X-ray Flux, Brilliance and Coherence of the Proposed Cornell Energy-recovery Synchrotron Source

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This document is a summary of the calculated properties of the proposed energy-recovery linac (ERL) source at Cornell. It is based on the exchanges during the ERL Science Workshop in December 2000 and on subsequent discussions with Don Bilderback, Ivan Bazarov, and Ken Finkelstein at CHESS. All calculations are performed using analytical formulae from the literature, mostly from Kwang-Je Kim's chapter in *X-ray Data Booklet* and from his article in *AIP Conference Proceedings* 189, 565-632 (1989). It is not intended to derive all the equations in this paper but rather to show how a desired quantity is calculated and how certain concepts and properties are inter-related. The main discussion is geared towards photon coherence and degeneracy, and how these compare with other sources.

Note: This is memo 01-001 on February 2, 2001, revised by the addition of Figure7.

Introduction

All modern synchrotron radiation sources, including the proposed ERL and FEL sources, are based on x-ray undulators. An undulator x-ray source can be characterized by the following three categories of parameters:

(1) *Single-electron based (or integrated) properties*: total flux F_n is the fundamental quantity in this category. It describes the total number of photons per second of the *n*th harmonic in a given energy bandwidth (usually in unit of 0.1%). Another quantity is brightness B_r, defined as the flux per unit area (intensity) or flux per solid angle. It basically describes how flux F_n is distributed in space. Both F_n and brightness B_r are only functions of machine energy E_G, undulator period l_u and length L, deflection parameter K which is determined by peak magnetic field, and electron beam current I.

(2) *Emittance based properties*: brilliance B is the fundamental quantity in this category. It is defined as the photon flux per unit transverse phase-space area, which is called the source emittance. The best physical meaning or physically measurable quantity related to the brilliance is the transversely coherent photon flux F_c. In order to calculate these quantities, one needs to know the electron beam size and divergence in both x and y directions. Brightness B_r may also include the effect from the electron beam divergences.

(3) *Single-pulse based properties*: the basic quantity in this category is the peak brilliance $\hat{B}$, which is defined as the photon flux per unit transverse *and* longitudinal phase-space volume. It describes the brilliance for a single pulse, and for this reason brilliance B is often called the average brilliance. A closely related physical quantity is the number of photons in the three-dimensional coherent volume, which is called the degeneracy parameter d_D. These quantities require the additional knowledge of electron bunch length (pulse length) t and the average duty frequency f (number of bunches per second) which can be calculated as the number of bunches in a machine lattice divided by revolution time.

Another way of cataloging various parameters is to realize that the above categories respectively describe the integral, the transverse, and the longitudinal properties of a pulsed x-ray source. In fact, one can make use of the pulsed nature of a source independently from the transverse properties. Therefore for any time-averaged photon parameter A, there can be a corresponding per-pulse parameter A_p given by A/f and a peak parameter $\hat{A}$ given by A_p/t or $A/(t\,f)$. The table below summarizes the most commonly used quantities:

	Average property A	Per-pulse property A/f	Peak property $A/(t\,f)$
Basic parameter	Average brilliance B	–	Peak brilliance $\hat{B}$
Angle-integrated flux	Average flux F_n	Photons per pulse F_p	Peak flux $\hat{F}_n$
Transverse coherence	Average coherent flux F_c	Coherent photons / pulse	Peak coherent flux $\hat{F}_c$
Photon degeneracy	Average degenerate flux	Degenerate parameter d_D	Peak degenerate flux

Furthermore, one can obtain the corresponding power or energy parameter by multiplying each flux or photon quantity in the above table by the photon energy. For example, multiplying photon energy to the row on 'transverse coherence' would yield average coherent power, coherent energy per pulse, and peak coherent power, respectively.

Flux, Brightness, and Power

For an undulator of $N_u = L/l_u$ periods, the angle integrated flux F_n of its nth harmonic is given by [Kim, 1986]:

$$F_n \text{ [ph/s/0.1\%]} = 1.431\text{x}10^{14} N_u Q_n I\text{[A]}, \tag{1}$$

where I is electron beam current in Amperes, and Q_n is a function of K, the deflection parameter, which is given by the peak magnetic field B_0 and period l_u of the undulator:

$$K = 0.934\, l_u[\text{cm}] B_0[\text{T}]\,. \tag{2}$$

For the fundamental peak $n = 1$, $Q_n(K)$ ranges from 0 at $K = 0$ to ~0.5 at $K = 1$ and approaches unity for $K > 4$. Deflection parameter K can be tuned by opening up the gap of an undulator thus changing its peak magnetic field B_0. The tuning range, e.g. $0.2<K<2$, provides a corresponding energy range for all undulator harmonics $E_n = nE_1$ in its x-ray spectrum, with the fundamental E_1 given by

$$E_1[\text{keV}] = \frac{0.95 \cdot E_G^2[\text{GeV}]}{(1+K^2/2)\, l_u[\text{cm}]}\,. \tag{3}$$

The bandwidth at the nth harmonic is given by

$$\frac{\Delta E}{E} = \frac{\Delta l}{l} = \frac{1}{nN_u}\,. \tag{4}$$

The central cone of the nth harmonic has an angular rms width of

$$s_{r'} = \sqrt{\frac{l}{2L}}\,, \tag{5}$$

where L is the length of the undulator [Kim, 1989]. With $s_{r'}$ the on-axis spectral brightness B_r in units of ph/s/0.1%/mr^2 is given by

$$B_r = \frac{F_n}{2p s_{r'}^2}\,.$$

With the electron beam divergences $s_{x'}$ and $s_{y'}$ taken into account, B_r is given by

$$B_r = \frac{F_n}{2p\sqrt{s_{r'}^2 + s_{x'}^2}\sqrt{s_{r'}^2 + s_{y'}^2}}\,. \tag{6}$$

Integrating photon flux spectrum over all energies, we obtain the total power P_0 emitted by an undulator source:

$$P_0[\text{kW}] = 0.633 E_G^2[\text{GeV}] B_0^2[\text{T}] L[\text{m}] I[\text{A}]\,.$$

Similarly, integrating on-axis brightness over energy yields the on-axis power density (power per solid angle):

$$\left.\frac{dP}{d\Omega}\right|_0 [\text{W/mr}^2] = 10.84 B_0[\text{T}] E_G^4[\text{GeV}] I[\text{A}] N_u G(K) ,$$

where $G(K) \sim 1$ for $K > 0.8$, and the electron source divergences have been ignored.

Transverse and Longitudinal Emittances

Source emittance usually refers to the transverse emittance of the electron beam in an accelerator or a storage ring. The emittance is defined in each transverse direction as the product of the rms source size times the rms source divergence: $e_x = s_x s_{x'}$ and $e_y = s_y s_{y'}$. While the emittance remains the same at any point in the machine, source size and divergence do change at different locations and are determined by b-tron oscillations:

$$s_x = \sqrt{e_x b_x}\ , \quad s_{x'} = \sqrt{e_x / b_x} \quad \text{and} \quad s_y = \sqrt{e_y b_y}\ , \quad s_{y'} = \sqrt{e_y / b_y}\ .$$

By choosing a proper b for a given undulator location, one can trade source size with divergence and vice versa within the constraint of a fixed machine emittance.

