PARTICLE PHYSICS IN THE RANDALL-SUNDRUM FRAMEWORK

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In this dissertation, several aspects of particle physics in the Randall-Sundrum framework are discussed. We view the Randall-Sundum framework as a model for the dual of a strongly interacting technicolor theory of electroweak symmetry breaking (EWSB). First, we consider extra dimensional descriptions of models where there are two separate strongly interacting EWSB sectors ("topcolor" type models). Such models can help alleviate the tension between the large top quark mass and the correct value of the $Zb\bar{b}$ couplings in ordinary Higgsless models. A necessary consequence is the appearance of additional pseudo-Goldstone bosons ("top-pions"), which would be strongly coupled to the third generation. Second, we examine extra-dimensional theories as "AdS/QCD" models of hadrons, pointing out that the infrared physics can be developed in a more systematic manner by exploiting backreaction of the nonperturbative condensates. We also show how asymptotic freedom can be incorporated into the theory, and the substantial effect it has on the glueball spectrum and gluon condensate of the theory. Finally, we study the S parameter, considering especially its sign, in models with fermions localized near the UV brane. We show that for EWSB in the bulk by a Higgs VEV, S is positive for arbitrary metric and Higgs profile, assuming that the effects from higher-dimensional operators in the 5D theory are sub-leading and can therefore be neglected. Our work strongly suggests that S is positive in calculable models in extra dimensions.

BIOGRAPHICAL SKETCH

Matt Reece was born in Louisville, Kentucky on January 24, 1982, to Beverly and Kenneth Reece. He grew up in Louisville, where he attended duPont Manual High School and got his first taste of research in a summer working with electrical engineers at the University of Louisville. As an undergraduate he attended the University of Chicago, concentrating in physics and mathematics. While there he worked on the CDF experiment, which helped to foster his interest in particle physics but also convinced him that he would be more useful as a theorist than an experimentalist. His interest in building theoretical models that could be discovered by future experiments led him to Cornell, where he has spent four years trying to understand what to expect from the upcoming LHC experiments. He hopes the results will, nonetheless, surprise him.

This dissertation is dedicated to my parents.

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Chapter 1

Introduction

1.1 The Standard Model and EWSB

The Standard Model of particle physics is extraordinarily well-established and well-tested. It describes all of the known non-gravitational forces and matter through a gauge group, $SU(3) \times SU(2)_L \times U(1)_Y$, and a set of fermionic matter fields Q, U, D, L, E with appropriate charges under the gauge group. Despite its success, there is one subtlety: we know that in the real world $SU(2)_L \times U(1)_Y$ is broken spontaneously to $U(1)_{EM}$, the gauge group of electromagnetism. This is known as "electroweak symmetry breaking" or EWSB for short. The Standard Model describes this in the simplest way possible: via a scalar doublet Higgs boson H that gets a vacuum expectation value $v \sim 246$ GeV at the minimum of its potential $V(H^\dagger H)$. In the Standard Model a potential is simply added by hand, $V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$.

The Higgs boson has not yet been discovered; it must be around 115 GeV or heavier, or the Standard Model must be modified in such a way that it decays in a manner that could have escaped detection at LEP. Searches are underway at the Tevatron (and in a narrow mass range near 160 GeV are quite close to excluding a Standard Model Higgs). The Large Hadron Collider should be turning on within the next year, and eventually if the Standard Model is correct it will discover the Higgs.

There are several reasons to think that this is not the whole story. For one, the real world has gravity, which can be added to the Standard Model as a low-energy effective theory but breaks down at high energies (near the Planck scale $M_{Pl} \sim 10^{19}$ GeV). This is outside the scope of this thesis, although it is conceivable that string theory explains quantum gravity and also gives insight into lower-energy physics. Another is the observation of neutrino masses, which are suggestive of physics at higher energy scales, but likely not relevant to the LHC. Yet another is dark matter, the observed properties of which could be explained by a weakly interacting massive particle ("WIMP") with mass at the TeV scale. This is perhaps the most compelling *experimental* reason to expect physics beyond the Standard Model to make its presence known at the LHC.

However, there is a yet more compelling *theoretical* reason which, in the opinion of most particle physicists, leads us to expect physics beyond the Standard Model at the LHC. This is the so-called "hierarchy problem," and can be viewed as the question of why the electroweak symmetry breaking scale v is so much smaller than the fundamental quantum gravity scale M_{Pl}. Another way of looking at this question is: what is the origin of the physics driving the Higgs to get a vacuum expectation value? While it's conceivable that there really is a fundamental potential $V(H^\dagger H)$, unexplained by any deeper principle, it's more appealing to think that there is other physics determining the potential and the scale v. From a technical standpoint, the main aspect of the hierarchy is its lack of technical naturalness. If one begins with a classical mass for the Higgs, quantum effects will produce "corrections" to the mass of order the cutoff scale of the theory, $\delta m^2 \sim \Lambda^2$. This quadratic divergence implies that there is a fine-tuning in the theory; classical and quantum masses must nearly balance in order for the final, physical mass to be much less than the Planck scale. Such considerations lead many to think that fundamental scalars should not exist in nature, absent some mechanism to protect them from these severe quantum effects.

1.2 Options for the Hierarchy Problem

There are many attitudes one can take toward the hierarchy problem, as it is to some extent a theorist's problem, not an experimental one. It is entirely possible that we just happen to live in a universe with a Higgs with some appropriately finely-tuned potential. This has led to anthropic arguments for the hierarchy: if the Higgs vacuum expectation value were too large, physics in our universe would be very different, and the formation of structure could be altered, for instance. Similar arguments are currently the *only* way we have of understanding the smallness of the cosmological constant Λ (which, together with the Higgs mass, is one of the only relevant operators in the Standard Model). Since we have at present no way of addressing the Λ problem other than anthropics, it might not be unreasonable to appeal to the same thing to explain EWSB. This isn't very satisfying, from a theoretical point of view, and from a phenomenological one, it doesn't tell us what we should expect from upcoming experiments like the LHC. (Variations on this theme, like "split supersymmetry", do make LHC predictions.) While it's always worth keeping this in the back of our minds as a possibility we might have to face if the data don't show us anything new, for now we will only discuss theoretical approaches that explain the hierarchy through some physical mechanism.

One theoretical option for addressing the hierarchy problem is to posit TeV-scale supersymmetry. Supersymmetry is a spacetime symmetry that mixes bosons with fermions. Fermion masses do not suffer the same severe quadratic divergences as scalar masses, due to chiral symmetry: the term $m\bar{\psi}\psi$ in a Lagrangian breaks the symmetry, so $m = 0$ is radiatively stable and corrections for finite m are only logarithmically divergent. Supersymmetry enforces that fermions and scalars have equal masses. This means that exact supersymmetry cancels quadratic divergences in scalar masses, making them also only logarithmically divergent. The world around us is not exactly supersymmetric; in fact, we have so far not observed the supersymmetric partner of any particle in the Standard Model. This implies that *if* physics is fundamentally supersymmetric, the extra symmetry is broken at a scale $M_{SUSY} \gtrsim 1\text{TeV}$. This scale cuts off the quadratic divergence and resolves the problem of technical naturalness.

Supersymmetry is the most-studied and, on theoretical grounds, perhaps the most appealing of options for solving the hierarchy problem. It arises in Calabi-Yau compactifications of string theory, where it is also naturally broken, frequently at the string scale. However, it does appear that there are viable models, both in which supersymmetry is "gravity-mediated" (which, e.g. in IIB strings, could mean that some Kähler modulus of the Calabi-Yau gets an F term and transmits the breaking to the Standard Model superpartners) or "gauge-mediated" (in which case the breaking is at a relatively low scale, and the physics is essentially all low-energy field theory insensitive to the UV completion). These options each have their own phenomenological problems, and it is safe to say that there is no existing model of supersymmetry breaking which is completely problem-free, consistent with experiment, and theoretically well-motivated. This is one reason to explore other options. Over the past decade, a number have arisen, including large extra dimensions [1] and warped extra dimensions [2]. Large extra dimensions transmute the hierarchy of energy scales into a very large number for the volume of an extra dimension. It's not clear that this geometrization of the hierarchy problem succeeds in really solving it, but it does make exciting and exotic predictions and provides an intriguing new way of thinking about what the hierarchy means.

Warped extra dimensions, on the other hand, are essentially a much older idea in disguise. Let's remind ourselves how the hierarchy works in the strong interactions. For instance, most of the proton mass (of about 1 GeV) arises not from the fundamental masses of the quarks (which come from EWSB) but from a "constituent quark mass" supplied by strong dynamics. This scale arises from chiral symmetry breaking, which is a nonperturbative property of QCD in which the composite operator $\bar{q}q$ gets a vacuum expectation value of order Λ_{QCD}^3, where

$$\Lambda_{QCD} \sim \mu e^{-\frac{8\pi^2}{g_{YM}^2(\mu)}}. \tag{1.1}$$

The running gauge coupling $g_{YM}^2(\mu)$ evaluated at a very high scale μ is very small, leading to an exponentially small number multiplying μ to create the effective scale Λ_{QCD}. This "dimensional transmutation" means that the smallness of the proton mass is not at all mysterious (assuming light quarks from the outset). Chiral symmetry breaking itself, while nonperturbative, can be understood fairly well, as confinement tends to create massive particles but we need massless pions to supply Wess-Zumino-Witten terms that match the ultraviolet anomalies of QCD. In a number of supersymmetric theories, such nonperturbative effects can be calculated reliably, and this general picture of how to create hierarchies from small couplings is well-understood. The chiral symmetry breaking pattern is (considering only the light up and down quarks) $SU(2)_L \times SU(2)_R \to SU(2)_D$, as $\bar{q}q$ is a bifundamental of $SU(2)_L \times SU(2)_R$.

Since nature chooses to use this solution to the hierarchy in QCD, maybe it is also the right explanation of the electroweak hierarchy. This idea was originally formulated by Weinberg [3] and Susskind [4, 5] thirty years ago. It goes by the name of "technicolor": one postulates a new gauge group and estimates its properties by scaling up the color interactions of QCD to also explain EWSB. The same symmetry breaking pattern, $SU(2)_L \times SU(2)_R \to SU(2)_D$, can be used to break electroweak symmetry, if we view electroweak symmetry as weakly gauging a subgroup of the flavor symmetries of the technicolor theory.

This idea sounds very nice, but the experimental data that we have are very precise and they imply strong constraints on a technicolor scenario. For instance, one can parametrize the low-energy physics with an electroweak chiral Lagrangian, similar to the chiral Lagrangian of QCD, for the Goldstone bosons that supply the longitudinal modes of the Z and $W^{\pm}$ bosons. Let τ denote $U\tau_3U^\dagger$ and $V_\mu = (D_\mu U)U^\dagger$. We can construct various gauge-invariant operators from these building blocks together with the field strengths $W_{\mu\nu}$ and $B_{\mu\nu}$. The coefficients of these operators are given names, and constrained by experiment [6]. For example, there are the famous Peskin-Takeuchi S, T, and U parameters: T relates to the coefficient of the $\mathcal{O}(p^2)$ operator $\left(\mathrm{Tr}(\tau V_\mu)\right)^2$, while S and U occur at $\mathcal{O}(p^4)$ and correspond to the operators $B_{\mu\nu}\mathrm{Tr}(\tau W^{\mu\nu})$ and $\left(\mathrm{Tr}(\tau W_{\mu\nu})\right)^2$. The operators T and U violate custodial symmetry and one can usually avoid constraints simply by constructing custodially symmetric models, but S is nonzero whenever electroweak symmetry is broken and it typically imposes severe constraints on technicolor.

