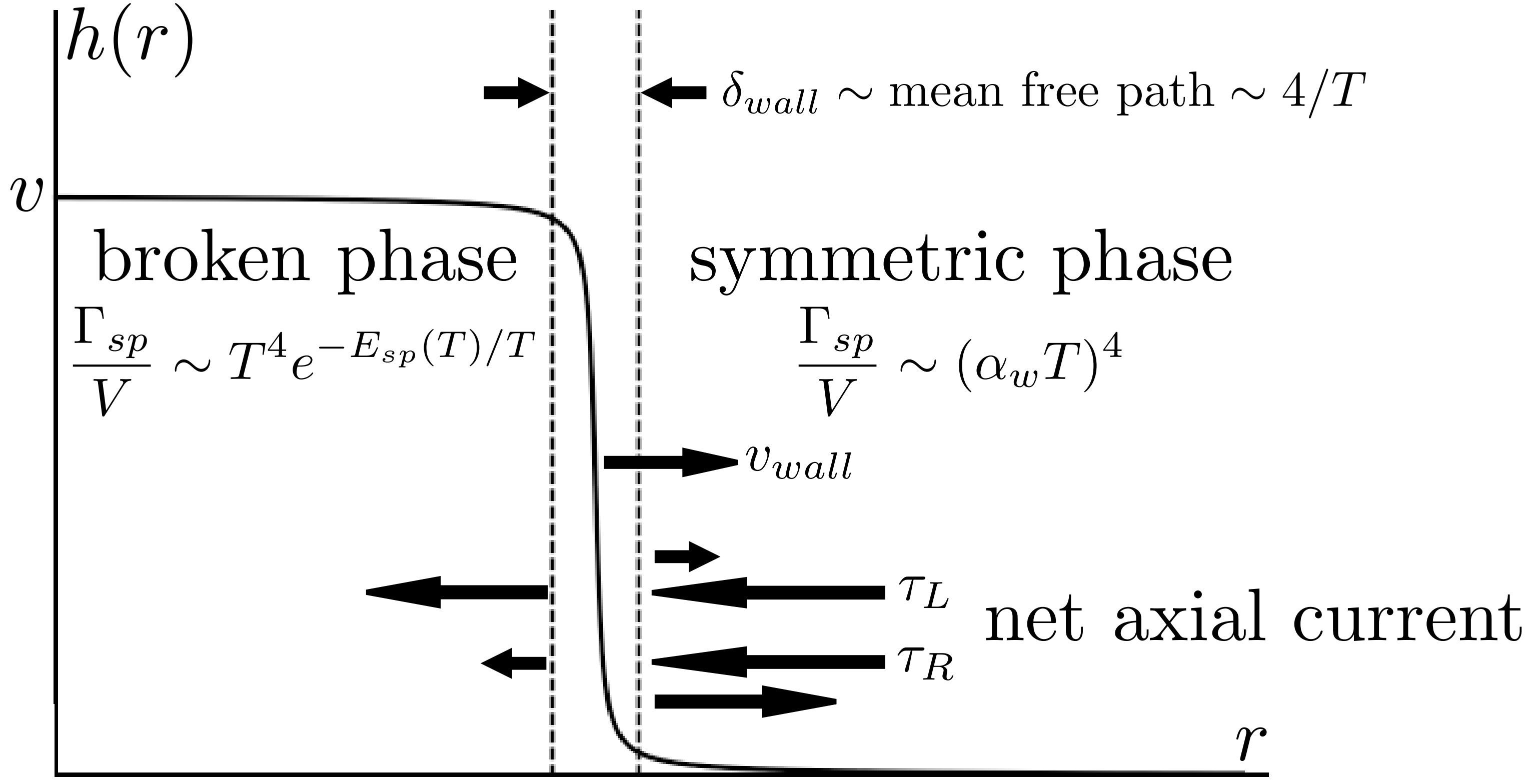
$$\frac{d(n_b - n_{\bar{b}})}{dt} \sim \frac{\Gamma_{sp}}{T}\mu_B$$



Non-local, Thin-Wall EWBG

(Cohen, Kaplan, Nelson 1992)