For a hard-x-ray undulator of a substantial length, it is possible that the angular central cone $s_{r'}$ of the radiation is so narrow that an 'apparent' source size s_r has to be taken into account, due to the uncertainty principle or the diffraction limit of $s_r s_{r'} = l/(4p)$:

$$s_r = \frac{1}{4p}\sqrt{2\,lL}\ . \tag{7}$$

These so-called radiative terms are often added to the true electron beam emittance $e_x = s_x s_{x'}$ and $e_y = s_y s_{y'}$ to form total emittance of the photon source:

$$\bar{e}_x = \sqrt{s_r^2 + s_x^2}\sqrt{s_{r'}^2 + s_{x'}^2} \quad \text{and} \quad \bar{e}_y = \sqrt{s_r^2 + s_y^2}\sqrt{s_{r'}^2 + s_{y'}^2}\ . \tag{8}$$

One should note that all emittances assume a Gaussian distribution in both source size and source divergence. It is helpful to review some basics about a Gaussian curve. In particular, the full width at half maximum (FWHM) of a Gaussian is $2.35s$, and the area under the Gaussian curve is $\sqrt{2p}\,s$. From an experiment point of view, it is often easier to think in terms of FWHM, which implies that the total area is $1*2.35s = 2.35s$, giving rise to an error of $\sqrt{2p}/2.35 = 1.07$. One way to remove this error is to replace all FWHM with $\sqrt{2p}\,s$ instead of $2.35s$ whenever phase space areas are calculated, which is what we will do in the following sections.

Finally, in addition to the commonly mentioned transverse emittances, one can also speak of a longitudinal emittance of a pulsed electron beam. The longitudinal emittance is defined by the product of rms electron energy spread with the rms electron bunch length, which is usually given in units of keV-mm for a multi-GeV machine. Similarly, a longitudinal phase space area or longitudinal emittance e_E of the photon beam is defined by the product of the rms bandwidth $s_E/E = (DE/E)/2.35$ with the rms pulse length $s_t = t/2.35$:

$$e_E = s_E \cdot s_t\ . \tag{9}$$

It should be noted that the undulator bandwidth, Eq.(4), is affected by the energy spread of electrons. One of the machine requirements is to have the electron energy spread s_g/g less than half the bandwidth DE/E of the undulator fundamental [Kim, 1992]. This requirement, together with the longitudinal emittance of the electrons, determines the *lower limit* of the rms electron and photon pulse length s_t. For example, a 1000-period undulator would require $s_g/g \sim 0.05\%$, which is 2.65 MeV for a 5.3 GeV machine. Thus for a longitudinal emittance of 32 keV-mm (a rough number suggested for the ERL), we obtain a minimum rms pulse length s_t ~ 32/2650 = 12 μm or 40 fs, which is t ~ 94 fs FWHM.

Brilliance and Peak Brilliance

The average on-axis spectral brilliance *B* of the *n*th harmonic of an undulator source is defined as the average photon flux per unit transverse phase-space volume:

$$B = \frac{F_n}{(2p)^2 \bar{e}_x \bar{e}_y}, \tag{10}$$

which is usually given in units of ph/s/0.1%/mm^2/mr^2. The word *spectral* refers to the fact that this is quantity for a given energy bandwidth.

Peak brilliance $\hat{B}$ is defined as the number of photons per pulse, $F_p = F_n / f$, per unit volume in the six-dimensional phase space that includes both transverse and longitudinal directions:

$$\hat{B} = \frac{F_p}{(2p)^3 \bar{e}_x \bar{e}_y e_E}. \tag{11}$$

Plugging in $F_p = F_n / f$ and using equations (9) and (10), it can be shown that the peak brilliance $\hat{B}$ is related to the average brilliance *B* as follows:

$$\hat{B} = \frac{(F_n / f) \cdot 2.35 \times 2.35}{(2p)^2 \bar{e}_x \bar{e}_y \cdot 2p\,(\Delta E / E) t} = \left(\frac{2.35^2}{2p}\right) \frac{B}{t \cdot f}. \tag{12}$$

Compared to the 'intuitive' or the more liberal definition in the table in Introduction, the extra factor of $2.35^2/2p$ is simply due to the conversions of conventional FWHMs *DE/E* and t to the Gaussian-normalized full widths as discussed in the last section. Peak brilliance $\hat{B}$ is usually expressed in the same units as the average brilliance *B*.

Transverse and Longitudinal Coherence

The undulator radiation from a finite source emittance does not have complete transverse coherence, but a portion of it within its central cone does. This portion is usually called the transverse coherence length or transverse coherence angle. There are basically two ways to figure out the coherence length. One is to use the uncertainty principle in position and momentum, and the other is to figure out the phase errors based on wave front propagation. We will follow the latter and leave the former to a later discussion on the concept of photon degeneracy. Obviously these two descriptions are completely equivalent [Arthur, 2000].

For a Gaussian source of full size $d = \sqrt{2p}\, s$, the optical-path difference between the waves emitted from the two ends is given by

$$d l = d \cdot dq\ ,$$

when viewed at an angle dq from its axis. When this difference reaches half of the x-ray wavelength, the two waves will be out of phase from each other. Thus we say that the two waves are transversely coherent within this angle dq:

$$dq = \frac{l}{2d}. \tag{13}$$

This angular range actually corresponds to the FWHM of the coherence angle since on one hand we need to consider both $+d\theta$ and $-d\theta$ but on the other hand we should let the phase error reach only half way to the complete out-of-phase condition. Transverse coherence length is simply equal to this coherent angle multiplied by the distance from the source.

One of the useful properties that is intrinsic for an undulator source is the coherent fraction p_c of its total emitted flux or the total coherent flux F_c. The coherent flux is calculated as the flux within the coherent solid angle, $d\theta_x$ times $d\theta_y$, and is given by the solid angle multiplied by the source brightness Eq.(6):

$$F_c = B_r \cdot d\theta_x d\theta_y = \frac{F_n \cdot d\theta_x d\theta_y}{2\pi\sqrt{\sigma_{r'}^2+\sigma_{x'}^2}\sqrt{\sigma_{r'}^2+\sigma_{y'}^2}} = \frac{F_n \cdot (\lambda/2)^2}{2\pi\sqrt{\sigma_{r'}^2+\sigma_{x'}^2}\sqrt{\sigma_{r'}^2+\sigma_{y'}^2}\cdot\delta_x\delta_y}\,.$$

Converting the Gaussian-normalized FWHMs to the total rms source sizes, and using the brilliance definition Eq.(10), it is easy to see that the above equation is

$$F_c = \left(\frac{\lambda}{2}\right)^2 B\,. \tag{14}$$

In practice, Eq.(14) needs to take the units of B into account, and becomes

$$F_c\,[\mathrm{ph/s/0.1\%}] = 10^{-8}\cdot\left(\frac{\lambda[\text{Å}]}{2}\right)^2 B[\mathrm{ph/s/0.1\%/mm^2/mr^2}]\,. \tag{15}$$

Using Eq.(14) and Eq.(10), coherent fraction p_c can be expressed as:

$$p_c = \frac{F_c}{F_n} = \frac{\lambda^2}{(4\pi)^2\bar{\varepsilon}_x\bar{\varepsilon}_y}\,, \tag{16}$$

which is completely determined by the total transverse emittance and the x-ray wavelength. One should note that the emittances in Eq.(16) include both the true electron beam emittances and the radiative contributions (5) and (7), and therefore p_c approaches unity only when electron emittances vanish in both x and y directions. In addition, the word ‘coherent’ here refers to transverse (spatial) coherence only and does not imply anything about the longitudinal (temporal) coherence.

The longitudinal coherence length l_c of an undulator radiation is determined by its bandwidth $\Delta\lambda/\lambda$ and is given by

$$l_c = \lambda^2/\Delta\lambda\,. \tag{17}$$

In most cases, the bandwidth is further modified (narrowed) by x-ray optics. However, if an undulator has thousands of periods, then its intrinsic bandwidth Eq.(4) may be a determining factor of the longitudinal coherence length. It may be noted that radiation of a given wavelength λ emitted from higher harmonics, e.g. a 5th order harmonic at a lower-energy machine, may have a better longitudinal coherence length than the fundamental radiation at higher-energy machines, but of course its flux may be much less.