Another problem with technicolor is that, while it can quite easily explain electroweak symmetry breaking and the masses of the $W^{\pm}$ and Z bosons, explaining fermion masses is more difficult. This is because one needs to generate a Yukawa coupling, e.g. $yHQU$, but now H is really a composite operator $\bar{\psi}_{tc}\psi_{tc}$ of technicolor fermions ψ_{tc}, so this is a nonrenormalizable operator and some model is needed to explain how it is generated. An alternative is the Georgi-Kaplan mechanism, in which the fermion masses are assumed to originate by a direct coupling of some elementary fermion to a fermionic operator of the technicolor sector. As we will see, this is the approach that is at work in the Randall-Sundrum framework. Once one has a mechanism of generating fermion masses, additional constraints can apply, e.g. bounds on flavor-changing neutral currents. The combination of constraints from S and constraints from flavor seem to make the prospects for technicolor fairly bleak, and the model-building difficulties are compounded by the difficulty of calculating in strongly-coupled field theory. This last point is where Randall-Sundrum models have changed the nature of the model-building game.

1.3 Technicolor and Randall-Sundrum

The Randall-Sundrum solution to the hierarchy is to imagine that spacetime is a five-dimensional slice of anti deSitter space (AdS), cut off at two boundaries, $z = z_{IR}$ and $z = z_{UV} \ll z_{IR}$. The natural mass scale for a field localized near z_{IR} is $\frac{1}{z_{IR}}$, but the proper distance between the two boundaries is $R \log \frac{z_{IR}}{z_{UV}}$, with R the AdS curvature radius. Thus a small five-dimensional distance can easily explain an exponentially large hierarchy in energy scales. This is, in some sense, a toy version of the story of dimensional transmutation. The AdS/CFT correspondence (Reference [7]) suggests that the Randall-Sundrum scenario is dual to a conformal field theory in which conformal invariance is broken at the ultraviolet end (which corresponds to a UV cutoff on the field theory, and to coupling the field theory to dynamical gravity) and at the infrared end [8]. The infrared cutoff is essentially a simple model of confinement. The separation between z_{IR} and z_{UV} can be stabilized by the Goldberger-Wise mechanism [9], among others.

If we wish to think of Randall-Sundrum as dual, in the AdS/CFT sense, to something like a technicolor model, then the Higgs boson should be localized near z_{IR} (either on the brane, i.e. exactly at z_{IR}, or as a bulk field with a profile that grows quickly with z). On the other hand, the global symmetries of the CFT become gauge symmetries in AdS, so the technicolor global symmetry becomes a massless gauge field in the Randall-Sundrum bulk. The symmetries of the Standard Model, at least, should be weakly gauged, which can be accommodated by boundary conditions at z_{UV} that allow a zero mode to exist. If the boundary conditions at z_{IR} then give this mode a mass, we can think of it as a "Higgsless" model, which is still a form of spontaneous breaking of the global symmetry [10, 11].

The major advantage of Randall-Sundrum models over traditional technicolor scenarios is calculability. A tree-level five-dimensional calculation gives a spectrum of narrow resonances, as one expects to find in any large-N_c field theory. However, in actual four-dimensional field theories, we remain completely unable to calculate precisely what the masses, decay constants, and couplings of such narrow resonances are. In Randall-Sundrum models, all this and more is calculable.

1.4 Contents

The second chapter of this thesis, "Top and Bottom: A Brane of Their Own," concerns a variation on the Randall-Sundrum scenario in which one considers the dual of two separate strongly-coupled sectors each coupled to the Standard Model fields. Here there are two global symmetries containing the electroweak symmetry group, and the diagonal is weakly gauged. One of the strongly-coupled sectors communicates with the first two fermion generations while the other communicates with the third generation, giving some explanation for why the top quark is so much heavier than the other fermions. This model allows a solution to a problem that plagued the original Higgsless models, in which it was problematic to have a heavy enough top mass without distorting the coupling of the left-handed bottom quark to the Z boson. On the other hand, in certain regimes the "brane of their own" theory becomes difficult to calculate in.

The third chapter of this thesis, "A Braneless Approach to Holographic QCD," deals with how well a modification of the Randall-Sundrum scenario can be used to model the properties of QCD itself. (Such a model would also apply to technicolor scenarios.) A five-dimensional model is constructed in which the generation of the QCD scale (dually, of the cutoff z_{IR}) is accomplished by a potential for a scalar field in the bulk. This potential is engineered to give logarithmic running of the gauge coupling in the UV, so that the model is essentially a dual of the story of dimensional transmutation. The properties are found to be very similar to those of Randall-Sundrum. Some aspects of QCD are modeled well, but the overall spectrum of massive excitations is very different. The reasons for this are understandable: AdS/CFT works well for theories at large 't Hooft coupling, where almost every operator gets a large anomalous dimension. In the dual gravity theory this means that almost every field is a heavy string mode and can be ignored. On the other hand, in an asymptotically free theory like QCD, most operators have fairly small anomalous dimensions (at least over a wide range of energy scales) and one would expect that an accurate dual would need to keep fields corresponding to every operator. Such a theory would rapidly become intractable.

The fourth chapter of this thesis, "The S-parameter in Holographic Technicolor Models," focuses on the Peskin-Takeuchi S parameter and in particular its sign. This is one of the most dangerous constraints on the original technicolor theories and it should come as no surprise that it is dangerous for Randall-Sundrum scenarios as well. A way of tuning S away by delocalizing fermions is known; it essentially cancels a significant "pure technicolor" positive S against a negative contribution arising from the details of how the Standard Model fermions couple to fermionic operators of the technicolor sector. The question arose of whether the "pure technicolor" contribution, related to a vector minus axial spectral integral, could ever be negative. We argue that in any calculable Randall-Sundrum-like model, this contribution to S is strictly positive. Unfortunately a completely general proof of this for all field theories remains elusive.

In chapter 5 we offer a brief summary of the results and some concluding remarks about the future prospects of Randall-Sundrum models and open questions about them.

Chapter 2

Top and Bottom: A Brane of Their Own

2.1 Introduction

There has been a tremendous explosion of new models of electroweak symmetry breaking including large extra dimensions [1], Randall-Sundrum [2], gauge field Higgs [12], gauge extensions of the minimal supersymmetric standard model (MSSM) [13], little Higgs [14], and the fat Higgs [15]. All of these new models share one feature in common: a light Higgs. With the realization that unitarity can be preserved in an extra-dimensional model by Kaluza-Klein (KK) towers of gauge fields rather than a scalar [16, 10] a more radical idea has emerged: higgsless models [11, 17, 18, 19]. The most naive implementations of these models face a number of phenomenological challenges, mostly related to avoiding strong coupling while satisfying bounds from precision electroweak measurements [18, 20, 21, 22, 23, 24, 25, 26, 27, 28]. Collider signatures of higgsless models have been studied in [29, 30], further discussions on unitarity can be found in [31, 32, 33], while other ideas related to higgsless models can be found in [34, 35, 36]. However it has gradually emerged that, in a slice of 5 dimensional anti de Sitter space (AdS_5) with a large enough curvature radius and light fermions almost evenly distributed in the bulk, WW scattering is perturbative because the KK gauge bosons can be below 1 TeV, and the S parameter and most other experimental constraints are satisfied because the coupling of the light fermions to the KK gauge bosons is small [37, 38, 39]. The outstanding problem is how to obtain a large enough top quark mass without messing up the left-handed top and bottom gauge couplings or the W and Z gauge boson masses themselves. The tension arises [40, 41] because in order to get a large mass it would seem that the top quark must be close to the TeV brane where electroweak symmetry is broken by boundary conditions. However this implies that the top and (hence) the left-handed bottom have large couplings to the KK gauge bosons, and thus have large corrections to their gauge couplings. Furthermore this arrangement leads to a large amount of isospin breaking in the KK modes of the top and bottom which then feeds into the W and Z masses through vacuum polarization at one loop [40].

It was previously suggested [37] that a possible solution to this problem would be for the third generation to live in a separate AdS_5. In terms of the AdS/conformal field theory (CFT) correspondence [7, 42, 8, 44] this means that the top and bottom (as well as τ and ν_τ) would couple to a different (approximate) CFT sector than the one which provides masses to the W and Z as well as the light generations. There is a long history of models where the mechanism of electroweak symmetry breaking is different for the third generation. This is most often implemented as a Higgs boson that couples preferentially to the third generation (a.k.a. a "top-Higgs") but has also appeared in other guises such as top-color-assisted-technicolor (TC^2) where top color [45] produces the top and bottom quark masses and technicolor produces all the other masses. From the point of view of AdS/CFT, double CFT sectors have been considered for a variety of reasons. The setup is usually taken to two slices of AdS_5 "back-to-back" with a shared Planck brane. This is intended to approximately describe the situation of two strongly coupled CFT's that both couple to the same weakly coupled sector such as would arise when two conifold singularities are near each other in a higher dimensional space. The tunneling between the two AdS "wells" was considered in [46] in order to generate hierarchies. More recently inflationary models [47] have been based on one or more AdS wells. For other extra dimensional implementation of topcolor-type models see [48, 49].

In this paper we will consider models of electroweak symmetry breaking with two "back-to-back" AdS_5's in detail. The motivation for these models is to be able to separate the dynamics responsible for the large top quark mass from that giving rise to most of electroweak symmetry breaking. Thus we will assume that the light fermions propagate in an AdS_5 sector that is essentially like the higgsless model described in [37], while the third generation quarks would propagate in the new AdS_5 bulk. To analyze such theories we first discuss in detail what the appropriate boundary and matching conditions are in such models. Then we consider the different possibilities for electroweak symmetry breaking on the two IR branes (Higgs—top-Higgs, higgsless—top-Higgs and higgsless—higgsless) and derive the respective formulae for the gauge boson masses. We then discuss the CFT interpretation of all of these results. Also from the CFT interpretation we find that there have to exist uneaten light pseudo-Goldstone bosons ("top-pions") in this setup. This is due to the fact that doubling the CFT sectors implies a larger global symmetry group, while the number of broken gauge symmetries remains unchanged.