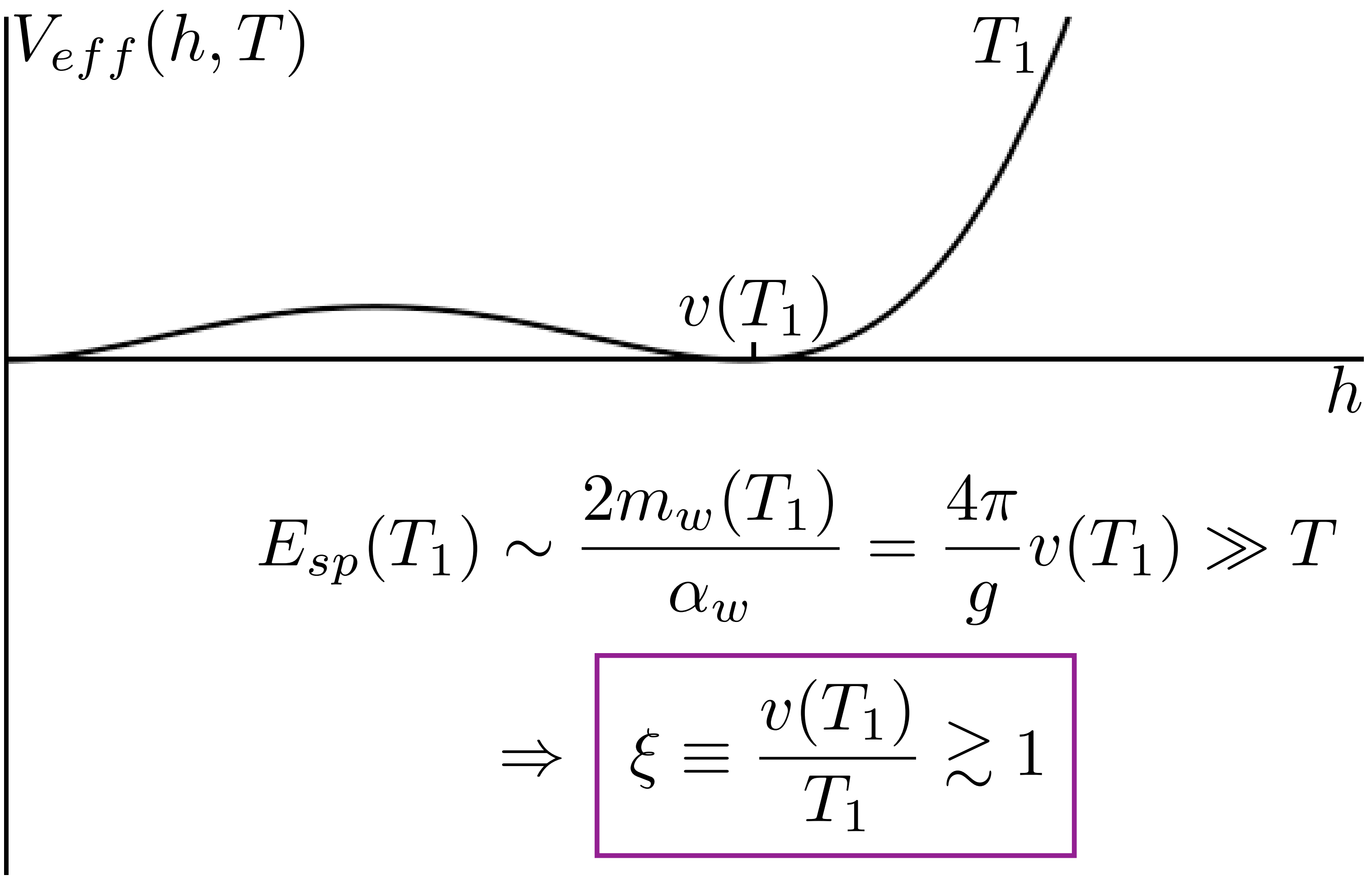
$$\frac{d(n_b - n_{\bar{b}})}{dt} \sim \frac{\Gamma_{sp}}{T}\mu_B$$



A baryon asymmetry is generated in front of the bubble wall then consumed. If $E_{sp}(T) \gg T$, $\Gamma_{sp} \to 0$ inside the bubble, and washout can be avoided.

Non-local, Thin-Wall EWBG

(Cohen, Kaplan, Nelson 1992)



$$E_{sp}(T_1) \sim \frac{2m_w(T_1)}{\alpha_w} = \frac{4\pi}{g} v(T_1) \gg T$$

$$\Rightarrow \boxed{\xi \equiv \frac{v(T_1)}{T_1} \gtrsim 1}$$

MSSM: A Narrow Window

(Carena, Quiros, Wagner 1998)

- Violation of B: Inherited from SM.

- Violation of C: Inherited from SM.
  Violation of CP: $\mathcal{O}(1)$ from gaugino masses, μ , etc.

- Nonequilibrium dynamics:
  For $m_h < 120\mathrm{GeV}$ and $m_{\tilde{t}_R} < m_t$, the phase transition can be first order due to an enhancement in the cubic coupling of the effective potential.

Generic BSM Scenario

- Violation of B: Inherited from SM.
- Violation of C: Inherited from SM.
  Violation of CP: $\mathcal{O}(1)$ a possibility in many models.
- Nonequilibrium dynamics:
  The enlarged parameter space may allow for a first order phase transition.

EWBG Phenomenology

- A precision measurement of the full TeV Lagrangian (masses, couplings, mixings, etc.) would allow us to calculate the viability of various EWBG mechanisms.

- Lacking that, how much can we determine from the least data?
  - New CP violating sectors are highly model dependent and difficult to probe.
  - How about signatures of nonequilibrium dynamics?

    -Astrophysics: Gravitational relics may be accessible to LISA. (Grojean and Servant, 2006)

    -Collider Physics: Search for simple observables correlated to the order of the phase transition.

The Higgs Effective Potential

Zero Temperature

$$Z[j] \equiv \int [\mathcal{D}\phi] \exp\left[i(S[\phi] + j\phi)\right]$$

$$S_{eff}[\phi_{cl}] \equiv -i \log Z[j] - j\phi_{cl}, \quad \text{where} \quad \phi_{cl} \equiv \langle \Omega | \phi(x) | \Omega \rangle_J$$

$$S_{eff}[\phi_{cl}] \equiv \int d^4x \left[-V_{eff}(\phi_{cl}) + A(\phi_{cl})(\partial_\mu \phi_{cl})^2 + \cdots\right]$$

$$\left.\frac{\delta V_{eff}(\phi_{cl})}{\delta \phi_{cl}}\right|_{J=0} = 0$$

From here on, $h \equiv \phi_{cl}$.

Zero Temperature

$$
\begin{aligned}
V_{eff}(h, T=0) &= V^t + V_0^l \\
&= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \sum_i n_i \int \frac{d^4 k_E}{(2\pi)^4} \log\left(k_E^2 + m_i^2(h)\right) \\
&= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \sum_i n_i \frac{m_i^4(h)}{64\pi^2}\left(\log\frac{m_i^2(h)}{\mu^2} + \text{const.}\right)
\end{aligned}
$$

$$\text{where } i \,\epsilon \,\{t, W, Z, h, G, BSM\}$$

$$m_i^2(h) = m_{0i}^2 + a h^2 \text{ in a renormalizable theory}$$

The Goldstones

Problem: $m_G^2(h) \leq 0$ for $h \leq v$.

Solution: Use *on-shell* renormalization conditions.