According to Born and Wolf (1980), one may also speak of ‘coherence time’ Δt_c, which is the time that a photon takes to travel through its longitudinal coherence length l_c and is given by l_c divided by speed of light c:

$$\Delta t_c = \frac{\lambda^2}{c\cdot\Delta\lambda}\,. \tag{18}$$

A source can be considered fully coherent in the longitudinal direction if its pulse duration τ is comparable to or less than the coherent time Δt_c. For a typical x-ray wavelength 1 Å and bandwidth of 10^{-4}, we have l_c = 1 μm and Δt_c = 3.3 fs. Thus a pulse FWHM of $\tau \le 3$ fs after a silicon monochromator from an undulator source is longitudinally coherent. So far no existing or proposed synchrotron sources can offer pulse lengths less than ~100 fs, and thus all sources, including the FELs [Kim, 1997], are only partially coherent in the longitudinal direction. However, these ~100 fs sources, including the ERL, can have *substantial temporal coherence* if filtered through a 30-meV x-ray monochromator. Here we avoid using the word '*fully*' coherent because is requires a more precise definition of coherence.

Photon Degeneracy

Based on the discussion at the end of last section, we see that a photon pulse with its pulse length defined by Eq.(18) can be considered coherent in the longitudinal direction. A question that may come to mind is the following. If a pulse is substantially longer than that specified by Eq.(18), then on average how many photons within the pulse can be considered longitudinally coherent? The answer is simple. For a pulse of FWHM duration t, only the fraction Dt_c/t is longitudinally coherent.

A similar question can be asked for the transverse coherence. In fact we already went through the answer in the section from Eq.(13) to Eq.(14), and obtained the number of photons per second F_c that are transversely coherent. The number of coherent photons per pulse is simply given by $F_{cp} = F_c/f$, where f is the number of pulses per second.

Combining the answers to the above two questions, we can speak of *the number of photons in a single pulse that are both transversely and longitudinally coherent,* which is called the photon degeneracy parameter $\boldsymbol{d}_D$ [Mandel, 1961]:

$$d_D = F_{cp} \cdot \frac{\Delta t_c}{t} = F_{cp} \cdot \frac{l^2}{c\Delta l \cdot t} = \frac{F_{cp}\, l}{ct}\left(\frac{l}{\Delta l}\right). \tag{19}$$

Strictly speaking, the pulse width and the bandwidth in Eq.(19) should be the Gaussian normalized $\sqrt{2p}\,s_t$ = $\left(\sqrt{2p}/2.35\right)t$ and $\left(\sqrt{2p}/2.35\right)\Delta l/l$, so it becomes:

$$d_D = \left(\frac{2.35^2}{2p}\right)\frac{F_{cp}\, l}{c\,t}\left(\frac{l}{\Delta l}\right), \tag{20}$$

or in practical units,

$$d_D = \frac{F_{cp}\,[\text{ph}/0.1\%] \cdot l\,[\text{Å}]}{3.41 \times t\ [\text{fs}]}. \tag{21}$$

Using $F_{cp} = F_c/f$, and Eqs.(14) and (12), it can be shown that Eq.(20) leads to the following relation between the photon degeneracy parameter $\boldsymbol{d}_D$ and the peak brilliance $\hat{B}$:

$$d_D = \hat{B} \cdot \frac{l^3}{4c}\left(\frac{l}{\Delta l}\right). \tag{22}$$

This agrees with those found in the literature [Brinkmann, et al., 1997; LCLS, 1998], although the bandwidth factor is usually only implied and often omitted. In practice, again the units need to be straightened out and the formula becomes:

$$d_D = \hat{B}\,[\text{ph}/\text{s}/0.1\%/\text{mm}^2/\text{mr}^2] \cdot \frac{l\,[\text{Å}]^3}{1.2 \times 10^{24}}. \tag{23}$$

Eq.(23) tells us right away that for hard x-rays of l ~ 1 Å , the peak brilliance needs to be much larger than 10^{24} in order to have d_D significantly greater than unity. As indicated in Table 1, to date no existing synchrotron sources provide degeneracy parameters close to or greater than unity, and thus these existing sources are not laser-like sources. This situation would be changed with the proposed ERL or FEL sources. How to exploit this type of degenerate sources for novel x-ray applications is a wide-open area that the ERL could contribute significantly.

Quantum Modes for Photons

Another way of deriving the photon coherence and degeneracy is to use the uncertainty principles in position-momentum and energy-time space. The idea here is that each photon is identified or labeled by (*x*, *y*, *t*) or by (k_x, k_y, *E*). Note that *z* is not an independent variable since an electromagnetic wave is a transverse wave. Because of the uncertainty principle, these labels on photons can only be determined simultaneously within certain intervals that cannot be arbitrarily small. The smallest intervals thus define a photon quantum mode, which can also be interpreted as the fundamental volume element in photon phase space, as given by the following uncertainty relationship:

$$s_x\, s_{kx} = 1/2\,, \quad s_y\, s_{ky} = 1/2\,, \quad s_t\, s_E = \hbar\,.$$

Since $s_k = (2\pi/\lambda) s_\theta$, the above equations are equivalent to the following in the photon position-angle and time-energy phase space:

$$s_x\, s_{x'} = l/4\pi, \quad s_y\, s_{y'} = l/4\pi, \quad s_t\, s_E = \hbar\,.$$

Obviously the first two equations are related to the transverse coherence and the last one is related to the longitudinal coherence discussed in the previous sections. In fact, one can derive the above relations precisely from Eqs.(13) and (18). The first two relations also define a *diffraction-limited* source. Considering the volume under a two-dimensional Gaussain curve for each of the phase-space pairs, we obtain the *fundamental volume units for each single quantum mode* of a photon:

$$2ps_x s_{x'} = l/2\,, \quad 2ps_y s_{y'} = l/2\,, \quad 2ps_t s_E = 2p\hbar\,. \tag{24}$$

From this point of view, the degeneracy parameter d_D is defined as the number of photons in a pulse within a single quantum mode in phase space. We now consider the transverse modes first. For a source with an average brilliance *B*, the number of photons (per second) in a single transverse mode (transversely coherent) is given by

$$F_c = B \cdot 2ps_x s_{x'} \cdot 2ps_y s_{y'} = \left(\frac{l}{2}\right)^2 B\,, \tag{25}$$

which is the same as Eq.(14), as expected.

To take the longitudinal mode into account, we realize that a synchrotron source is pulsed, and it is the number of photons in each single pulse, F_{cp}, that needs to satisfy the uncertainty principle. For a photon pulse of FWHM duration *t* and energy FWHM *DE*, the number of quantum modes in the pulse is $[2p(t/2.35)(DE/2.35)]/(2ps_t s_E)$ = $(tDE)/(5.52\,\hbar)$. In another word, the phase-space unit (FWHM) of a single longitudinal mode is $5.52\,\hbar$, which equals to 3.63 eV-fs, since $\hbar = 6.582\times10^{-16}\,\text{eV sec}$. Thus the number of photons per quantum mode is

$$d_D = F_{cp} \cdot \frac{5.52\hbar}{t \cdot \Delta E}\,. \tag{26}$$

or:

$$d_D = \frac{3.63 \times F_{cp}\,[\text{ph}/0.1\%]}{t\,[\text{fs}] \cdot \Delta E\,[\text{eV}]}\,. \tag{27}$$

As an example, for LCLS (1998), F_{cp} = 2x10^{12} photons per pulse, t = 277 fs, DE = 8 eV at 8 keV for 0.1% bandwidth, so the degeneracy parameter for LCLS is

$$d_D = \frac{3.63 \times 2 \times 10^{12}}{277 \times 8} = 3.3 \times 10^9 ,$$

which is a very large number comparable to optical lasers.