After the general discussion of models with two IR branes we focus on those that can potentially solve the issues related to the third generation quarks in higgsless models. A fairly simple way to eliminate these problems is by considering the higgsless—top-Higgs case, that is when most of electroweak symmetry breaking originates from the higgsless sector, but the top quark gets its mass from a top-Higgs on the other TeV brane (which also gives a small contribution to the W mass). The only potential issue is that since we assume the top-Higgs vacuum expectation value (VEV) v_t on this brane to be significantly smaller than the

Standard Model (SM) Higgs VEV, the top Yukawa coupling needs to be larger and thus non-perturbative. Also, the coupling of the top-pions to $t\bar{t}$ and $t\bar{b}$ will be of order m_t/v_t. Ideally, one would like to also eliminate the top-Higgs sector arising from the new TeV (IR) brane. In this case in order to get a very heavy top one needs to take the IR cutoff scale on the new side much bigger than on the old side ($\sim$ TeV), while keeping the top and bottom sufficiently far away from the new IR brane (in order to ensure that the bottom couplings are not much corrected). However, to make sure that most of the contributions to electroweak symmetry breaking are still coming from the old side one needs to choose a smaller AdS radius for the side where the top lives. In this case perturbative unitarity in WW scattering is still maintained. However, it will also imply that the new side of the 5D gauge theory is strongly coupled for all energies. Electroweak precision observables are shielded from these contributions by at least a one-loop electroweak suppression, however it is not clear that the KK spectrum of particles mostly localized on the new side would not get order one corrections and thus modify results for third generation physics significantly.

Finally we analyze some phenomenological aspects of this class of models. An interesting prediction is the presence of a scalar isotriplet (top-pions), and eventually a top-Higgs. The top-pions get a mass at loop level from gauge interactions, and the mass scale is set by the cutoff scale on the new TeV brane so that they can be quite heavy. The main feature that allows us to distinguish such scalars from the SM or MSSM Higgses is that they couple strongly with the top (and bottom) quarks, but have sensibly small couplings with the massive gauge bosons and light quarks. They are expected to be abundantly produced at the Large Hadron Collider (LHC), via the enhanced gluon or top fusion mechanisms. If heavy, the main decay channel is in (multi) top pairs, although the golden channel for the discovery is in $\gamma\gamma$ and ZZ. Thus, a heavy resonance in $\gamma\gamma$ and $l^+l^-l^+l^-$, together with an anomalously large rate of multi-top events, would be a striking hint for this models.

2.2 Warmup: Boundary Conditions for a $U(1)$ on an Interval

As an introduction to the later sections, we will in this section first present a discussion on how to obtain the boundary conditions (BCs) for a $U(1)$ gauge group on an interval, broken on both ends by two localized Higgses. The major focus will be on explaining the effects of the localized Higgs fields on the boundary conditions for the A_5 bulk field, and possible effects of mixings among the scalars, and the identification of possible uneaten physical scalar fields. We will also be allowing here a very general gauge kinetic function $\mathcal{K}(z)$ in the bulk, which will mimic both the effect of possible warping, and also the presence of a Planck brane separating two bulks with different curvatures and gauge couplings. We use a $U(1)$ so that we keep the discussion as simple as possible, while in the later sections we will use straightforward generalizations of the results obtained here for more complicated groups.

We assume that without the Higgs field being turned on the BCs are Neumann for the A_μ and Dirichlet for the A_5, as in the usual orbifold projection. This is a possible BC allowed by requiring the boundary variations of the action to be vanishing [10]. With this choice, before the Higgs VEVs are turned on, there is no zero mode for the A_5 and all the massive degrees of freedom are eaten by the massive vector KK modes. In order to be able to clearly separate the effects of the original boundary conditions from those of the localized fields added on the boundary, we will add the localized fields a small distance ϵ away from the boundary. Of course later on we will be taking the limit $\epsilon \to 0$.

Thus the Lagrangian we consider is:

$$\mathcal{L} = \int_{L_1}^{L_2} dz \left\{ -\mathcal{K}(z) \frac{1}{4g_5^2} F_{MN}^2 + \mathcal{L}_1 \delta(z - L_1 - \epsilon) + \mathcal{L}_2 \delta(z - L_2 + \epsilon) \right\} . \quad (2.1)$$

As explained above, the generic function $\mathcal{K}$ encodes both the eventual warping of the space and a possible z-dependent kinetic term corresponding to a different g_5 on the two sides of the Planck brane. We also assume that any eventual discontinuity is regularized such that $\mathcal{K}$ is continuous and non-vanishing.

The localized Lagrangians are the usual Lagrangians for the Higgs field in 4D [1] $(i = 1, 2)$:

$$\mathcal{L}_i = |\mathcal{D}_\mu \phi_i|^2 - \frac{\lambda_i}{2} \left(|\phi_i|^2 - \frac{1}{2} v_i^2 \right)^2 \quad (2.2)$$

and they will induce non vanishing VEVs v_i for the Higgses, around which we expand:

$$\phi_i = \frac{1}{\sqrt{2}} (v_i + h_i) \, e^{i\pi_i / v_i} \, . \quad (2.3)$$

The above Lagrangian contains some mixing terms involving A_μ that we want to cancel out with a generalized R_ξ gauge fixing term. Expanding up to bilinear terms:

$$\begin{aligned} \mathcal{L} = \int_{L_1}^{L_2} dz \, \Bigg\{ & \frac{\mathcal{K}(z)}{g_5^2} \left(-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_z A_\mu)^2 + \frac{1}{2} (\partial_\mu A_5)^2 - \partial_\mu A_5 \partial_5 A^\mu \right) \\ & + \left[\frac{1}{2} (\partial_\mu h_1)^2 - \frac{1}{2} \lambda_1 v_1^2 h_1^2 + \frac{1}{2} (\partial_\mu \pi_1 - v_1 A_\mu)^2 + ... \right] \delta(z - L_1 - \epsilon) \\ & + \left[\frac{1}{2} (\partial_\mu h_2)^2 - \frac{1}{2} \lambda_2 v_2^2 h_2^2 + \frac{1}{2} (\partial_\mu \pi_2 - v_2 A_\mu)^2 + ... \right] \delta(z - L_2 + \epsilon) \Bigg\} . \end{aligned} \quad (2.4)$$

[1] Note that warp factors are usually added in the localized lagrangians, so that the scale v is naturally of order $1/R$, and λ of order 1. Such factors are not relevant for the discussion at this point, so we will neglect them for the moment.

The crucial point now is the integration by parts of the mixing term in the bulk. As a consequence of the displacement of the localized Lagrangians $\mathcal{L}_{1,2}$, the integral splits into three regions limited by the regularized branes where the Higgs interactions are localized. The contributions of the edges vanish in the limit $\epsilon \to 0$, so that the BC on the true boundaries are effectively "screened", and a mixing between A_μ and A_5 on the branes is generated:

$$\left(\int_{L_1}^{L_1+\epsilon} + \int_{L_1+\epsilon}^{L_2-\epsilon} + \int_{L_2-\epsilon}^{L_2}\right) dz\, \mathcal{K} A_5 \partial_5 \partial_\mu A^\mu =$$
$$\left(\int_{L_1}^{L_1+\epsilon} + \int_{L_2-\epsilon}^{L_2}\right) dz(...) - \int_{L_1+\epsilon}^{L_2-\epsilon} dz\, \partial_5 \left(\mathcal{K} A_5\right) \partial_\mu A^\mu + \left[\mathcal{K} A_5 \partial_\mu A^\mu\right]_{L_1+\epsilon}^{L_2-\epsilon} . \quad (2.5)$$

Note that the boundary terms would vanish on the true boundaries $L_{1,2}$, however they don't vanish on the branes at $L_1+\epsilon$ and $L_2-\epsilon$, and thus the minimization of the action will require that A_5 has to be non-zero on the branes. In other words, in the limit $\epsilon \to 0$ the A_5 field will develop a discontinuity on the boundaries.

We can now add a bulk and two brane gauge fixing Lagrangians[2]:

$$\mathcal{L}_{GF} = \int_{L_1}^{L_2} dz \left\{ -\frac{1}{g_5^2}\frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi \partial_z \left(\mathcal{K} A_5\right)\right)^2 - \frac{1}{2\xi_1}\left(\partial_\mu A^\mu + \xi_1 \left(v_1 \pi_1 - \frac{\mathcal{K}}{g_5^2} A_5\right)\right)^2 \cdot \right.$$
$$\left. \delta(z-L_1) - \frac{1}{2\xi_2}\left(\partial_\mu A^\mu + \xi_2 \left(v_2 \pi_2 + \frac{\mathcal{K}}{g_5^2} A_5\right)\right)^2 \delta(z-L_2) \right\} , \quad (2.6)$$

where the three gauge fixing parameters are completely arbitrary, and the unitary gauge is realized in the limit where all the ξ's are sent to infinity.[3] This gauge fixing term is devised such that all the mixing terms between A_μ and $A_5, \pi_{1,2}$ cancel out. The full Lagrangian then leads to the following equation of motion for A_μ (in the

[2]In [50] the authors considered a similar situation, but adding gauge fixing terms in the KK basis. As it has to be expected, our approach leads to the same results.

[3]Note that we have chosen a bulk gauge fixing term with a different z dependence than the bulk gauge kinetic term, i.e., we have included a z dependence in the gauge fixing parameter ξ. This allows us to obtain a simple equation of motion for A_5, at the price of a non-conventional form of the gauge propagator that will not be well suited for a warped space loop calculation in general ξ gauge. In the unitary gauge that we will use in this paper, all the ξ dependence vanishes (and thus also the z dependence of ξ will be irrelevant).

unitary gauge):

$$\frac{1}{\mathcal{K}}\partial_z\left(\mathcal{K}\partial_z A_\mu + m^2 A_\mu\right) = 0\,, \tag{2.7}$$

while the BCs, fixed by requiring the vanishing of the boundary variation terms in Eq. (2.4), are:

$$\frac{\mathcal{K}(L_{1,2})}{g_5^2}\partial_z A_\mu \mp v_{1,2}^2 A_\mu = 0\,. \tag{2.8}$$

The bulk equation of motion for the scalar field A_5 will result in:

$$\partial_z^2\left(\mathcal{K}A_5\right) + \frac{m^2}{\xi}A_5 = 0\ , \tag{2.9}$$

and the π's obey the following equations of motion on the branes:

$$\begin{aligned} \left(\tfrac{m^2}{\xi_1} - v_1^2\right)\pi_1 + v_1\tfrac{\mathcal{K}(L_1)}{g_5^2}\ A_5\big|_{L_1} = 0\,, \\ \left(\tfrac{m^2}{\xi_2} - v_2^2\right)\pi_2 - v_2\tfrac{\mathcal{K}(L_2)}{g_5^2}\ A_5\big|_{L_2} = 0\,. \end{aligned} \tag{2.10}$$

These last equations fix the values of $\pi_{1,2}$ in terms of the boundary values of A_5. Finally, requiring that the boundary variations of the full action with respect to A_5 vanish, combined with the above expression (2.10) for the π's, will give the desired BCs for A_5:

$$\left\{\begin{aligned} \partial_z\left(\mathcal{K}A_5\right) - \frac{\xi_1}{\xi}\frac{\mathcal{K}(L_1)}{g_5^2}\frac{m^2/\xi_1}{m^2/\xi_1 - v_1^2}A_5\bigg|_{L_1} = 0\,, \\ \partial_z\left(\mathcal{K}A_5\right) + \frac{\xi_2}{\xi}\frac{\mathcal{K}(L_2)}{g_5^2}\frac{m^2/\xi_2}{m^2/\xi_2 - v_2^2}A_5\bigg|_{L_2} = 0\,. \end{aligned}\right. \tag{2.11}$$