(Delaunay, Grojean, Wells, 2006)

$$\frac{dV_{eff}(h, T=0)}{dh}\Big|_{h=v} = 0$$

$$\frac{d^2V_{eff}(h, T=0)}{dh^2}\Big|_{h=v} = m_h^2 - \Delta\Sigma$$

$$V_{eff}(h, T=0) = -\frac{m_h^2}{4}h^2 + \frac{m_h^2}{8v^2}h^4$$

$$+\sum_i \frac{n_i}{64\pi^2}\left(m_i^4(h)\left(\log\frac{m_i^2(h)}{m_i^2(v)} - \frac{3}{2}\right) + 2m_i^2(v)m_i^2(h)\right)$$

Finite Temperature

Rotate to Euclidean time: $x^0 = -ix_E^0$

Compactify on a circle: $0 \le x_E^0 < 2\pi R, \text{ where } T \equiv 1/2\pi R$

Require field configurations to be static.

$$Z[j] = \int [\mathcal{D}\phi] \exp\left[-\int d^4x_E \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V_0(\phi) + j\phi\right)\right]$$

$$Z[j] = \int [\mathcal{D}\phi] \exp\left[-\frac{1}{T}\int d^3x \left(\frac{1}{2}\partial_i\phi\partial^i\phi + V_0(\phi) + j\phi\right)\right]$$

$$Z[j=0] = \int [\mathcal{D}\phi]\, e^{-\frac{E[\phi]}{T}} \sim \sum_{S=\text{all states}} e^{-E_S/T}$$

The Perscription

$$\int \frac{dk_0}{2\pi} f(k_0) \to T \sum_{n=-\infty}^{\infty} f(k_0 = -i\omega_n)$$

Statistics on a circle of compactified time:

Bosons are periodic, so $\omega_n = 2n\pi T$.

Fermions are anti-periodic, so $\omega_n = (2n+1)\pi T$.

The Potential

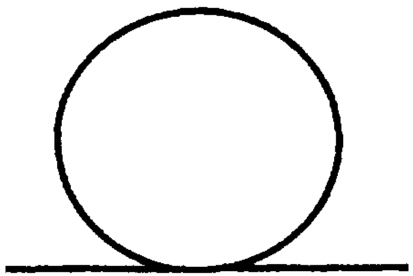
$$V_{eff}(h,T)$$

$$= -\frac{\mu^2}{2}h^2 + \frac{\lambda}{4}h^4 + \sum_i \frac{n_i T}{2} \sum_{n=-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3} \log\left(k^2 + \omega_n^2 + m_i^2(h)\right)$$

$$= V_{eff}(h, T=0) + \sum_i \frac{n_i T}{2\pi} \int dk k^2 \log\left(1 \mp \exp\left(-\frac{1}{T}\sqrt{k^2 + m_i^2(h)}\right)\right)$$

Pheno note:
The zero temperature potential completely determines the finite temperature potential.

Thermal IR Divergences



$$\sim \lambda T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + k^2 + m^2}$$

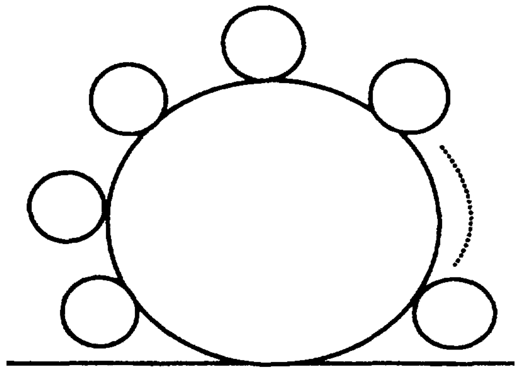
For boson loops, with m<<T,
the integral diverges for n=0, k=0.

Underlying problems:

1. We have a double expansion in both λ and λ T/M,
2. We lose perturbative control in the high-T limit.

Resummation

(Carrington, 1992)



$$\sum_i \frac{n_i T}{12\pi}\left(m_i^3(h) - \left(m_i^2(h) + \Pi_i(T^2)\right)^{3/2}\right)$$

Resumming these "ring" or "daisy" diagrams, the leading two-loop contributions to the effective potential, cancels imaginary, and unphysical, contributions of the Goldstones to the finite temperature potential.

Low-T Expansion: m>>T

$$V_{eff}(h,T) = V_{eff}(h,T=0) + \sum_i n_i T^4 \left(\frac{m_i(h)}{2\pi T}\right)^{3/2} e^{-m_i(h)/T}$$

For a phase transition at T~100GeV,
only weak-scale states will effect the dynamics.

High-T Expansion: m<<T

$$V_{eff}(h,T) \;=\; D(T^2 - T_2^2)h^2 - ETh^3 + \frac{\lambda(T)}{4}h^4$$

$$\xi \equiv \frac{v(T_1)}{T_1} \;=\; \frac{2E}{\lambda(T_1)}$$

If new *scalar* d.o.f. couple to the Higgs such that

$$m_i^2(h) \;=\; m_{0i}^2 + ah^2$$

their contributions to $V_{eff}(h, T \neq 0)$ enhance E, and hence

$\boxed{\xi}$

while their loop contributions to $V_{eff}(h, T = 0)$ enhance

$\boxed{\lambda_3}$

The
Higgs Cubic
And
EWBG

A Proposal for EWBG Pheno

- Phenomenologically interesting BSM physics scenarios replace the ad hoc SM Higgs potential with a realistic mechanism for EWSB.
- This new Higgs physics modifies the shape of $V_{eff}(h,T)$ at the EW phase transition and may allow for a strong first order phase transition, i.e. one where $\xi \gtrsim 1$.
- The same new physics modifies $V_{eff}(h,T=0)$, leading to deviations in λ_3 from its SM value.

Our Claim: Models possessing a strong, first order Electroweak Phase Transition (EWPT) exhibit large (typically 20-100%) deviations of the Higgs cubic coupling from its SM value.

Our Evidence

We demonstrate the correlation between ξ and λ_3 by analyzing a series of toy models that can be matched onto a broad range of realistic BSM Higgs scenarios with weakly coupled physics at the TeV scale.