Energy Flow and Electric Field

As already mentioned in Introduction, energy flow and power characteristics of a quasi-monochromatic x-ray beam can be calculated directly by multiplying photon energy with the corresponding flux numbers. For some applications, it would be interesting to know the transversely coherent power P_c in a beam and its coherent power density S_c. For a beam with coherent flux F_c, coherent power in the beam is simply

$$P_c = hn \cdot F_c .$$

To calculate the power density, one usually needs to know the distance to the source. For simplicity, we assume a perfect 1:1 focusing, by e.g. a zone plate, so that the area illuminated by the beam is equal to the source area:

$$A = 2.35^2 \sqrt{s_x^2 + s_r^2} \sqrt{s_y^2 + s_r^2} .$$

The coherent power density, which is the magnitude of the time-averaged Poynting vector S_c, is therefore given by

$$S_c = hn \cdot F_c / A . \qquad (28)$$

Since the Poynting vector is related to the electric field E_0 [Jackson, 1975]:

$$S_c = \frac{c}{8p} |E_0|^2 ,$$

we obtain a time-averaged electric field E_0, expressed in practical units:

$$E_0[\text{V/m}] = 2.746 \times 10^4 \sqrt{S_c[\text{W/mm}^2]} . \qquad (29)$$

Even though one could use Eq.(29) to convert any power density into some kind of electric-field strength, one has to be careful about its physical meaning, because strictly speaking Eq.(29) is only valid for plane electromagnetic waves of a single frequency which is a fully coherent wave. A detailed discussion in this area is beyond the scope of this document.

As Table 1 indicates, focused peak (coherent) electric fields for the ERL can be in the 10^7-10^8 V/m range, making it comparable to those from visible laser sources. Thus a possible new research area for the ERL would be in the field of nonlinear x-ray interactions with condensed matter. This could be a unique regime that the ERL (or spontaneous radiation from any long linac) can serve very well because the peak fields from 3rd generation sources are usually not strong enough to excite nonlinear interactions, and those from 4th generation FEL sources on the other hand would be above the threshold of electron binding field in atoms which is on the order of 10^{10} V/m. Further discussions will be given in a future technical memo.

Results and Conclusions

Using the analytical formulae listed in the previous sections, we have calculated the various parameters that characterize the proposed ERL source at Cornell, and compared them with those at the existing and some proposed synchrotron sources. These results are summarized in Table 1, and shown in Figures 1-5. The comparison table is based on the suggestions [Shenoy & Arthur, 2000] at the *ERL Science Workshop* held at Cornell in December 2000. Two options are chosen for the ERL with slight modifications from the white paper [Gruner et al., 2000]. One is based on a machine current of 100 mA and a transverse emittance of 0.15 nm. The other is based on an ultra-high emittance of 0.015nm but a low current of 10 mA. Both assume a FWHM bunch duration of 300 fs [Kraft, 2000], and a machine energy of 5.3 GeV. All calculations assume perfect machine and undulator conditions, without any field errors, nor electron energy spreads. For simplicity, only the first and the third harmonic curves are shown in the Figures.

All flux and brilliance calculations are performed at 8 keV fundamental energy except for the proposed FEL sources LCLS and TESLA. Thus in some cases the value of brilliance may be somewhat lower than the value at the peak fundamental energy of the undulator. This is done for the purpose of proper coherence comparisons since coherence is very sensitive to x-ray wavelength. The FEL numbers are obtained from the LCLS (1998) and the TESLA (Brinkmann, et al., 1997) design reports. Power density numbers at 20m for SASE are for average coherent power only and include the effects from source size and beam divergence, while all other numbers for that row are calculated using the formula listed at the end of "Flux, Brightness and Power".

All calculated results are for high-duty cycle operations only (APS: 6+21 singlets; ESRF: 2x1/3 filling of 992 RF buckets; SPring8: 2/3 filling of 2436 RF buckets). A separate table for single-bunch oriented running and performances are being assembled and may be added to this document at a later date. These and additional machine parameters are obtained from the respective web sites. APS parameters are obtained from http://www.aps.anl.gov/xfd/calendar/fp_2000-4.html on 1/8/01, and from e-mail communications with Dr. Dennis Mills. Upgraded parameters for APS are based on Arthur (2000). ESRF parameters are obtained from http://www.esrf.fr/machine/myweb/machine/brill.html on 12/18/00, and from communications with Dr. Pascal Elleaume (2000). SPring8 parameters are obtained from *SPring8 Annual Report 1998*, available at http://www.spring8.jp/, and from Don Bilderback's personal communication with Dr. Kitamura at SPring8.

The following reasoning is used to justify the bunch length for the ERL 10mA option, based on the proposed transverse and longitudinal emittance for TESLA. Assuming that similar emittances could be achieved with the DC gun for the ERL, then scaling charge per bunch from 1 nC to 0.008 nC means a bunch volume reduction by a factor 125 or a factor of 5 in each direction. Thus $\varepsilon_x = \varepsilon_y = 0.02$ nm for TESLA becomes 0.004 nm for ERL. However, TESLA is a 25 GeV machine, so ERL can only reach $0.004 \times 25/5.3 = 0.019$ nm. What this means is that to reach 0.015 nm transverse emittances, we have to reduce the longitudinal factor of 5 to $5 \times (0.015/0.019)^2 = 3.1$. Assuming that the ERL can achieve the same longitudinal rms emittance of 27 keV-mm at 1 nC as that for TESLA, and has an energy spread of 1×10^{-4} (rms) or $\sigma_E = 5.3 \times 10^5$ eV at 5.3 GeV, then the bunch length for 1 nC is FWHM $= 2.35 \times 27000/(5.3 \times 10^5) = 120$ µm, which is 400 fs. Now going back to use the reduction factor of 3.1 due to 0.008 nC/bunch yields a FWHM bunch length of 130 fs. Because several assumptions are involved in the above argument, we have adopted a more conservative 300 fs as the proposed value for the ERL.

A couple of rounds of optimizations are performed for the Cornell ERL, in an effort to obtain the best conditions for highest brilliance and thus best coherent properties.

The first is on the choice of the β functions. Because of the radiative terms, Eq.(5) and (7), in the total emittance, horizontal and vertical β functions have noticeable effects on photon brilliance, especially when electron emittance is approaching the diffraction limit. For a given undulator length L and a fixed electron emittance, the smallest total emittance in each direction, Eq.(8), is obtained when the electron source-size to divergence ratio is equal to the radiative size to divergence ratio. This then defines the best β for the highest photon brilliance for a given L:

$$\beta_{x,y} = \frac{\sigma_{x,y}}{\sigma_{x',y'}} = \frac{\sigma_r}{\sigma_{r'}} = \frac{L}{2\pi} \quad . \tag{30}$$

Based on this idea, the high-coherence option for the ERL is calculated using $b = 4$ m for a 25 m undulator. Whether this is feasible in practice would of course depend on a lot of other things such as space for segmentation due to focusing and defocusing requirements.

The second optimization is about the best electron emittance for peak photon brilliance, given a certain dependence on beam current. These results are summarized in Figure 6. Two types of current dependence are assumed, a linear dependence $e_{x,y} \propto I$ in (a) and a 2/3 power dependence $e_{x,y} \propto I^{2/3}$ in (b). According to Ivan Bazarov, the latter may be closer to reality, although no one seems to know for sure. If that is the case, then only a low-b undulator would be able to take advantage of a lower emittance, as Fig.6(b) suggests. In another word it may not be advantageous to go for ultra-low emittance with sacrifices in beam current. In (c) we show brilliance for different fundamental energies for an optimal-brilliance undulator with b given by Eq.(30).

The basic conclusion from this exercise is that the Cornell ERL can be an extremely brilliant source of x-rays. With a long, short-period undulator at a low-b section, it can offer roughly one orders of magnitude more average brilliance and coherent flux than the best from the existing 3rd generation storage rings, making them comparable to that from prototype 4th generation sources. The combination of its high brilliance with its short pulses in the high-duty-cycle mode would mean two-to-three-orders-of-magnitude higher peak brilliance, peak coherent flux, and peak photon degeneracy, making it an outstanding leader in these fields before the 4th generation sources come along. Even with the 4th generation prototype sources, the high-duty-cycle frequency from the ERL would make it well balanced in all three categories of source characteristics: high flux, high coherence, and high peak properties.

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Table 1 : Comparison of the Cornell ERL source with other existing and proposed synchrotron light sources.