From Eq. (2.10) one can see that π is not independent of A_5. In the unitary gauge ($\xi \to \infty$) it is also clear from Eq. (2.9) that all the massive modes of A_5 are removed. This is simply expressing the fact that A_5 and the π's are the sources of the longitudinal components of the massive KK modes, and will be eaten. The only possible exception for the existence of a physical mode is if there is a massless state in A_5. Without the Higgses on the boundaries this would not be possible due to the Dirichlet BC. However, we have seen above that the BC for the A_5 is significantly changed in the presence of the localized Higgs, and now a massless state is possible. Physically, this expresses the fact that there are "enough" modes in A_5 to provide all the longitudinal components for the massive KK modes. If we add some localized Higgs fields, then there may be some massless modes left over uneaten. We will loosely refer to such modes as pions, to emphasize that these are physical (pseudo-)Goldstone bosons. In the case of a massless physical pion mode, the BCs for A_5 simplify to:

$$\partial_z \left(\mathcal{K} A_5\right)\big|_{L_1 \text{ and } L_2} = 0\,. \tag{2.12}$$

The solution to the equation of motion for the zero mode is of the form:

$$A_5 = \frac{g_5^2 d}{\mathcal{K}(z)}, \tag{2.13}$$

and using (2.10) we also get that

$$\pi_1 = \frac{d}{v_1}, \quad \pi_2 = -\frac{d}{v_2}\,. \tag{2.14}$$

As expected in the higgsless limit (namely $v_i \to \infty$), the π's vanish. However, a massless scalar is still left in the spectrum if one chooses to break the gauge symmetry on both ends of the interval. It is also interesting to note that in the limit where the function $\mathcal{K}$ is discontinuous, the solution A_5 develops a discontinuity as well. But, the function $\mathcal{K} A_5$ is still continuous, so that no divergent term appears in the action for such a solution.

Finally, the spectrum also contains two scalars localized on the branes, corresponding to the physical Higgs fields. As in the usual 4D Higgs mechanism, they will pick a mass proportional to the quartic coupling, $m_{h1,2}^2 = \lambda v_{1,2}^2$ (and decouple in the higgsless limits $v_i \to \infty$).

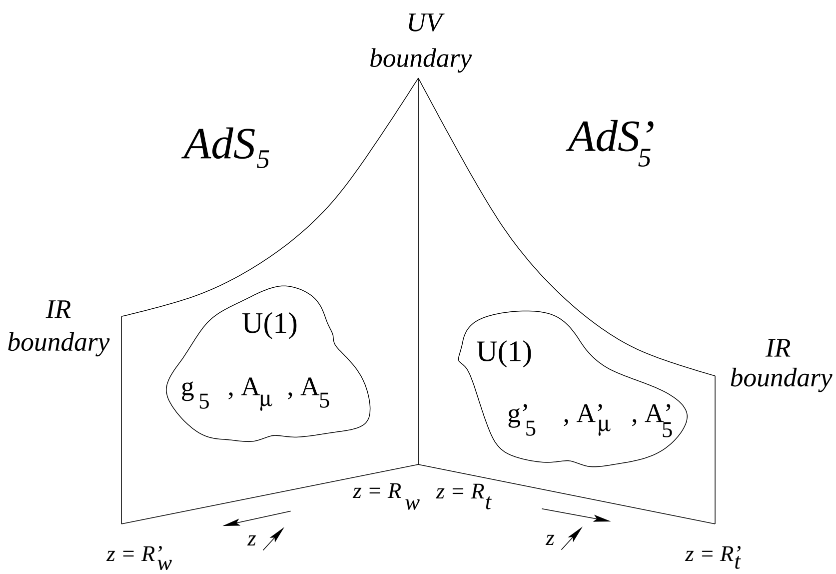

Figure 2.1: Schematic view of the double AdS space that we consider.

2.2.1 Double AdS case

The physical setup we are actually interested in consists of two AdS_5 spaces intersecting along a codimension one surface (Planck brane) that would serve as a UV cutoff of the two AdS spaces. The whole picture can be seen as two Randall-Sundrum (RS) models glued together along their Planck boundary, as in Figure 2.1. The two AdS spaces are characterized by their own curvature scale, R_w and R_t. We define two conformal coordinate systems on the two spaces, namely ($i = w, t$):

$$ds^2 = \left(\frac{R_i}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right). \tag{2.15}$$

The common UV boundary is located at the point $z = R_i$ in the coordinate system associated to each brane. Each AdS space is also cut by an IR boundary located respectively at $z = R'_w$ and $z = R'_t$.

Alternatively, we can also think of the two AdS spaces as an interval with boundaries given by the IR branes, and the Planck brane as a singular point in the bulk. From this point of view we can apply the formalism developed above, in particular we can define the function $\mathcal{K}$ in the two spaces:

$$\mathcal{K} = \begin{cases} \dfrac{R_w}{z} & \text{for} \quad R_w \leq z \leq R'_w \,, \\ \dfrac{g_5^2}{g_5'^2} \dfrac{R_t}{z'} & \text{for} \quad R_t \leq z' \leq R'_t \,, \end{cases} \tag{2.16}$$

where we have allowed for a different value of the bulk gauge coupling on the two sides if $g_5 \neq g'_5$. In order to maintain the traditional form of the metric in both sides of the bulk, we have chosen a peculiar coordinate system where z is growing from the Planck brane towards both the left and the right. This way most formulae from RS physics will have simple generalizations, however it will also imply some unexpected extra minus signs. Note that if $g_5 = g'_5$, the function $\mathcal{K}$ is continuous on the Planck brane. However, a discontinuity is generated if we define different gauge couplings on the two sides. As noted before, the only effect of such choice will be a discontinuity in the wave function of the scalar field A_5 on the Planck brane.

The Lagrangians localized on the two IR branes that cut the two spaces are ($i = w, t$):

$$\mathcal{L}_i = \left(\frac{R_i}{R'_i} \right)^2 \left\{ |\mathcal{D}_\mu \phi_i|^2 - \left(\frac{R_i}{R'_i} \right)^2 \frac{\lambda_i}{2} \left(|\phi_i|^2 - \frac{1}{2} v_i^2 \right)^2 \right\} , \tag{2.17}$$

where, introducing the above warp factors, all the scales and constants have natural values, namely $\lambda_i \sim 1$ and $v_i \sim 1/R_i$.

For the vectors, Eq. (2.7) reduces to the usual RS equation of motion in the two bulks, with BCs given by the mass terms on the respective IR branes:

$$\partial_z A_\mu(z)\big|_{R'_w} + \frac{g_5^2}{R_w} \frac{(v_w R_w)^2}{R'_w} A_\mu(R'_w) \quad = \quad 0 \,, \tag{2.18}$$

$$\partial_z A'_\mu(z)\big|_{R'_t} + \frac{g_5'^2}{R_t} \frac{(v_t R_t)^2}{R'_t} A'_\mu(R'_t) \quad = \quad 0 \,. \tag{2.19}$$

The equation of motion (2.7) is satisfied on the Planck brane as well, so we can translate it into matching conditions for the solutions in the two bulks. In particular, assuming that there are no interactions localized on the Planck brane, Eq. (2.7) implies that the functions $A_\mu(z)$ and $\mathcal{K}(z)\partial_z A_\mu(z)$ are continuous:

$$
\begin{aligned}
A_\mu(R_w) &= A'_\mu(R_t)\,, && (2.20)\\
\frac{1}{g_5^2}\,\partial_z A_\mu(z)\big|_{R_w} &= -\frac{1}{{g'_5}^2}\,\partial_z A'_\mu(z)\big|_{R_t}\,. && (2.21)
\end{aligned}
$$

The minus sign in the derivative matching Eq. (2.21) comes from the coordinate systems that we are using. Note that the condition (2.21) would be modified by localized terms: for instance, if we add a mass term for A_μ, generated by a localized Higgs, the matching becomes:

$$
\frac{1}{g_5^2}\,\partial_z A_\mu(z)\big|_{R_w} + \frac{1}{{g'_5}^2}\,\partial_z A'_\mu(z)\big|_{R_t} + v_{Planck}^2 A_\mu = 0\,. \qquad (2.22)
$$

In particular, in the large VEV limit, it is equivalent to the vanishing of A_μ (and of A'_μ).

Let us next consider the equations determining possible massless scalar pion modes. As discussed above, from Eq. (2.9) we can read off that the continuity condition has to be applied to the functions $\mathcal{K}(z)A_5(z)$ and $\partial_z(\mathcal{K}(z)A_5(z))$:

$$
\begin{aligned}
\frac{1}{g_5^2}A_5(R_w) &= \frac{1}{{g'_5}^2}A'_5(R_t)\,, && (2.23)\\
\frac{R_w}{g_5^2}\,\partial_z\frac{A_5(z)}{z}\bigg|_{R_w} &= -\frac{R_t}{{g'_5}^2}\,\partial_z\frac{A'_5(z)}{z}\bigg|_{R_t}\,. && (2.24)
\end{aligned}
$$

The solution then is:

$$
\begin{aligned}
A_5(z) &= g_5^2 d \tfrac{z}{R_w}\,, \qquad & \pi_w &= -\left(\tfrac{R'_w}{R_w}\right)^2 \tfrac{d}{v_w}\,,\\
A'_5(z') &= {g'_5}^2 d \tfrac{z'}{R_t}\,, \qquad & \pi_t &= -\left(\tfrac{R'_t}{R_t}\right)^2 \tfrac{d}{v_t}\,.
\end{aligned} \qquad (2.25)
$$

Finally, the two Higgses localized on the IR branes get a mass proportional to the quartic couplings in the potentials:

$$m_{h_w}^2 = \lambda_w \frac{(v_w R_w)^2}{R_w'^2}\,, \quad m_{h_t}^2 = \lambda_t \frac{(v_t R_t)^2}{R_t'^2}\,. \tag{2.26}$$

Their masses are naturally of order $R_w'^{-1}$ or $R_t'^{-1}$ respectively, and have an upper bound given by the breakdown of perturbative unitarity. Such limits will be much looser than in the SM, due to the contribution of the gauge boson KK modes, as we will see in the following sections.

2.3 The Standard Model in Two Bulks: Gauge Sector

We will now use our general formalism developed above to put the Standard Model in two AdS_5 bulks. Our goal will be to separate electroweak symmetry breaking from the physics of the third generation fermions. In each AdS bulk, we have a full $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, so that a custodial symmetry is protecting [40] the ρ parameter. We want to specify the boundary and matching conditions according to the following symmetry breaking pattern: on the common UV brane only $SU(2)_L \times U(1)_Y$ survives, while on the IR boundaries two Higgses break the gauge group to the $SU(2)_D \times U(1)_{B-L}$ subgroup.