- Toy Model I:
  Loop Modified, Unmixed Higgs.

- Toy Model II:
  Tree-Level Modified, Unmixed Higgs.

- Toy Model III:
  Tree-Level Modified, Mixed Higgs.

I: Loop Modified, Unmixed h

Add a single BSM real scalar field
(inspired by Little Higgs models).

$$\Delta V_{SM} = \frac{1}{2} M_{0,S}^2 S^2 + a|H|^2 S^2$$

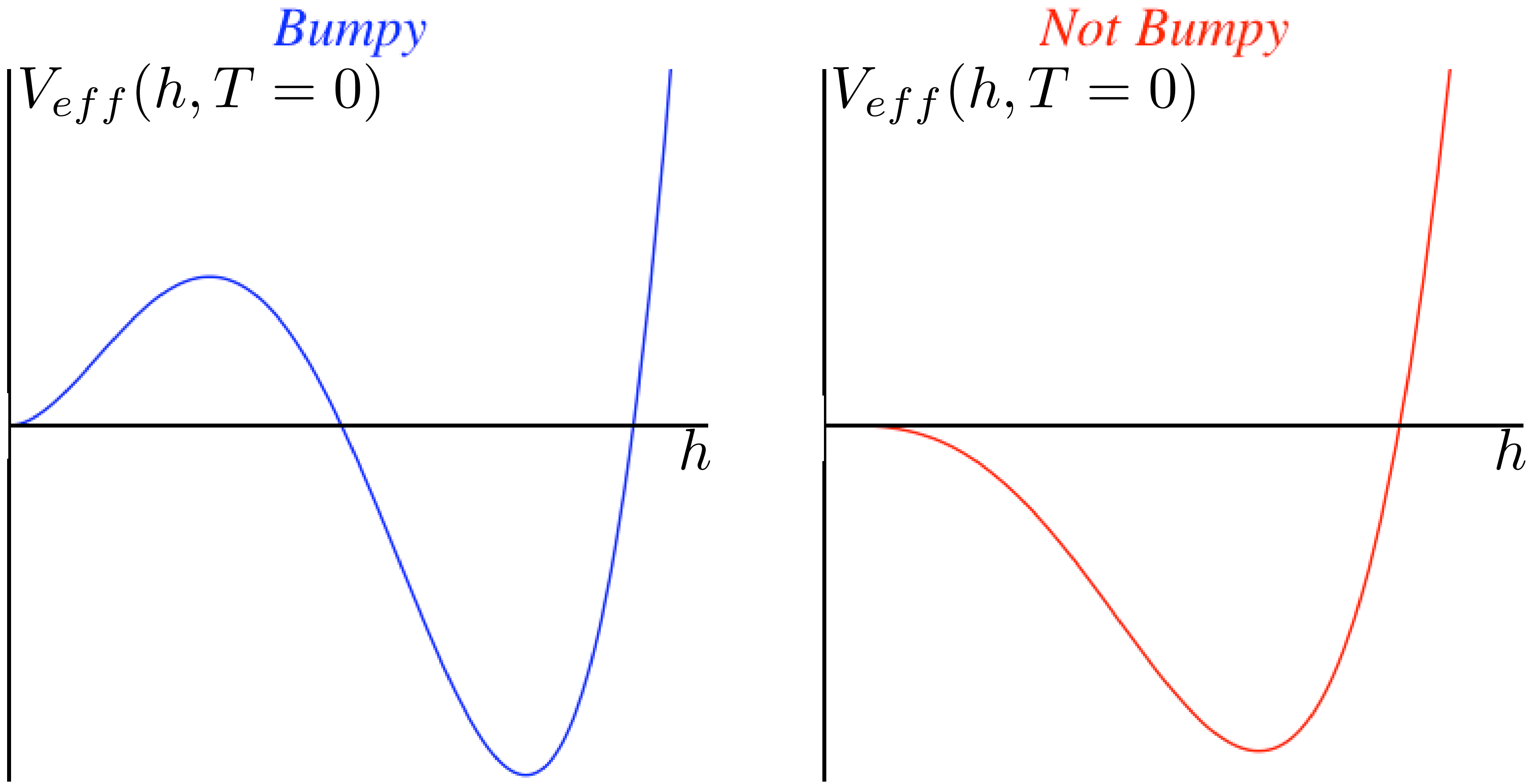
$$\Downarrow$$

$$\Delta V_{eff}(h, T=0) = \frac{1}{64\pi^2} m_S^4(h) \log \frac{m_S^2(h)}{m_S^2(v)} + \cdots$$

- $M_{0,S}^2 > 0$ ensures $\langle S \rangle = 0$.
- Most general interaction after imposing a symmetry $S \to -S$ to prevent mixing.