	Assuming high duty-cycle operations	ERL hi-flux	ERL hi-coh.	APS und. A	APS upgrade	ESRF U35	Spring8 5m	Spring8 25m	LCLS spont.	LCLS SASE	TESLA spont.	TESLA SASE
Machine design	Energy E_G (GeV)	5.3	5.3	7	7	6	8	8	15	15	25	25
	Current I (mA)	100	10	100	300	200	100	100	$72\cdot10^{-6}$	$72\cdot10^{-6}$	0.063	0.063
	Charge q (nC/bunch)	0.077	0.008	14	14	0.85	0.29	0.29	1	1	1	1
	e_x (nm-rad)	0.15	0.015	8	3.5	4	6	6	0.05	0.05	0.02	0.02
	e_y (nm-rad)	0.15	0.015	0.08	0.0035	0.01	0.003	0.003	0.05	0.05	0.02	0.02
	Bunch fwhm t (ps)	0.3	0.3	73	73	35	36	36	0.23	0.23	0.188	0.090
	# of bunches f (Hz)	$1.3\cdot10^{9}$	$1.3\cdot10^{9}$	$7.3\cdot10^{6}$	$22\cdot10^{6}$	$2.3\cdot10^{8}$	$3.4\cdot10^{8}$	$3.4\cdot10^{8}$	120	120	56575	56575
Insertion device	Undulator L (m)	25	25	2.4	4.8	5	4.5	25	100	100	30	87
	Period l_u (cm)	1.7	1.7	3.3	3.3	3.5	2.4	3.2	3	3	3.81	5
	# of period N_u	1470	1470	72	145	142	187	781	3300	3300	787	1740
	Horizontal b_x (m)	12.5	4.0	15.9	4.0	35	24	24	18	18	14.7	33.3
	Vertical b_y (m)	12.5	4.0	5.3	4.0	2.5	3.9	15	18	18	14.7	33.3
	Und. K (@ E_1)	1.38	1.38	1.24	1.24	0.67	2.08	1.66	3.9	3.9	2.28	4.14
	1^{st} harmonic E_1 (keV)	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.26	8.26	10	12.4
Beamline & optics	H. div. fwhm (μrad)	9.1	6.2	54.3	70.2	26.8	38.4	37.4	4.9	1	6.7	1.76
	V. div. fwhm (μrad)	9.1	6.2	16.2	9.7	10.4	10.0	4.3	4.9	1	6.7	1.76
	H. source fwhm (μm)	103	24.5	839	277	879	892	890	82	78	60	60
	V. source fwhm (μm)	103	24.5	48.6	11.4	13.9	10.6	22.8	82	78	60	60
	Power P_0 (kW)	33.9	3.4	1.2	7.2	1	15.7	31.2	0.0027	0.003	0.070	1.6
	dP/dA @20m (W/mm^2)	2600	260	180	1080	194	1830	4568	0.45	63	336	$2\cdot10^{5}$
DC experiments	Ave. flux F_n (p/s/0.1%)	$1.5\cdot10^{16}$	$1.5\cdot10^{15}$	$7.0\cdot10^{14}$	$4.2\cdot10^{15}$	$1.3\cdot10^{15}$	$2.4\cdot10^{15}$	$9.0\cdot10^{15}$	$3.3\cdot10^{10}$	$2.4\cdot10^{14}$	$6.4\cdot10^{12}$	$4\cdot10^{17}$
	Ave. brilliance B (p/s/0.1%/mm^2/mr^2)	$1.3\cdot10^{22}$	$5.2\cdot10^{22}$	$1.5\cdot10^{19}$	$1.5\cdot10^{21}$	$3.1\cdot10^{20}$	$5.0\cdot10^{20}$	$2.2\cdot10^{21}$	$1.6\cdot10^{17}$	$4.2\cdot10^{22}$	$3.6\cdot10^{19}$	$8\cdot10^{25}$
	Coh flux F_c (p/s/0.1%)	$8.1\cdot10^{13}$	$3.1\cdot10^{14}$	$0.9\cdot10^{11}$	$9.0\cdot10^{12}$	$1.8\cdot10^{12}$	$3.0\cdot10^{12}$	$1.3\cdot10^{13}$	$9.0\cdot10^{8}$	$2.4\cdot10^{14}$	$1.4\cdot10^{11}$	$4\cdot10^{17}$
	Coh. fraction p_c (%)	0.52	20	0.013	0.22	0.14	0.13	0.14	2.7	100	2.1	100
Pulsed expts.	Photons / bunch	$1.2\cdot10^{7}$	$1.2\cdot10^{6}$	$9.6\cdot10^{7}$	$1.9\cdot10^{8}$	$5.7\cdot10^{6}$	$7.1\cdot10^{6}$	$2.7\cdot10^{7}$	$2.8\cdot10^{8}$	$2\cdot10^{12}$	$1.1\cdot10^{8}$	$7\cdot10^{12}$
	Peak brilliance (p/s/0.1%/mm^2/mr^2)	$3.0\cdot10^{25}$	$1.2\cdot10^{26}$	$2.5\cdot10^{22}$	$8.3\cdot10^{23}$	$3.3\cdot10^{22}$	$3.6\cdot10^{22}$	$1.6\cdot10^{23}$	$4.8\cdot10^{27}$	$1.2\cdot10^{33}$	$3.4\cdot10^{27}$	$7\cdot10^{33}$
	Peak flux (p/s/0.1%)	$3.9\cdot10^{19}$	$3.9\cdot10^{18}$	$1.3\cdot10^{18}$	$2.6\cdot10^{18}$	$1.6\cdot10^{17}$	$1.9\cdot10^{17}$	$7.4\cdot10^{17}$	$1.2\cdot10^{21}$	$7.2\cdot10^{24}$	$6.0\cdot10^{20}$	$3\cdot10^{25}$
	Pk coh. flux (p/s/0.1%)	$2.1\cdot10^{17}$	$7.9\cdot10^{17}$	$1.7\cdot10^{14}$	$5.6\cdot10^{15}$	$2.2\cdot10^{14}$	$2.5\cdot10^{14}$	$1.1\cdot10^{15}$	$2.7\cdot10^{19}$	$7.2\cdot10^{24}$	$1.4\cdot10^{19}$	$3\cdot10^{25}$
	Peak degen. par. d_D	95	368	0.078	2.6	0.103	0.113	0.49	$1.3\cdot10^{4}$	$3.3\cdot10^{9}$	$4.7\cdot10^{3}$	$8\cdot10^{9}$
Nonlinear expts.	Ave. coh. power (W)	0.10	0.40	$1.2\cdot10^{-4}$	0.011	0.0023	0.0038	0.017	$1.2\cdot10^{-6}$	0.32	$2.2\cdot10^{-4}$	794
	Peak coh. power (W)	269	1011	0.22	7.2	0.28	0.32	1.4	$3.8\cdot10^{4}$	$9\cdot10^{9}$	$2.2\cdot10^{4}$	$60\cdot10^{9}$
	A coh dP/dA (W/mm^2)	12.0	848	0.0029	3.5	0.19	0.40	0.84	$2.3\cdot10^{-4}$	0.0077	0.078	$2.8\cdot10^{5}$
	P coh dP/dA (W/mm^2)	$3.2\cdot10^{4}$	$2.2\cdot10^{6}$	5.4	2280	22.9	33.8	69.0	$7.2\cdot10^{6}$	$1.9\cdot10^{12}$	$7.8\cdot10^{6}$	$2.1\cdot10^{13}$
	Ave. E-field (V/m)	$1.0\cdot10^{5}$	$8.0\cdot10^{5}$	1479	$5.1\cdot10^{4}$	$1.2\cdot10^{4}$	$1.7\cdot10^{4}$	$2.5\cdot10^{4}$	416	2410	7670	$1.5\cdot10^{7}$
	Peak E-field (V/m)	$4.9\cdot10^{6}$	$4.1\cdot10^{7}$	$6.4\cdot10^{4}$	$1.3\cdot10^{6}$	$1.3\cdot10^{5}$	$1.6\cdot10^{5}$	$2.3\cdot10^{5}$	$7.4\cdot10^{7}$	$3.8\cdot10^{10}$	$7.7\cdot10^{7}$	$1.3\cdot10^{11}$