The UV brane matching conditions arise from considering an $SU(2)_R$ scalar doublet with a $B-L$ charge $1/2$, that acquires a VEV $(0, v)$. As discussed above, all of the gauge fields A_μ^L, A_μ^R, and B_μ are continuous. The Higgs, in the limit of large VEV, forces the gauge bosons of the broken generators to vanish on the Planck brane: $A_\mu^{R1,2} = 0$ and $B_\mu - A_\mu^{R3} = 0$, thus breaking $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. On the other hand, on the unbroken gauge fields A_μ^{La} and ($g_{5L}, g_{5R}, \tilde{g}_5$ are the 5D gauge couplings of $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$ and $\mathrm{U}(1)_{B-L}$)

$$B^Y_\mu = \frac{g_{5R}^2 \tilde{g}_5^2}{g_{5R}^2 + \tilde{g}_5^2} \left(\frac{1}{g_{5R}^2} A^{R3}_\mu + \frac{1}{\tilde{g}_5^2} B_\mu \right) , \tag{2.27}$$

we need to impose the continuity condition in Eq. (2.21). The complete set of Planck brane matching conditions then reads

$$\text{at} \quad \begin{matrix} z = R_w \\ z' = R_t \end{matrix} \quad \begin{cases} A^{L\,a}_\mu = A'^{L\,a}_\mu \, , A^{R\,a}_\mu = A'^{R\,a}_\mu \, , B_\mu = B'_\mu \, , \\ A^{R\,1,2}_\mu = 0 \, , B_\mu - A^{R\,3}_\mu = 0 \, , \\ \partial_z A^{L\,a}_\mu + \partial_z A'^{L\,a}_\mu = 0 \, , \\ \frac{1}{g_{5R}^2} \partial_z A^{R\,3}_\mu + \frac{1}{g_{5R}^2} \partial_z A'^{R\,3}_\mu + \frac{1}{\tilde{g}_5^2} \partial_z B_\mu + \frac{1}{\tilde{g}_5^2} \partial_z B'_\mu = 0 \, . \end{cases} \tag{2.28}$$

Here we are assuming equal 5D gauge couplings on the two sides for simplicity. In case of different g_5's, Eqs. (2.28) will be modified according to the rules given in the previous section. However, this restricted set of parameters is sufficient for our purposes, because, as we will comment later, in the gauge sector the effect of different 5D couplings is equivalent to different AdS curvature scales on the two sides.

Regarding the IR breaking, we will study three limits: that in which it comes from a Higgs on both IR branes ("Higgs—top-Higgs"), that in which most electroweak symmetry breaking is from a higgsless breaking while the top gets its mass from a Higgs ("higgsless—top-Higgs"), and that in which we have higgsless boundary conditions on both IR branes ("higgsless—higgsless").

For future convenience, we first recall the expressions for M_W in the Randall-Sundrum scenario with an IR brane Higgs [2, 51] and in the higgsless case [11], in the limit $g_{5L} = g_{5R}$ at leading log order:

$$M^2_{W;Higgs} \approx \frac{g_5^2}{R \log \frac{R'}{R}} \frac{R^2 v^2}{4R'^2} \, , \tag{2.29}$$

$$M^2_{W;higgsless} \approx \frac{1}{R'^2 \log \frac{R'}{R}} \, . \tag{2.30}$$

Also, independent of the IR brane boundary conditions, we can express the 4D gauge couplings in terms of the 5D parameters:

$$\frac{1}{e^2} = \left(\frac{1}{g_{5L}^2} + \frac{1}{g_{5R}^2} + \frac{1}{\tilde{g}_5^2}\right)\left(R_w \log\frac{R'_w}{R_w} + R_t \log\frac{R'_t}{R_t}\right), \tag{2.31}$$

$$\tan\theta_W^2 = \frac{g_{5R}^2 \tilde{g}_5^2}{g_{5L}^2 (g_{5R}^2 + \tilde{g}_5^2)}, \tag{2.32}$$

$$g^2 = \frac{g_{5L}^2}{R_w \log\frac{R'_w}{R_w} + R_t \log\frac{R'_t}{R_t}}. \tag{2.33}$$

Note that the the 5D gauge couplings are related to the 4D ones via the total volume of the space, namely the sum of the two AdS spaces. Moreover, as we have two sets of "Planck" and "TeV" scales, we are assuming the logs to be of the same order in the expansion.

2.3.1 Higgs—top-Higgs

We first assume that on each IR brane we have a scalar Higgs field, transforming as a bifundamental under $SU(2)_L \times SU(2)_R$. They develop VEVs v_w and v_t that break this group to $SU(2)_D$, leaving $U(1)_{B-L}$ unbroken. We also take the localized Lagrangians in the form (2.17), so that $v_w \sim 1/R_w$ and $v_t \sim 1/R_t$. The VEVs will generate a mass term for the combination $A_\mu^L - A_\mu^R$, while the fields related to the unbroken subgroup

$$A_\mu^{Da} = \frac{g_{5L}^2 g_{5R}^2}{g_{5L}^2 + g_{5R}^2}\left(\tfrac{1}{g_{5L}^2} A_\mu^{La} + \tfrac{1}{g_{5R}^2} A_\mu^{Ra}\right), \tag{2.34}$$

and B_μ have Neumann BCs. The complete set of BCs is:

$$\text{at } z = R'_w \quad \begin{cases} \partial_z(\frac{1}{g_{5L}^2} A_\mu^{L\,a} + \frac{1}{g_{5R}^2} A_\mu^{R\,a}) = 0\,, \\ \partial_z(A_\mu^{L\,a} - A_\mu^{R\,a}) = -\frac{v_w^2}{4}\frac{R_w}{R'_w}\,(g_{5L}^2 + g_{5R}^2)\,(A_\mu^{L\,a} - A_\mu^{R\,a})\,, \\ \partial_z B_\mu = 0\,, \end{cases} \tag{2.35}$$

and similarly on the other IR brane.

To determine the spectrum in this case, we use the expansion of the Bessel function for small argument (assuming $M_A R' \ll 1$),

$$\psi^{(A)}(z) \approx c_0^{(A)} + M_A^2 z^2 \left(c_1^{(A)} - \frac{c_0^{(A)}}{2} \log \frac{z}{R} \right) \tag{2.36}$$

and solve (much as in [11]). We assume that $v_i R_i$ is small. We find for the W mass:

$$\begin{aligned} M_W^2 &\approx \frac{g_{5L}^2}{R_w \log \frac{R'_w}{R_w} + R_t \log \frac{R'_t}{R_t}} \left(\left(\frac{R_w}{R'_w} \right)^2 \frac{v_w^2}{4} + \left(\frac{R_t}{R'_t} \right)^2 \frac{v_t^2}{4} \right) \\ &= \frac{g^2}{4} \left(\left(\frac{R_w}{R'_w} \right)^2 v_w^2 + \left(\frac{R_t}{R'_t} \right)^2 v_t^2 \right) . \end{aligned} \tag{2.37}$$

This is of the form one would expect for a gauge boson obtaining its mass from two Higgs bosons. Note that the natural scale of the two contributions is $\frac{1}{R_w'^2}$ and $\frac{1}{R_t'^2}$ respectively.

We will not discuss this case at any length, as viable Randall-Sundrum models with Higgs boson exist [40]. We simply note that one can construct a variety of models analogous to two-Higgs doublet models, in which one has distinct KK spectra for particles coupling to different Higgs bosons.

2.3.2 Higgsless—top-Higgs

Next we consider a case in which the IR brane at R'_w has a higgsless boundary condition and is responsible for most of the electroweak symmetry breaking, while a top-Higgs on the brane at R'_t makes some smaller contribution to electroweak symmetry breaking. In this case we have distinct BC's:

$$\text{at } z = R'_w \quad \begin{cases} \partial_z(\frac{1}{g_{5L}^2} A_\mu^{L\,a} + \frac{1}{g_{5R}^2} A_\mu^{R\,a}) = 0\,, \\ A_\mu^{L\,a} - A_\mu^{R\,a} = 0,\ \partial_z B_\mu = 0\,, \end{cases} \tag{2.38}$$

while at $z = R'_t$ we have the same BCs as in Eq. (2.35).

Solving as above, we find:

$$\begin{aligned} M_W^2 &\approx \frac{g_{5L}^2}{R_w \log \frac{R'_w}{R_w} + R_t \log \frac{R'_t}{R_t}} \left(\frac{R_w}{R'^2_w} \frac{2}{g_{5L}^2 + g_{5R}^2} + \left(\frac{R_t}{R'_t}\right)^2 \frac{v_t^2}{4} \right) \\ &= g^2 \left(\frac{2R_w}{g_{5L}^2 + g_{5R}^2} \frac{1}{R'^2_w} + \left(\frac{R_t}{R'_t}\right)^2 \frac{v_t^2}{4} \right). \end{aligned} \tag{2.39}$$

This again takes the form of a sum of squares, with one term of the form found in the usual higgsless models (2.30) and one term in the form of an ordinary contribution from a Higgs VEV (2.29). Our boundary condition is a limit as $v_w \to \infty$ of the previous case, so we expect that for intermediate values of v_w, its contribution will level off smoothly to a constant value.

If we want to disentangle the top mass from the electroweak symmetry breaking sector, we assume that $v_t R_t \ll 1$ so that the Wmass comes mostly from the higgsless AdS. In this case the perturbative unitarity in longitudinal Wscattering will be restored by the gauge boson resonances. The physical top-Higgs, arising from the AdS$_t$, will not contribute much to it and will mostly couple to the top quark. We will come back to the physics of the third generation in Section 2.6.

2.3.3 Higgsless—higgsless

Finally, we consider a case in which both IR branes have higgsless boundary conditions Eq (2.38). In this case, as expected, we find:

$$\begin{aligned} M_W^2 &\approx \frac{g_{5L}^2}{R_w \log \frac{R'_w}{R_w} + R_t \log \frac{R'_t}{R_t}} \frac{2}{g_{5L}^2 + g_{5R}^2} \left(\frac{R_w}{R_w'^2} + \frac{R_t}{R_t'^2} \right) \\ &= g^2 \left(\frac{2R_w}{g_{5L}^2 + g_{5R}^2} \frac{1}{R_w'^2} + \frac{2R_t}{g_{5L}^2 + g_{5R}^2} \frac{1}{R_t'^2} \right), \end{aligned} \tag{2.40}$$

where we have grouped the terms for later convenience in discussing the holographic interpretation. Note that in the symmetric limit $R_w = R_t = R$, $R'_t = R'_w = R'$, we recover the usual (one bulk) higgsless result (2.30). Our expression can be reformulated in another useful way:

$$M_W^2 = \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \left(\frac{1}{R_w'^2} + \frac{R_t}{R_w} \frac{1}{R_t'^2} \right) \frac{1}{\log \frac{R'_w}{R_w} + \frac{R_t}{R_w} \log \frac{R'_t}{R_t}}, \tag{2.41}$$

In this formulation there is a manifest limit where the contribution of the AdS_t is small, namely if the volume of the new space is smaller that the volume of the old one: $R_t \ll R_w$. In this case the contribution of the top-sector to M_W is suppressed. It is interesting to note that this property is actually related to the relative size of the 5D gauge coupling and the warping factor. From the matching conditions, we find that g_5^2 is of order to the total volume of the space. The limit we are interested in is in fact when the 5D gauge coupling is larger than the warp factor in the second AdS, namely $g_5^2 \approx R_w \gg R_t$. On the other hand, if we assume that there also are different gauge couplings on the two spaces, each one of the order of the local curvature, the decoupling effect disappears. In this sense, at the level of the gauge bosons, a hierarchy between the curvatures is equivalent to a hierarchy between the bulk gauge couplings.