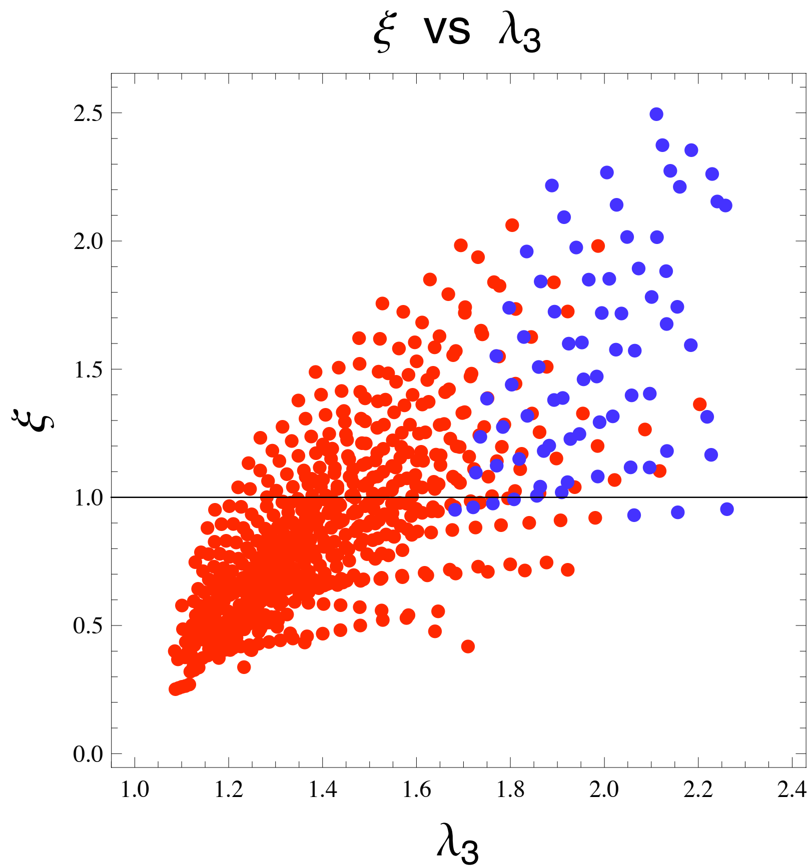
'Bumpy' Higgs Potentials

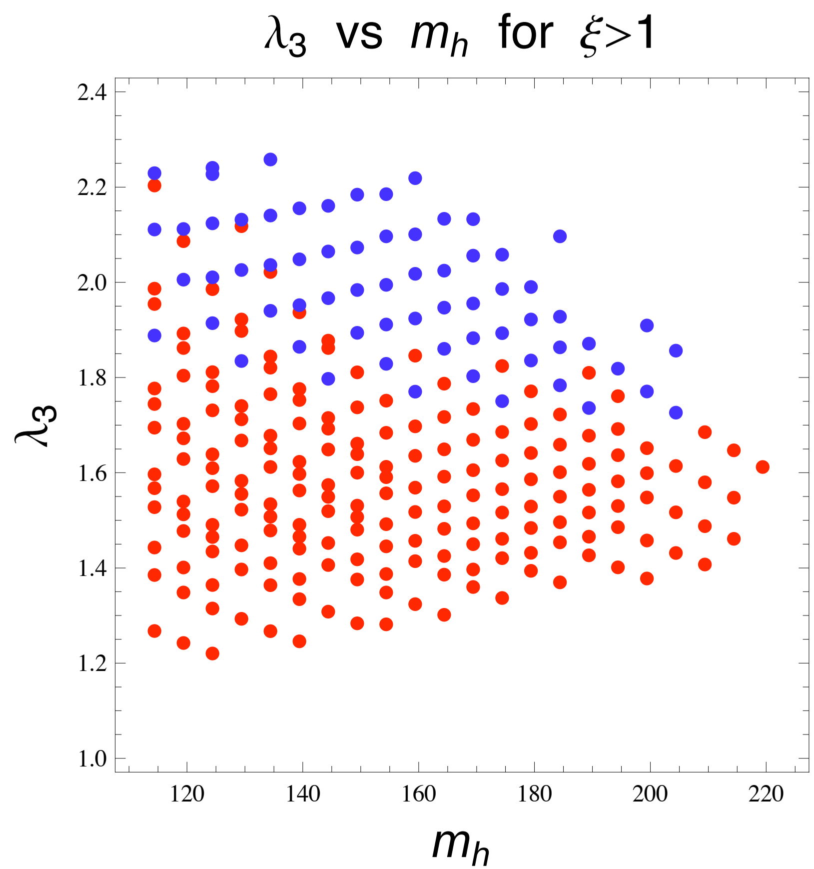
BSM couplings may induce a 'bump' in the zero temperature potential. This bump generally persists at finite temperature, allowing for a strong EWPT.



I: Loop Modified, Unmixed h

Add a single BSM real scalar field.





Blue = Bumpy at T=0

Expt. Prospects:

20% for a <140GeV Higgs at a 500GeV ILC (Djouadi, et. al., 2007)

20-30% for 160-180GeV Higgs at SLHC (Baur, et. al., 2002)

8-25% for 150-200GeV Higgs at 200TeV VLHC (Baur, et. al., 2002)

I: Loop Modified, Unmixed h

Multiple BSM scalars.

- The same conclusions apply to models with N real (or N/2 complex) identical scalars by a simple scaling argument.

- We checked that the pattern continues to hold for 2 non-identical scalars. A conjecture that it holds for N independent scalars seems reasonable.

- The one-loop analysis is independent of the scalars' gauge charges. They could be stops in the MSSM decoupling limit (one unmixed Higgs), weak triplets, etc.

I: Loop Modified, Unmixed h

Add a BSM boson-fermion pair (as in SUSY).
We choose a Dirac fermion and four identical real scalars.

$$\Delta V_{SM} = \sum_i \left(\frac{1}{2} M_{0,S}^2 S_i^2 + a|H|^2 S_i^2\right) + \left(M_{0,\Psi} + \frac{a}{M_{0,\Psi}}|H|^2\right)\Psi^\dagger \Psi$$