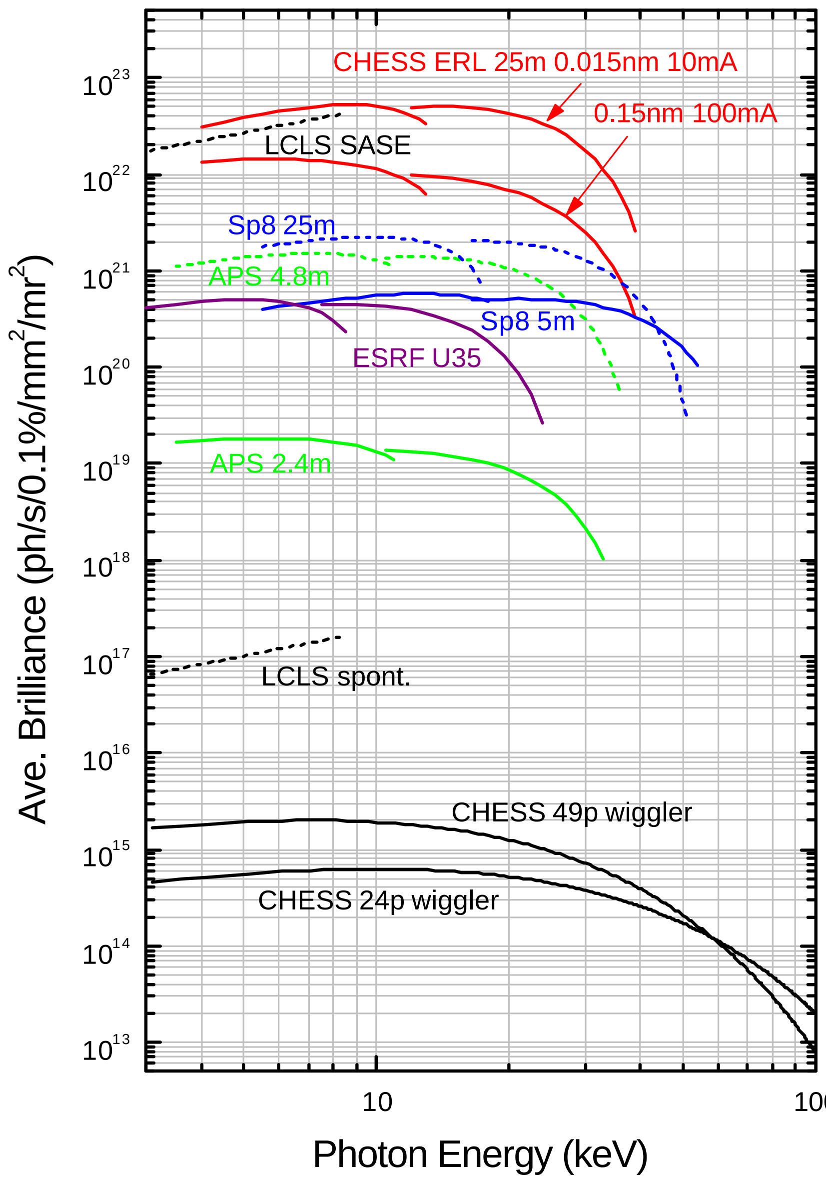

Figure 1: Comparison of calculated average spectral brilliance of the various sources. The parameters used for undulator sources are listed in Table 1. The CHESS wiggler sources assume 5.3 GeV 300 mA operation and a FWHM source size of d_x = 5.5 mm and d_y = 0.9 mm for the 24-pole wiggler at F-line and d_x = 3.3 mm and d_y = 0.85 mm for the 49-pole wiggler at A/G-line.

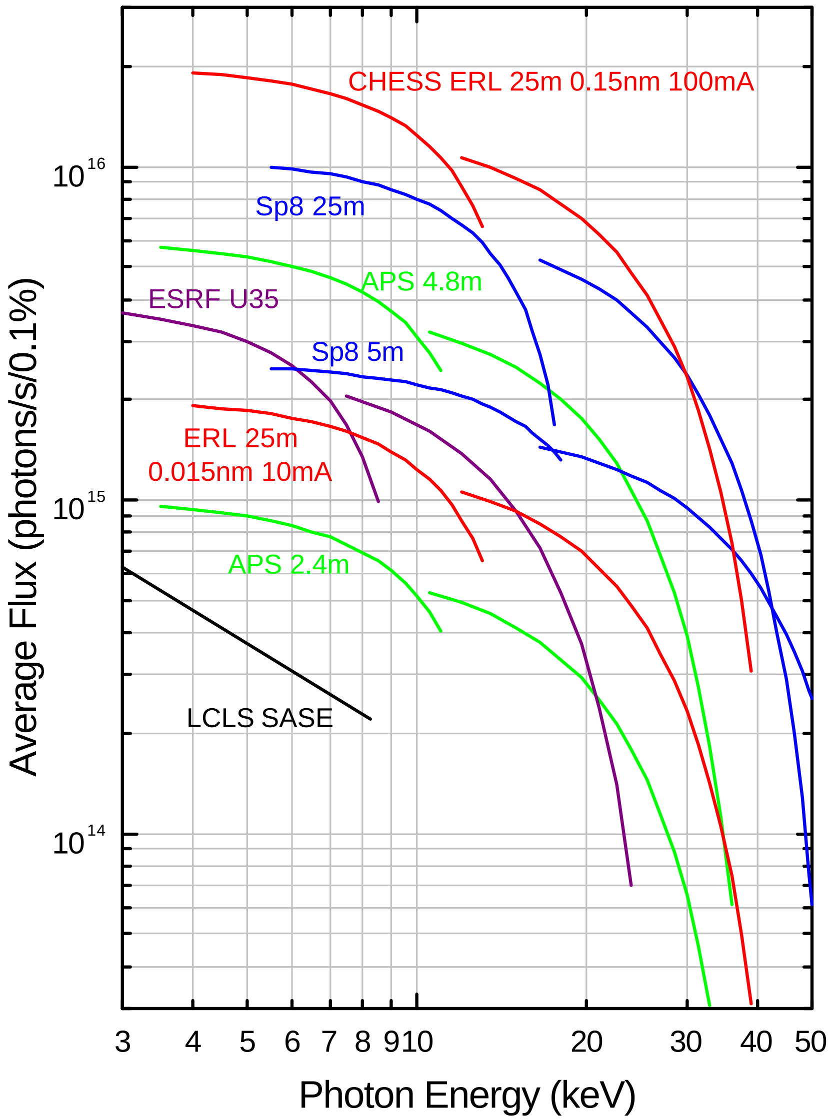

Figure 2: Comparison of calculated average photon flux of the various sources. The parameters used for all sources are listed in Table 1.