It is also interesting to study the spectrum of the resonances: at leading order in the log-expansion, they decouple into two towers of states proportional to the two IR scales. Namely:

$$M_{w'}^{(n)} \approx \frac{\mu_{0,1}^{(n)}}{R'_w}\,, \quad M_{t'}^{(n)} \approx \frac{\mu_{0,1}^{(n)}}{R'_t}\,, \tag{2.42}$$

where the numbers $\mu_{0,1}^{(n)}$ are respectively the zeros of the Bessel functions $J_0(x)$ and $J_1(x)$. This is true irrespective of the BC's on the TeV brane. In the higgsless case, with $R_t \ll R_w$, the tower of states proportional to $1/R'_w$ receives corrections suppressed by a log:

$$\begin{aligned} M_{(0)}^{(n)} &\approx \tfrac{1}{R'_w}\left(\mu_0^{(n)} + \tfrac{\pi}{2}\tfrac{g_{5R}^2}{g_{5L}^2+g_{5R}^2}\tfrac{Y_0(\mu_0^{(n)})}{J_1(\mu_0^{(n)})}\tfrac{1}{\log\frac{R'_w}{R_w}} + \mathcal{O}(\log^{-2})\right)\,, \\ M_{(1)}^{(n)} &\approx \tfrac{1}{R'_w}\left(\mu_1^{(n)} + \tfrac{\pi}{2}\tfrac{g_{5L}^2}{g_{5L}^2+g_{5R}^2}\tfrac{Y_1(\mu_1^{(n)})}{J_2(\mu_1^{(n)})-J_0(\mu_1^{(n)})}\tfrac{1}{\log\frac{R'_w}{R_w}} + \mathcal{O}(\log^{-2})\right)\,. \end{aligned} \tag{2.43}$$

This is equivalent to the states of a one brane model, up to corrections suppressed by R_t/R_w. On the other hand, the corrections to the tower proportional to $1/R'_t$ are always suppressed by R_t/R_w.

Finally, we can compute the oblique observables. Due to the custodial symmetry, we find $T \approx 0$, and for the case when the light fermions are localized close to the Planck brane the S-parameter is:

$$S \approx \frac{6\pi}{g^2}\frac{2g_{5L}^2}{g_{5L}^2+g_{5R}^2}\frac{R_w+R_t}{R_w\log\frac{R'_w}{R_w}+R_t\log\frac{R'_t}{R_t}}\,. \tag{2.44}$$

Also in this case, the contribution from the additional AdS_t is suppressed by the ratio R_t/R_w. Just as for the simple higgsless case the contribution to the S-parameter can be suppressed by moving the light fermions into the bulk and thus reducing their couplings to the KK gauge bosons [37, 38].

2.4 The CFT Interpretation

We would now like to interpret the formulae in the previous section in the 4D CFT language. Since we now have two bulks and two IR branes, it is natural to assume that there would be two separate CFT's corresponding to this system. Each of these CFT's has its own set of global symmetries, given by the gauge fields in each of the bulks. The gauge fields which vanish on the Planck brane correspond to genuine global symmetries, however the ones that are allowed to propagate through the UV brane will be weakly gauged. Since there is only one set of light (massless) modes for these fields, clearly only the diagonal subgroup of the two independent global symmetries of the two CFT's will be gauged.

The first test for this interpretation is in the spectrum of the KK modes. Indeed, in the limit where we remove the Planck brane, the only object that links the two AdS spaces, we should find two independent towers of states, each one given by the bound states of the two CFT's and with masses proportional to the two IR scales. This is exactly the structure we found in Eq. (2.42). Moreover, the W boson gets its mass via a mixing with the tower of KK modes, so that it can be interpreted as a mixture of the elementary field and the bound states. This mixing also introduces corrections to the simple spectrum described above, suppressed by the log. In the limit $R_t \ll R_w$ the W mass comes from the AdS$_w$ side, so that we expect large corrections to the states with mass proportional to $1/R'_w$ and small corrections to the states with mass $\sim 1/R'_t$. This is confirmed by Eqs. (2.43).

Let us now discuss in detail the interpretation of the electroweak symmetry breaking mechanisms described in the previous section. We know that the interpretation of a IR brane Higgs field is that the CFT is forming a composite scalar bound state which then triggers electroweak symmetry breaking, while the interpretation of the higgsless boundary conditions is that the CFT forms a condensate that gives rise to electroweak symmetry breaking (but no composite scalar). Thus these latter models can be viewed as extra dimensional duals of technicolor type models. What happens when we have the setup with two IR branes? Each of the two CFT's will break its own global $SU(2)_L \times SU(2)_R$ symmetries to the diagonal subgroup either via the composite Higgs or via the condensate. We can easily test these conjectures by deriving, just based on this correspondence, the formulae obtained in the previous section via explicitly solving the 5D equations of motion.

In order to be able to do that we need to find the explicit expression for the pion decay constant f_π of these CFT's. This can be most easily found by comparing the generic expression of the W-mass in higgsless models

$$M_W^2 = \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \frac{1}{R'^2 \log \frac{R'}{R}} \tag{2.45}$$

with the expression for the W-mass in a generic technicolor model with a single condensate

$$M_W^2 = g^2 f_\pi^2 \; . \tag{2.46}$$

Using the tree-level relation between g and g_{5L} in RS-type models we find that

$$f_\pi^2 = \frac{2R}{g_{5L}^2 + g_{5R}^2} \frac{1}{R'^2}. \tag{2.47}$$

The usual interpretation of this formula [21] in terms of large-N QCD theories is by comparing it to the relation

$$f_\pi \sim \frac{\sqrt{N}}{4\pi} m_\rho, \tag{2.48}$$

where m_ρ is the characteristic mass of the techni-hadrons, and N is the number of colors. In our case $m_\rho \sim 1/R'$, so the number of colors would be given by

$$N \sim \frac{32\pi^2 R}{g_{5L}^2 + g_{5R}^2}. \tag{2.49}$$

Note that one will start deviating from the large N limit once $\frac{R}{g_{5L}^2+g_{5R}^2} \ll 1$.

For the case when there is a composite Higgs the effective VEV (as always in RS-type models) is nothing but the warped-down version of the Higgs VEV

$$v_{\mathit{eff}} = v\frac{R}{R'} \ . \tag{2.50}$$

Now we can use this formula to derive expressions for the $W$and Z masses in the general cases with two IR branes. Based on our correspondence both of the CFT's break the gauge symmetry, either via a composite Higgs or via the condensate. Since the gauge group is the diagonal subgroup of the two global symmetries, we simply need to add up the contribution of the two CFT's. So for the Higgs—top-Higgs case we would expect

$$M_W^2 = \frac{g^2}{4}(v_{\mathit{eff},w}^2 + v_{\mathit{eff},t}^2) = \frac{g^2}{4}\left(\left(\frac{R_w}{R'_w}\right)^2 v_w^2 + \left(\frac{R_t}{R'_t}\right)^2 v_t^2\right) \ , \tag{2.51}$$

which is in agreement with (2.37). In the mixed higgsless—top-Higgs case we expect the Wmass to be given by

$$M_W^2 = g^2(f_{\pi,w}^2 + \frac{v_{\mathit{eff},t}^2}{4}) = g^2\left(\frac{2R_w}{g_{5L}^2 + g_{5R}^2}\frac{1}{R'^2_w} + \left(\frac{R_t}{R'_t}\right)^2 \frac{v_t^2}{4}\right) \ , \tag{2.52}$$

which is again in agreement with Eq. (2.39). Finally, in the higgsless—higgsless case we expect the W mass to be given by

$$M_W^2 = g^2(f_{\pi,w}^2 + f_{\pi,t}^2) = g^2\left(\frac{2R_w}{g_{5L}^2 + g_{5R}^2}\frac{1}{R'^2_w} + \frac{2R_t}{g_{5L}^2 + g_{5R}^2}\frac{1}{R'^2_t}\right) \ , \tag{2.53}$$

again in agreement with the result of the explicit calculation (2.40).

2.5 Top-pions

2.5.1 Top-pions from the CFT correspondence

We can see from the match of the expressions of the W masses above that the CFT picture is reliable. However, the CFT picture has one additional very important prediction for this model: the existence of light pseudo-Goldstone bosons, which are usually referred to as top-pions in the topcolor literature. The emergence of these can be easily seen from the gauge and global symmetry breaking structure. We have seen that there are two separate CFT's, each of which has its own $SU(2)_L \times SU(2)_R$ global symmetry. Only the diagonal $SU(2)_L \times U(1)_Y$ is gauged. Both CFT's will break their respective global symmetries as $SU(2)_L \times SU(2)_R \to SU(2)_D$. Thus both CFT's will produce three Goldstone bosons, while the gauge symmetry breaking pattern is the usual one for the SM $SU(2)_L \times U(1)_Y \to U(1)_{QED}$, so only three gauge bosons can eat Goldstone bosons. Thus we will be left with three uneaten Goldstone modes, which will manifest themselves as light (compared to the resonances) isotriplet scalars. They will not be exactly massless, since the fact that only the diagonal $SU(2)_L \times U(1)_Y$ subgroup is gauged will explicitly break the full set of two $SU(2)_L \times SU(2)_R$ global symmetries. Thus we expect these top-pions to obtain mass from one-loop electroweak interactions. We can give a rough estimate for the loop-induced size of the top-pion mass. For this we need to know which linear combination of the two Goldstone modes arising from the two CFT's will be eaten. This is dictated by the Higgs mechanism, and the usual expression for the uneaten Goldstone boson is

$$\Phi_{top\pi} = \frac{f_{\pi,t}\Phi_w - f_{\pi,w}\Phi_t}{\sqrt{f_{\pi,w}^2 + f_{\pi,t}^2}} \ , \tag{2.54}$$

where $\Phi_{w,t}$ are the isotriplet Goldstone modes from the two CFTs, and the f_π's should be substituted by $f_{\pi,i} \to \frac{1}{2} v_{eff,i}$ $(i = w, t)$ in case we are not in the higgsless limit. Since the scale of the resonances in the two CFT's are given by $m_{\rho,i} = \frac{1}{R'_i}$, we can estimate the loop corrections to the top pion mass to be of order

$$m^2_{top\pi} \sim \frac{g^2}{16\pi^2(f^2_{\pi,w} + f^2_{\pi,t})} \left(\frac{f^2_{\pi,w}}{R'^2_w} + \frac{f^2_{\pi,t}}{R'^2_t} \right). \tag{2.55}$$

Experimentally, these top-pions should be heavier than ~100 GeV. We will provide a more detailed expression for their masses when we discuss them in the 5D picture.