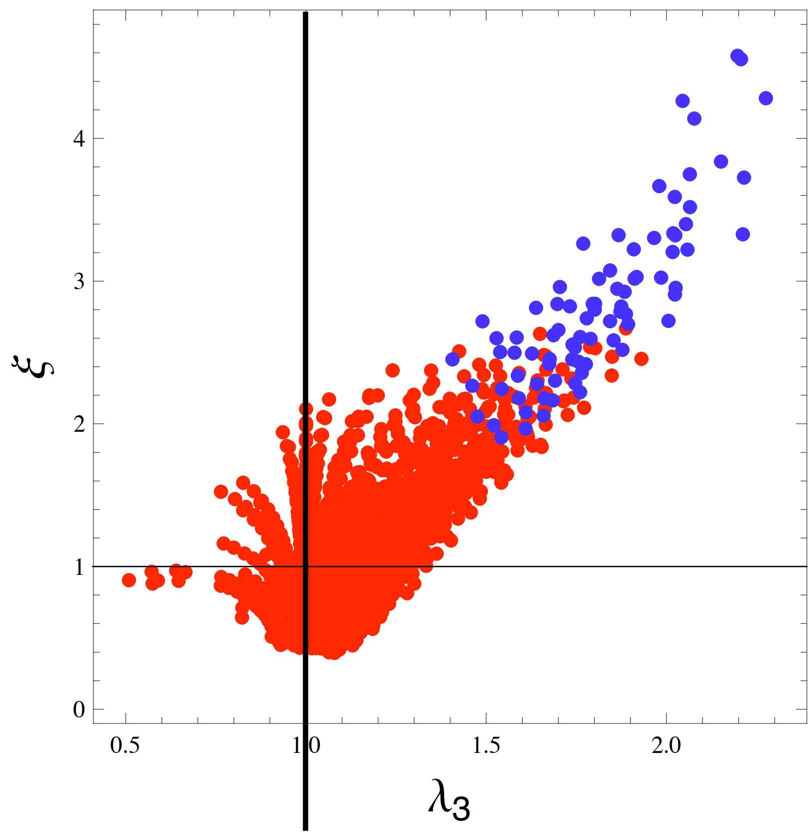
$$\Downarrow$$

$$\Delta V_{eff}(h, T=0) = \sum_i \frac{n_i}{64\pi^2} m_i^4(h) \log \frac{m_i^2(h)}{m_i^2(v)} + \cdots$$

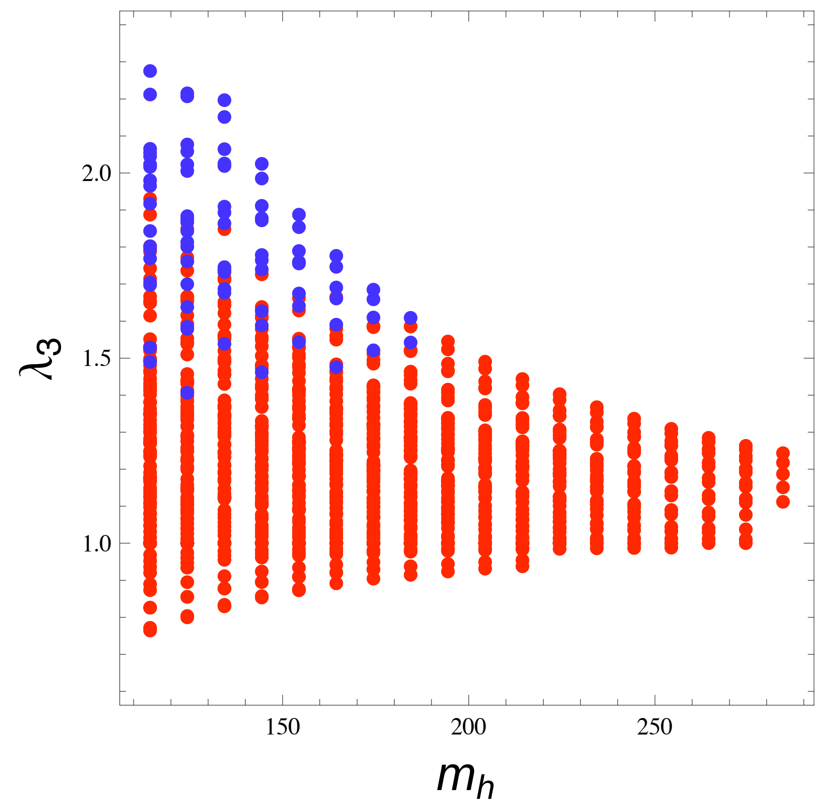
I: Loop Modified, Unmixed h

Add a BSM boson-fermion pair.

ξ vs λ_3



λ_3 vs m_h for $\xi > 1$



Blue = Bumpy at T=0

Accidental cancellations violate our claim!
For $M_{0,S} = M_{0,\Psi}$, the contributions of this supermultiplet to the zero temperature potential vanish, but not so in the finite temperature potential.

II: Tree-Level, Unmixed h

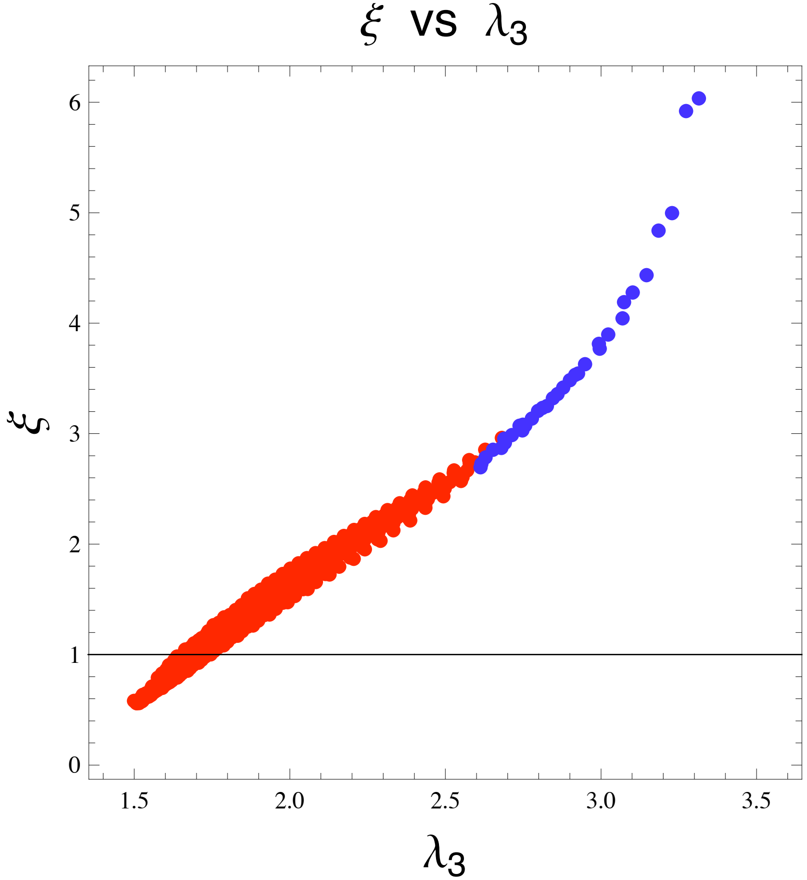
Consider the SM Higgs sector as an EFT and add the leading correction.