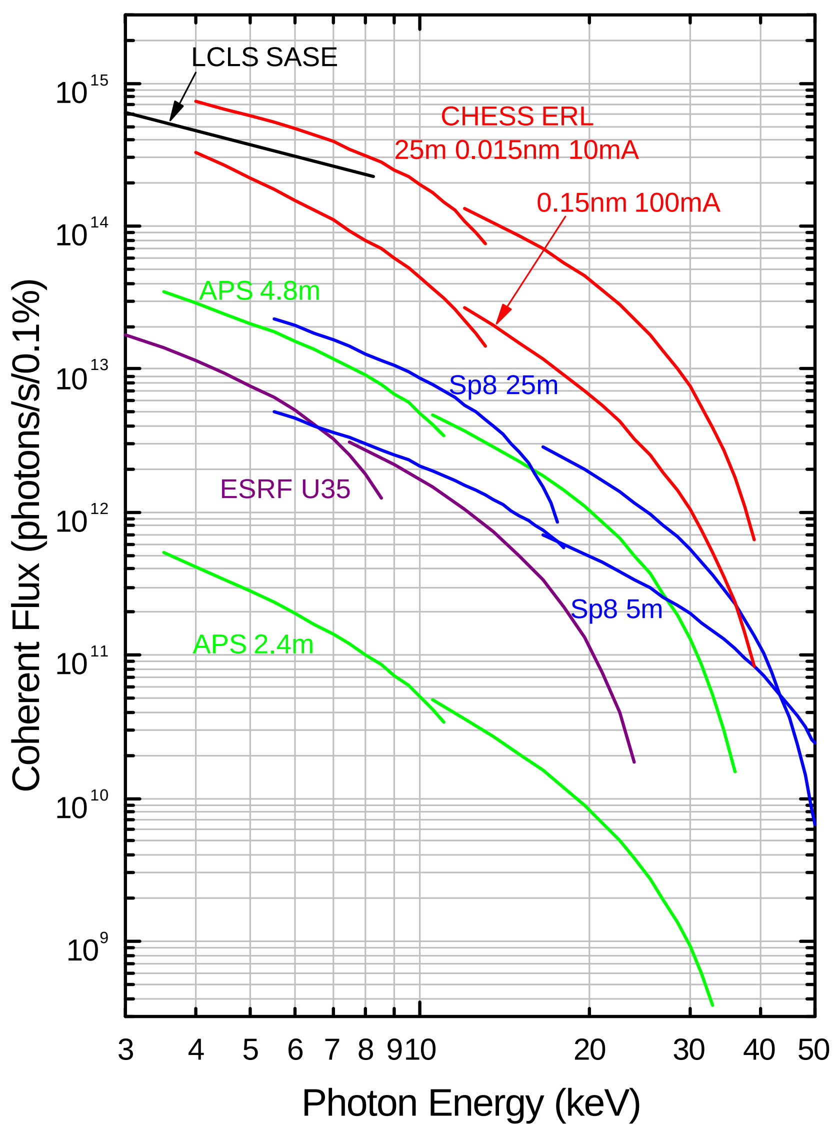

Figure 3: Comparison of calculated average coherent photon flux of the various sources. The parameters used for all sources are listed in Table 1.

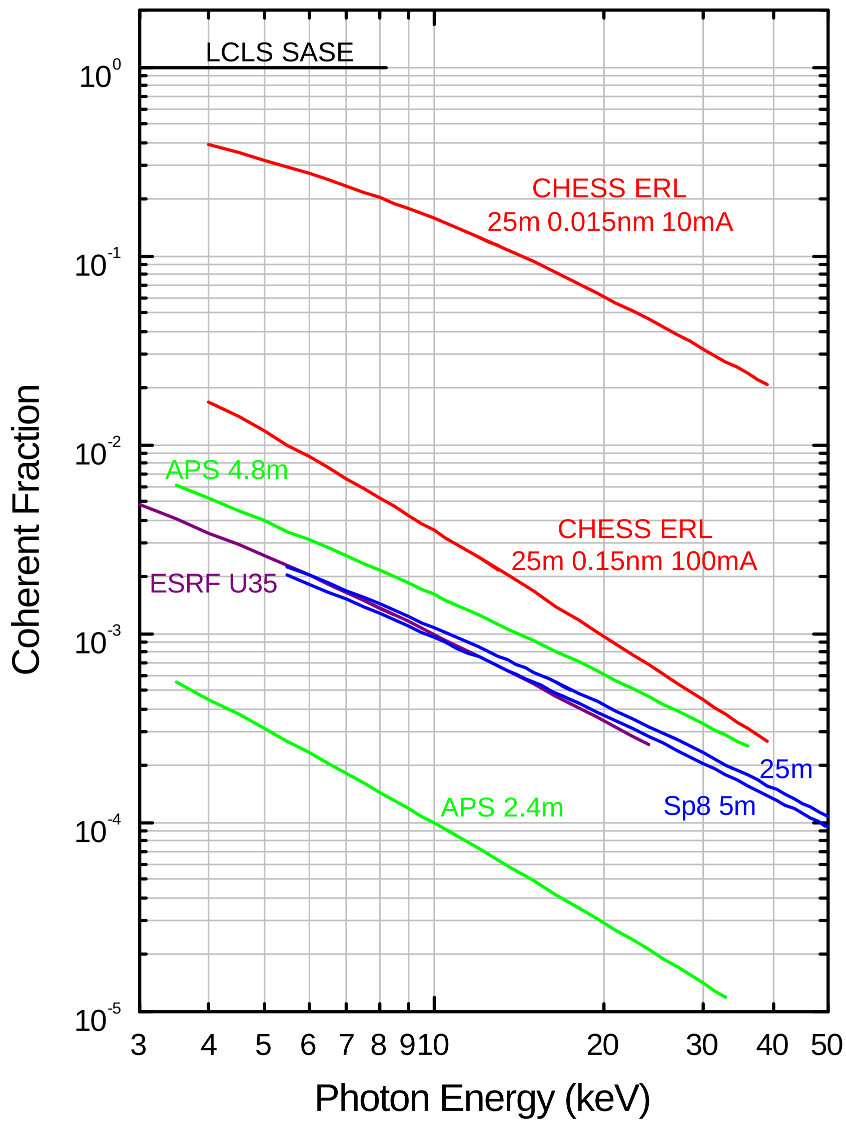

Figure 4: Comparison of calculated coherent fractions of the various sources. The parameters used for all sources are listed in Table 1.

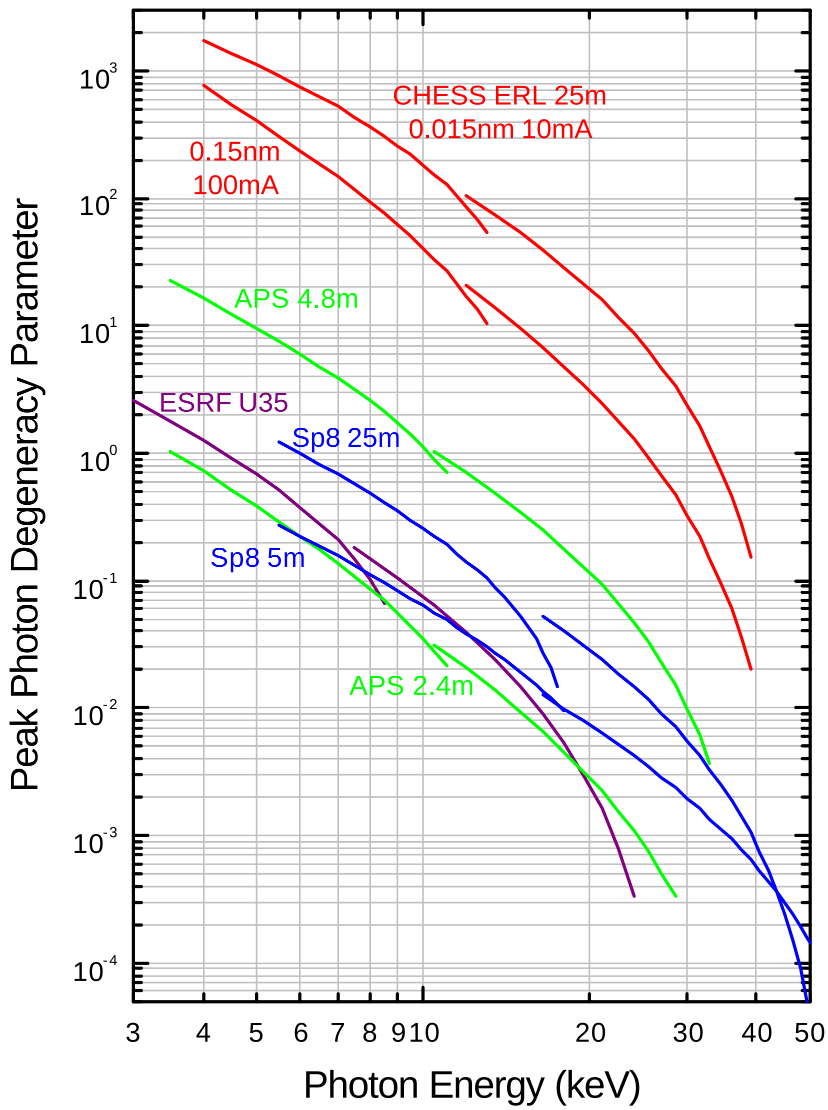

Figure 5: Comparison of calculated photon degeneracy parameter d_D of the various sources. The parameters used for all sources are listed in Table 1.