We can also estimate the coupling of these top-pions to the top and bottom quarks, assuming that the top pion lives mostly in the CFT that will give a rather small contribution to the W mass, but a large contribution to the top mass. The usual CFT interpretation of the top and bottom mass is the following [52]: the left-handed top and bottom are elementary fields living in a doublet of the (t_L, b_L). The right handed fields have a different nature: the top is a composite massless mode contained in a doublet under the local $SU(2)_R$, while the bottom is an elementary field weakly mixed with the CFT states to justify the lightness of the bottom. Assuming that a non-linear sigma model is a good description for the top-pions, we find that the top and bottom masses can be written in the following $SU(2)_L \times SU(2)_R$ invariant form:

$$(\bar{t}_R, \bar{b}_R/N^b_R) U^\dagger_R \begin{pmatrix} m_t & \\ & m_t \end{pmatrix} U_L \begin{pmatrix} t_L \\ b_L \end{pmatrix} + h.c. \tag{2.56}$$

Here the suppression factor N^b_R is due to the fact that only a small mixture of the right handed bottom is actually composite. To obtain the correct masses we will need $N^b_R \sim m_b/m_t$. If the top-pion is mostly the Goldstone boson from the CFT$_t$ that gives the top mass then $U_L = U^\dagger_R \sim e^{i\Phi^a \tau^a / 2 f_{\pi t}}$, which implies that the coupling of the top-pion to the top-bottom quarks will be of the form

$$\frac{m_t}{2f_{\pi,t}}(t_L\Phi^3\bar{t}_R+\sqrt{2}b_L\Phi^-\bar{t}_R)+\frac{m_b}{2f_{\pi,t}}(b_L\Phi^3\bar{b}_R+\sqrt{2}t_L\Phi^-\bar{b}_R)+h.c.\,. \tag{2.57}$$

Thus we can see that the couplings involving t_R are proportional to $m_t/f_{\pi,t}$, which will be large in the limit when the CFT does not contribute significantly to the W mass. A similar argument can be made in the limit when the electroweak symmetry breaking in CFT_t appears mostly from the VEV of a composite Higgs. In this case this Higgs VEV has to produce the top mass, and so we can show that the couplings of the top pions will be of order $m_t/v_{eff,t}$. Thus these couplings will be unavoidably large both in the higgsless and the higgs limit of the CFT_t, and one has to worry whether these couplings would induce additional shifts in the value of the top quark mass and the $Zb\bar{b}$ couplings.

2.5.2 Properties of the top-pion from the 5D picture.

We have shown in Section 2.2 how such massless modes appear in the 5D picture from the modified BC of the A_5 fields. We would like now to study in more detail their properties, already inferred from the CFT picture. The first check is to show how the strong coupling with the top and bottom arises in the 5D picture. Let us recall how the fermion masses are generated through brane localized interactions [18, 19]. The left- and right-handed fermions are organized in bulk doublets of $SU(2)_L$ and $SU(2)_R$ respectively, where specific boundary conditions are picked in order to leave chiral zero modes, and the localization of the zero modes is controlled by two bulk masses c_L and c_R in units of $1/R$. The Higgs localized on the IR brane allows one to write a Yukawa coupling linking the L and R doublets (in the higgsless limit this corresponds to a Dirac mass term): this gives a common mass to the up- and down- type quarks, due to the unbroken $SU(2)_D$ symmetry. The mass splitting can be then recovered adding a large kinetic term localized on the Planck brane for the $SU(2)_R$ component of the lighter quark [18]. A similar mechanism can be used to generate lepton masses.

In the higgsless—top-Higgs scenario, the localized Yukawa couplings can be written as:

$$\int dz \left(\frac{R_t}{R'_t}\right)^4 \delta(z - R'_t)\, \lambda_{top} R_t \, (\chi_L \phi \eta_R + h.c.) \,, \tag{2.58}$$

where λ_{top} is a dimensionless quantity, and the 5D fields χ_L (η_R) are the left- (right-) handed components that contain the top-bottom zero modes. Expanding the Higgs around the VEV:

$$\phi = \frac{v_t}{\sqrt{2}} \left(1 + \frac{R'_t}{R_t} \frac{h_t + i\pi_t^a \sigma^a}{v_t}\right) \,, \tag{2.59}$$

where the warp factor takes into account the normalization of the scalars, we can find the trilinear interactions involving the top-pion triplet π_t^a and the top-Higgs h_t. In the following we will assume that the fermion wave functions are given by the zero modes, basically neglecting the backreaction of the localized terms: this approximation is valid as long as the top mass is small with respect to the IR brane scale, namely $m_t R'_t < 1$. The wave functions are then

$$\chi_L = \frac{1}{\sqrt{R_t}} \left(\frac{z}{R_t}\right)^{2-c_L} \begin{pmatrix} t_l/N_L^t \\ b_l/N_L^b \end{pmatrix} \,, \quad \eta_R = \frac{1}{\sqrt{R_t}} \left(\frac{z}{R_t}\right)^{2+c_R} \begin{pmatrix} t_r/N_R^t \\ b_r/N_R^b \end{pmatrix} \,. \tag{2.60}$$

The normalizations for the L-fields are given by the bulk integral of the kinetic term, and are the same for top and bottom $N_L^t = N_L^b$. On the other hand, for the b_r field, the wave function is dominated by the localized kinetic term on the Planck brane. This is the source for the splitting between the top and bottom mass, so that

$$\frac{N_R^b}{N_R^t} = \frac{m_t}{m_b} \,. \tag{2.61}$$

With this in mind, we find that:

$$m_t = \frac{\lambda_{top} v_t R_t}{\sqrt{2} R'_t} \left(\frac{R'_t}{R_t} \right)^{1+c_R-c_L} \frac{1}{N_L N_R^t}\,, \tag{2.62}$$

and the couplings can be written as

$$\frac{m_t}{v_{\textit{eff},t}} \left(t_r, \frac{m_b}{m_t} b_r \right) (h_t + i\sigma^a \pi_t^a) \begin{pmatrix} t_l \\ b_l \end{pmatrix} . \tag{2.63}$$

It is clear that the $t_l t_r$ and $b_l t_r$ couplings are enhanced by the ratio $m_t / v_{\textit{eff},t}$, while the couplings involving the r-handed b will be suppressed by the bottom mass. This will lead to possibly large and incalculable contributions to the top-mass and the b_l coupling with the Z, however it is still plausible to have a heavy top in such models.

In the higgsless—higgsless limit the situation is more complicated: the top-pion is a massless mode of the A_5 of the broken generators of $SU(2)_L \times SU(2)_R$ (no Higgs is present in this limit) and its couplings are determined by the gauge interactions in the bulk, thus involving non trivial integrals of wave functions. However, as suggested by the CFT interpretation, we will get similar couplings, with $v_{\textit{eff},t}$ replaced by $f_{\pi,t}$, and again the requirement that $f_{\pi,t} \ll M_W$ will introduce strong coupling. This is a generic outcome from the requirement that the symmetry breaking scale that gives rise to the top mass is smaller than the electroweak scale.

An important, and in some sense related issue is the mass of the top-pion. As already mentioned, it will pick up a mass at loop level, generated by the interactions that break the two separate global symmetries. The top only couples to one CFT, so that its interactions cannot contribute to the top-pion mass. The net effect, although non-calculable due to strong coupling, is to renormalize the potential for the Higgs localized on the new IR brane. On the other hand, the only interactions that break the global symmetries are the gauge interactions that can propagate from one boundary to the other. In the higgsless case, assuming weak coupling, we expect a contribution of the form:

$$m_{A_5}^2 = \frac{C(r)}{\pi}\frac{g_{5L}^2 + g_{5R}^2}{R_t}\frac{1}{R_t'^2}F(R_t/R_t'), \tag{2.64}$$

where $F(R_t/R_t')$ is typically order 1 [53]. In the phenomenologically interesting region, this effect is not calculable, as the gauge KK modes are strongly coupled. However we still expect the mass scale to be set by $1/R_t'$. In the case of the top-Higgs, the gauge KK modes are weakly coupled to the localized Higgses, so a loop expansion makes sense and Eq. (2.64) is a good estimate of the mass.

Another interesting issue is the sign of the mass squared. Indeed, a negative mass square for the charged top-pion would signal a breakdown of the electromagnetic U$(1)_{QED}$, and would generate a mass for the photon. The gauge contribution is usually expected to be positive. In other words, we need to make sure that our symmetry breaking pattern is stable under radiative corrections. A useful way to think about it is the following: from the effective theory point of view, we have a two Higgs model. The tree level potential consists of two different and disconnected potentials for the two Higgses, so that two different $SU(2)_L \times SU(2)_R$ global symmetries can be defined. After the Higgses develop VEVs, we can use a gauge transformation to rotate away the phase of one of them, but a relative phase could be left. In other words the tree-level potential itself does not guarantee that the two VEVs are aligned and a $U(1)$ is left unbroken. Once we include the radiative contribution to the potential, the qualitative discussion does not change: some mixing terms will be generated by the gauge interactions, lifting the massless pseudo-Goldstone bosons, but in general a relative phase could be still present. So, we need to assume that the two VEVs are aligned, maybe by some physics in the UV. There is an analogous vacuum alignment problem that arises in the SM if we consider the limit of small u and d masses, where the dominant contribution to the mass of the $\pi^{\pm}$ comes from a photon loop. In that case, QCD spectral density sum rules can be used to show that $m_{\pi^{\pm}} > 0$ [54]. Thus in the higgsless—higgsless case it is possible that the dual CFT dynamics can ensure the correct vacuum alignment, as happens in QCD-like technicolor theories [54].

A possible extension of the model is considering a bulk Higgs instead of brane localized Higgses, in order to give an explicit mass to the top-pion in analogy with the QCD case studied in [55]. We imagine that we have a single Higgs stretching over the two bulks. The generic profile for the Higgs VEV along the AdS space will be [56]:

$$v(z) \sim a\left(\frac{z}{R}\right)^{\Delta_+} + b\left(\frac{z}{R}\right)^{\Delta_-} , \tag{2.65}$$

where the exponents $\Delta_\pm = 2 \pm \sqrt{4 + M_{bulk}^2 R^2}$ are determined by the bulk mass of the scalar. The bulk mass controls the localization of the VEV near the two IR branes, and in the large mass limit we recover the two Higgses case: all the resonances become very heavy and decouple, except for one triplet that becomes light and corresponds to the top-pion. Indeed, its mass will be proportional to the value of the VEV on the Planck brane, that is breaking the two global symmetries explicitly. In the CFT picture, the bulk VEV is an operator that connects the two CFTs and gives a tree level mass to the top-pion. However, the bulk tail will also contribute to the Wmass: we numerically checked in a simple case that in any interesting limit, when the bulk Higgs does not contribute to unitarity, the tree level mass is negligibly small. Nevertheless, this picture solves the photon mass issue: indeed we have only one Higgs. In other words, the connection on the Planck brane is forcing the two VEVs on the boundaries to be aligned.

To summarize, the top-pion will certainly get a mass at loop level, whose order of magnitude can be estimated to be at least one loop factor times $1/R_t'$. Moreover, a tiny explicit mixing between the two CFT, induced by a connections of the two VEVs on the Planck brane, would be enough to stabilize the symmetry breaking pattern and preserve the photon from getting a mass. It would be interesting to analyze more quantitatively these issues which we leave for further studies.