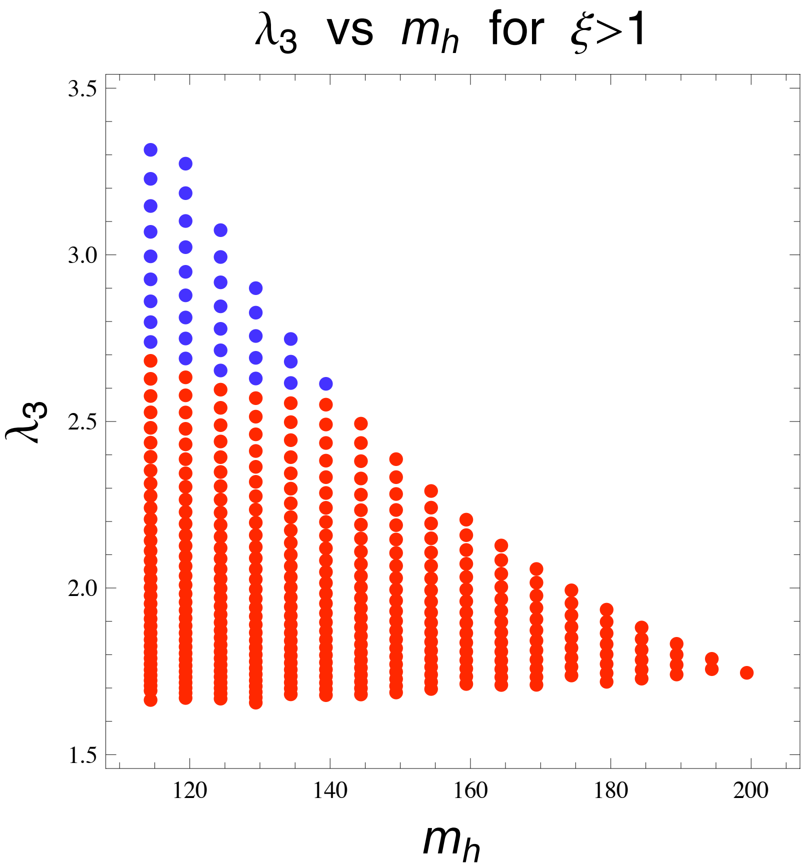
(Grojean, Servant, Wells, 2007)

$$\Delta V_{SM} = \frac{1}{\Lambda^2}|H|^6$$

II: Tree-Level, Unmixed h

Consider the SM Higgs sector as an EFT and add the leading correction.





Blue = Bumpy at T=0

$$\frac{\lambda_3^{\mathrm{GSW}}}{\lambda_3^{\mathrm{SM}}} = 1 + \frac{2v^4}{m_h^2 \Lambda^2}$$

III: Tree-Level, Mixed h

Consider the most general, renormalizable potential with one additional scalar

(as in the NMSSM or nMSSM).

$$\Delta V_{SM} = \frac{a_1}{2}|H|^2 S + \frac{a_2}{2}|H|^2 S^2 + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4$$

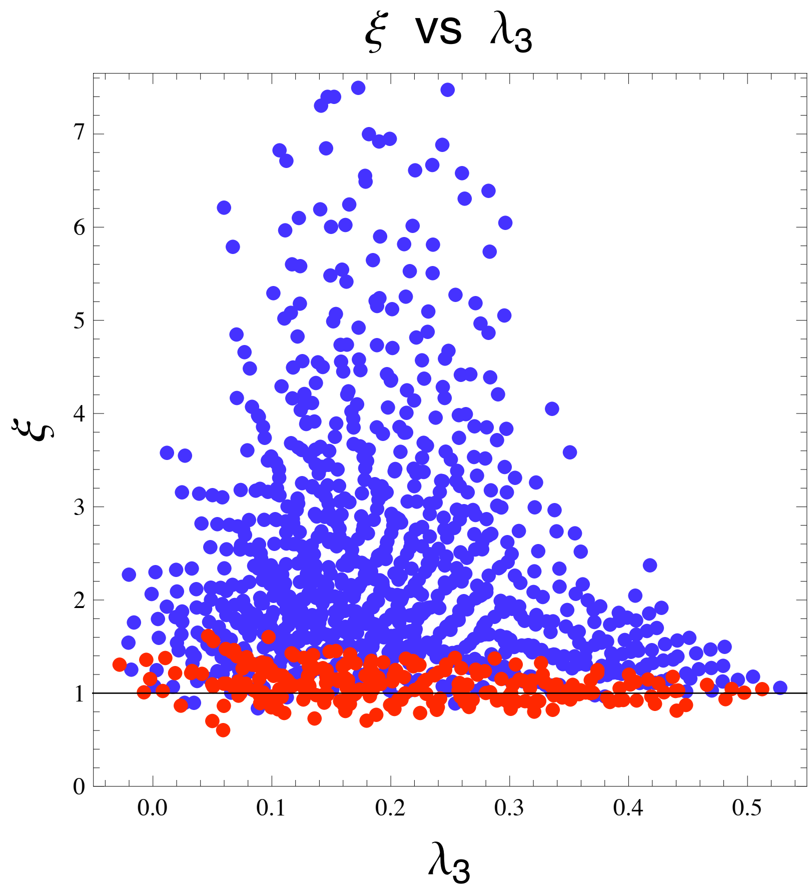
Mass eigenstates:

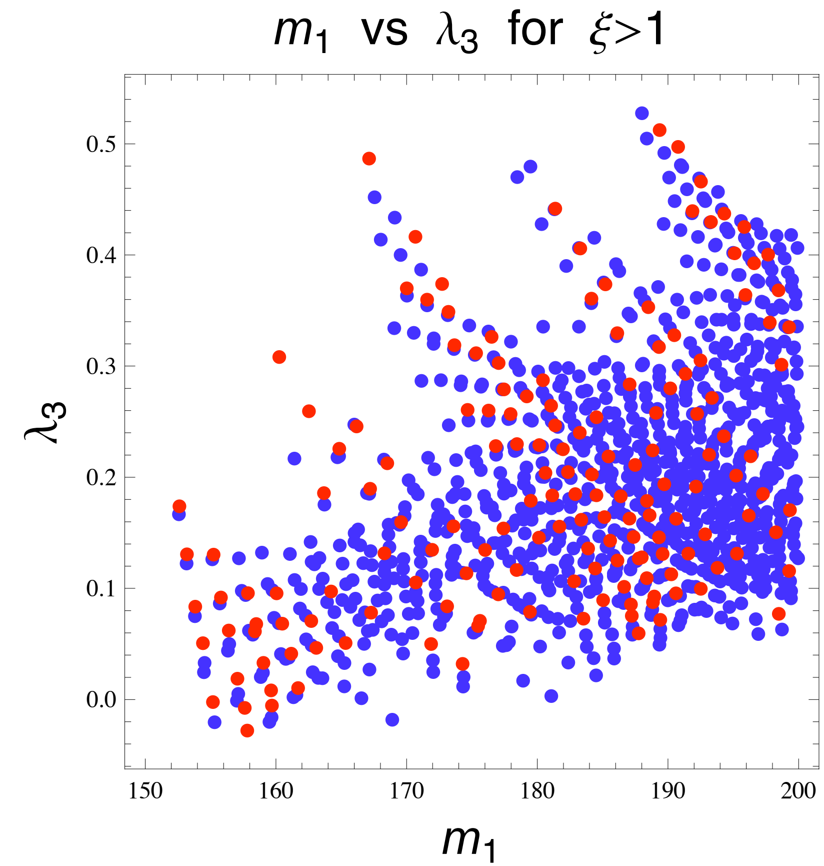
$$h_1 = \sin\theta \; s + \cos\theta \; h$$

$$h_2 = \cos\theta \; s - \sin\theta \; h$$

- Generically, H and S both acquire vevs, so the order parameter for the phase transition is a linear combination of two classical fields.
- Non-SM Yukawas.
- h_1 is the most doublet-like, so we consider its λ_3.

III: Tree-Level, Mixed h

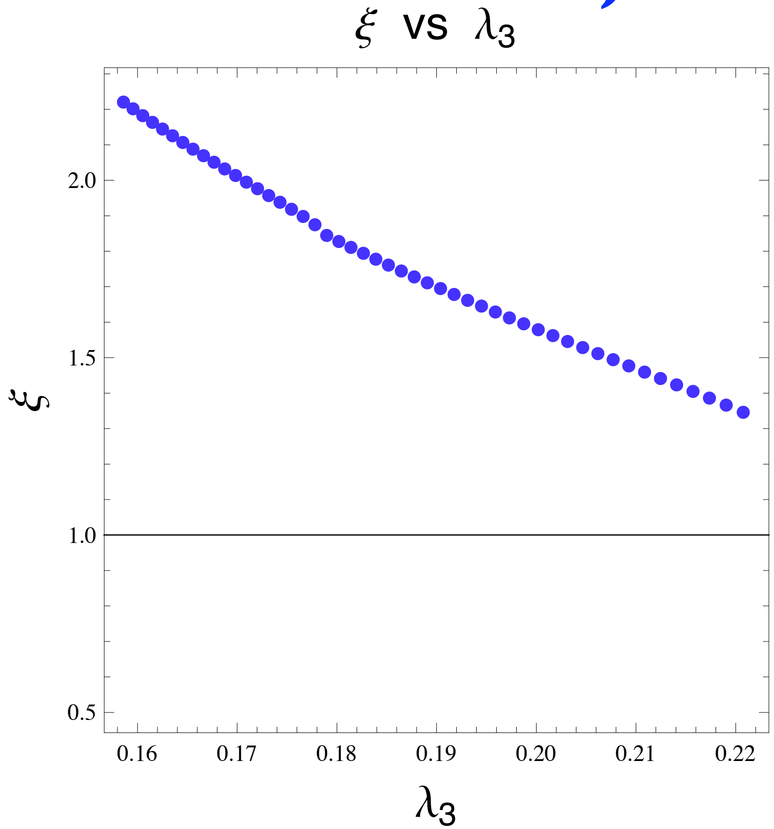




Blue = Bumpy at T=0

- A partial scan of the 6-dimensional parameter space roughly consistent with EW precision constraints.
- Both suppression and enhancement of λ_3 is possible.
- Small λ_3 corrections only occur due to accidental cancellations of two large contributions.

III: Tree-Level, Mixed h



Blue = Bumpy at T=0

- All parameters are fixed except for the mixing coefficient a_1.
- If the Higgs is mixed, deviations from the SM Higgs production x-section and braching ratios would be observed well before λ_3 is measured. Nevertheless, the correlation between ξ and λ_3 persists.

Conclusions

- Barring the possibility of accidental cancellations, there must be a large deviation in λ_3 from its SM value to achieve a strong first order EWPT and make EWBG viable.
- Large deviations in λ_3 are generic to BSM models exhibiting a strong EWPT.
- Typical deviations are large enough to be probed at the ILC and SLHC/VLHC.
- Future work: For specific models, could the order of the EWPT be determined from a small number of quantities measured to an accessible level of precision?