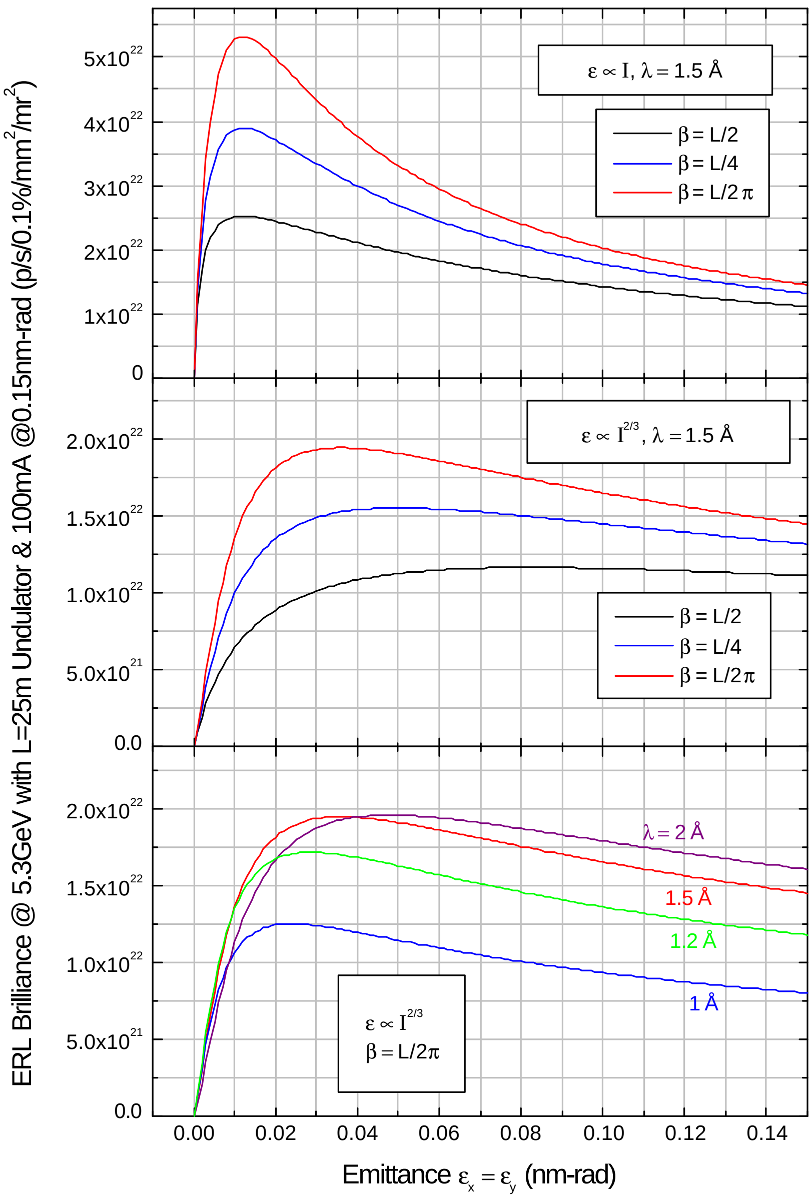

Figure 6: Calculated average spectral brilliance of the ERL as a function of electron beam emittance $e=e_x=e_y$ (round beam) and for different *b* values. Top panel (a): assuming emittance *e* grows linearly with beam current *I*. Middle panel (b): assuming *e* grows with $I^{2/3}$. Bottom panel (c): brilliance for fundamental energy set to different values for optimal *b*.

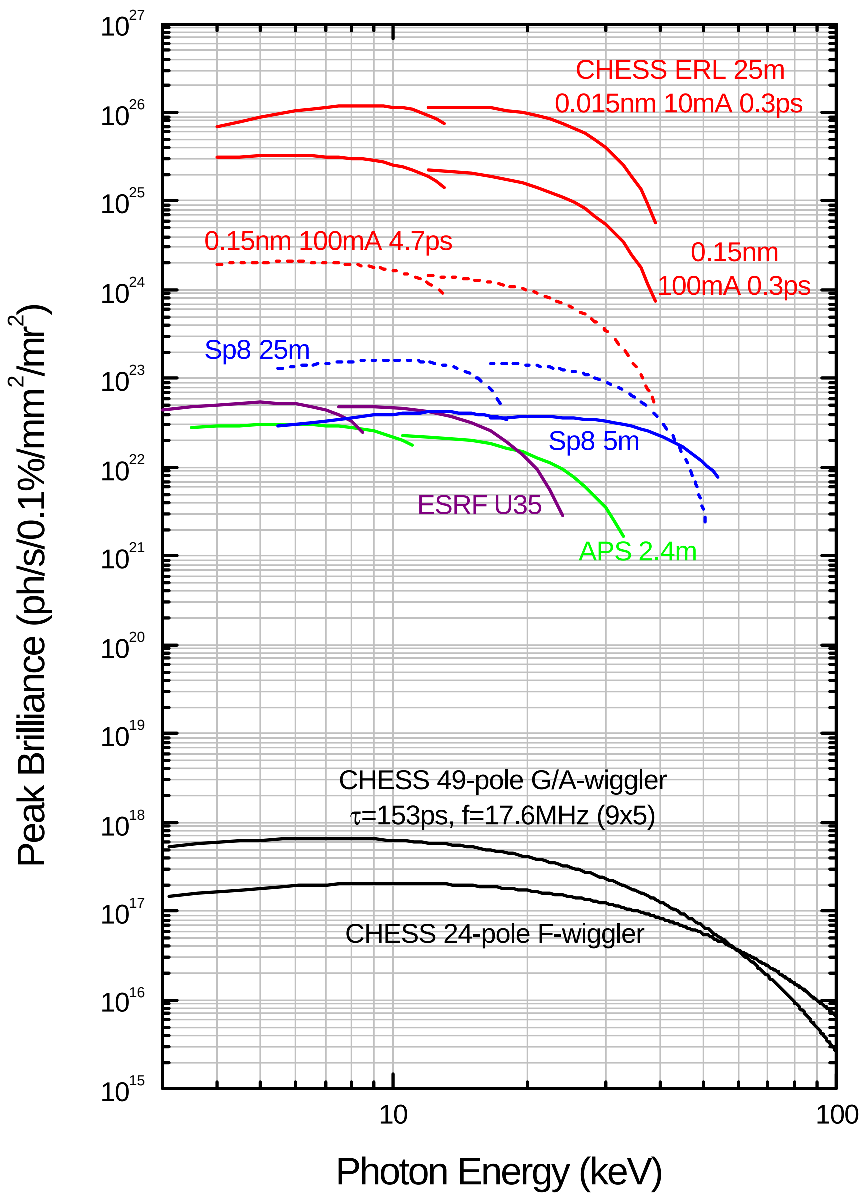

Figure 7: Comparison of the calculated peak spectral brilliance of the Cornell ERL with some existing synchrotron sources. The parameters used for the undulator sources are listed in Table 1. The CHESS wiggler sources assume 5.3 GeV 300 mA operation and a FWHM source size of d_x = 5.5 mm and d_y = 0.9 mm for the 24-pole wiggler at F-line and d_x = 3.3 mm and d_y = 0.85 mm for the 49-pole wiggler at A/G-line. A bunch length FWHM of 153 ps and a lattice of 9x5 in 2.56 μs revolution time, which represent the present operation mode, are used for the CHESS wigglers.