2.6 Phenomenology of the Two IR Brane Models

2.6.1 Overview of the various models

The main problematic aspect of higgsless models of electroweak symmetry breaking is the successful incorporation of a heavy top quark into the model without significantly deviating from the measured values of the $Zb_l\bar{b}_l$ coupling [37]. The reason behind this tension is that there is an upper bound on the mass of a fermion localized at least partly on the Planck brane given by

$$m_f^2 \leq \frac{2}{R'^2 \log \frac{R'}{R}} \ . \tag{2.66}$$

Since in the case of a single TeV brane the value of $R'^2 \log \frac{R'}{R}$ is determined by the W mass, the only way to overcome this bound is by localizing the third generation quarks on the TeV brane. However the region around the TeV brane is exactly the place where the wave functions of the $W$and Z bosons are significantly modified, thus leading to large corrections in the $Zb\bar{b}$ coupling.

The main motivation for considering the setups with two IR branes is to be able to separate the mass scales responsible for the generation of the W mass and the top mass. Thus as discussed before, we are imagining a setup where electroweak symmetry breaking is coming dominantly from a higgsless model like the one discussed in [37], while the new side is responsible for generating a heavy top quark. The gauge bosons would of course have to live in both sides, while of the fermions only the third generation quarks would be in the new side. We have seen that in the higgsless—higgsless limit the W mass is given by (2.53)

$$M_W^2 = g^2(f_{\pi,w}^2 + f_{\pi,t}^2) = \left(\frac{1}{R_w'^2} + \frac{R_t}{R_w}\frac{1}{R_t'^2}\right)\frac{1}{\log\frac{R_w'}{R_w} + \frac{R_t}{R_w}\log\frac{R_t'}{R_t}}\,. \tag{2.67}$$

In order to ensure that the dominant contribution to the Wmass arises from a higgsless model as in [37] we need to suppress the contribution of the new side by choosing $\frac{R_t}{R_w} \ll 1$, which in the CFT picture corresponds to $f_{\pi,t}^2 \ll f_{\pi,w}^2$. This way one can choose parameters on the "old side" similarly as in the usual higgsless models, that is $1/R_w' \sim 300$ GeV, $\log\frac{R_w'}{R_w} \sim 10$, resulting in a KK modes of the W and Z of about 700 GeV. The couplings will be slightly altered from the one-bulk higgsless model, but will remain close enough that we can maintain perturbative unitarity up to scales of about 10 TeV, provided the new side contributes only about 1% of the W mass. We will substantiate this claim numerically in a later section. All this can be achieved independently of the choice of $R_t'^{-1}$! Thus we can still make $1/R_t'$ quite a bit bigger than the TeV scale on the old side $1/R_w'$, making it possible to obtain a large top quark mass by circumventing the bound (2.66).

However, this framework is not without potential problems: as discussed in the previous section there is a light top-pion pseudo-Goldstone mode with a generically large coupling to the top quark, irrespective of the value of the VEV of the Higgs on the TeV brane. A more worrisome problem arises from the limit $f_{\pi,t}^2 \ll f_{\pi,w}^2$. In general, a condition for a trustworthy 5D effective field theory is that $b_{CFT} = 8\pi^2\frac{R}{g_5^2} \gg 1$ [8], a condition that will be violated in the new bulk when $f_{\pi,t}^2$ is too small. Violation of this condition will result in large 4D couplings among the KK modes of the AdS$_t$, whose masses are proportional to $1/R_t'$, which could potentially give rise to large incalculable shifts in the expected values of the masses and couplings of these modes. Note however, that the coupling of the light $W$and Z will never be strong to these KK modes: gauge invariance will make sure that the couplings of the $W$and Z are of the order g to all of these modes. Thus the expressions for the electroweak precision observables will be shielded by at least an electroweak loop suppression from the potentially strong couplings of the KK modes. Moreover, the unitarity in the longitudinal Wscattering is still maintained by the KK modes of the AdS_w, which are weakly coupled with the KK modes of the new AdS. This is again true as long as the Wmass mostly comes from the old side.

Thus there is a tension in the higgsless—higgsless case between constraints from perturbative unitarity of WW scattering which will want $f_{\pi,t}^2 \ll f_{\pi,w}^2$, and constraints from 5D effective field theory. Of course, we could evade the perturbative unitarity problem by lowering $R_t'^{-1}$ to near the scale $R_w'^{-1}$, but then the new side would merely be a copy of the old side and one would again start running into trouble with the third generation physics.

Alternatively, we can consider the higgsless—top-Higgs model, in which we can take $f_{\pi,w}^2 \approx f_{\pi,t}^2$ but v_t small. Then we again find that the new side contributes little to electroweak symmetry breaking and perturbative unitarity is safe, and we also have a reasonable 5D effective theory. In this case the only strong coupling is a large Yukawa coupling for the top quark. Of course, such a model is not genuinely "higgsless" in the sense that for small v_t the Higgs on the new side does not decouple from the SM fields. However, this surviving Higgs will have small couplings to the Standard Model gauge bosons and large coupling to the top, so it can be unusually heavy and will have interesting and distinct properties. We show a summary of the different constraints on the parameters R_t/R_w and v_t (assuming

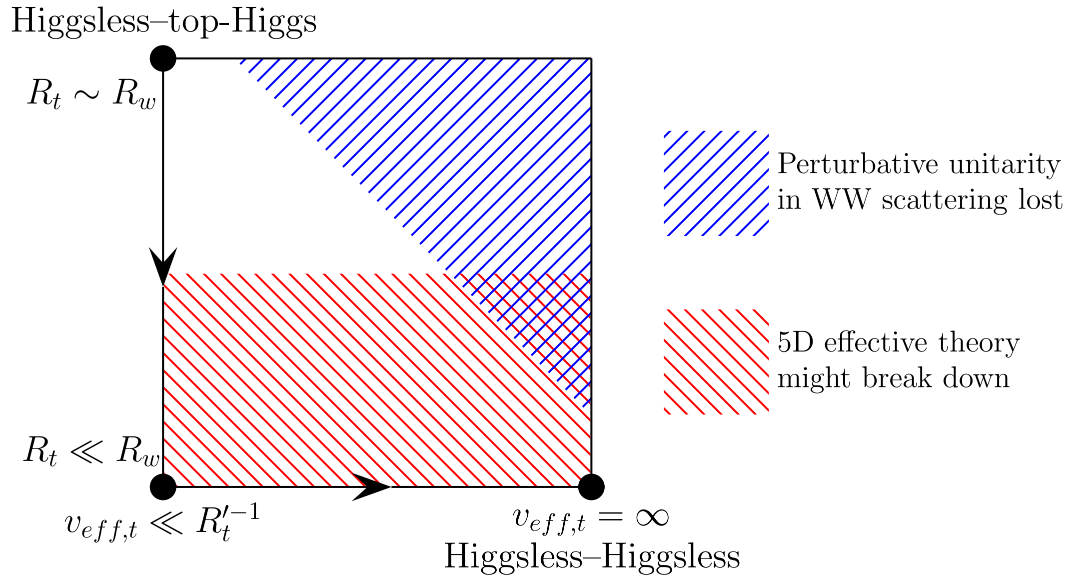

Figure 2.2: A visualization of constraints on the parameter space. The dot at the upper-left is the higgsless—top-Higgs theory in which only the top Yukawa is large. The dot at lower right is the higgsless—higgsless theory we would like to ideally reach to decouple all scalars from the SM fields. Moving along the arrow pointing right, from higgsless—top-Higgs to higgsless—higgsless, one can potentially run into perturbative unitarity breakdown. This is not a danger when $R_t \ll R_w$, but as one moves along the downward arrow toward small R_t/R_w, one faces increasingly strong coupling among all KK modes on the new side. This signals a potential breakdown of the 5D effective theory.

$1/R_t'$ large) in Figure 2.2.

2.6.2 Phenomenology of the higgsless—top-Higgs model

From the previous discussions we can see that the model that is mostly under perturbative control is the higgsless—top-Higgs model with $R_t \sim R_w$, $v_{eff,t} \lesssim 50$ GeV and $1/R_t'$ at a scale of 2 to 5 TeV. In the following we will be discussing some features of this model in detail.

We can place the third-generation fermions in the new bulk and give them

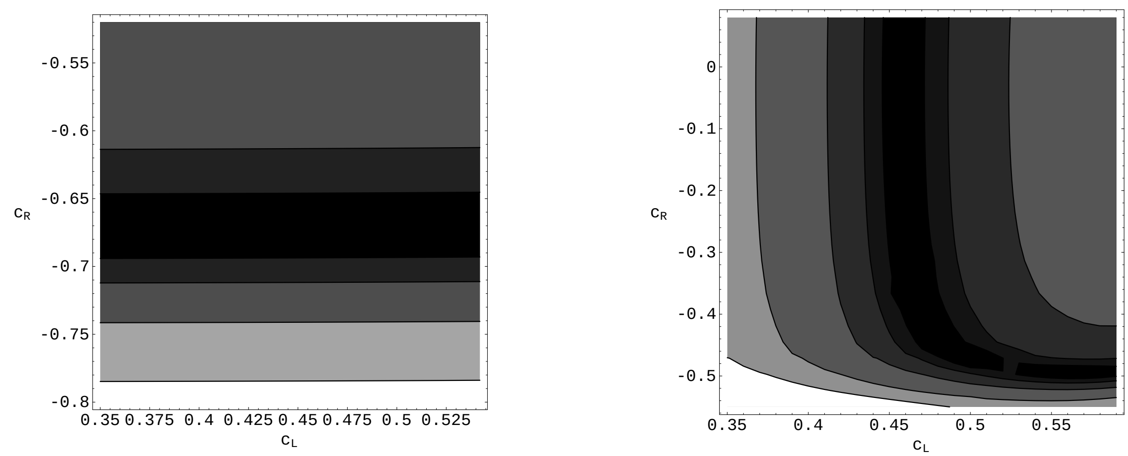

Figure 2.3: Deviation of $Zb_l\bar{b}_l$ from SM value, as a function of bulk mass parameters, in the higgsless—top-Higgs case in the plot on the left and in the higgsless—higgsless case on the right. The coupling decreases from bottom-to-top in the left plot and left-to-right in the right plot. The contours (darkest to lightest) are at .5%, 1%, 2%, 4%, and 6%.

masses by Yukawa coupling to the brane-localized top-Higgs. The wave functions of the W and Z in the new bulk will be approximately flat. Then, from the perspective of third-generation physics (quantities like m_t and the $Zb\bar{b}$ coupling), the physics in the new bulk looks essentially the same as that of the usual Randall-Sundrum model with custodial symmetry [40], with two important differences. The first difference is that the top-Higgs VEV v_t is small, so that the top Yukawa coupling must be large. The second is the presence of the top-pion scalar modes noted in the last section. Aside from this, the results must be much as in the usual Randall-Sundrum model. We find that we do not have to take either the left- or right-handed top quark extremely close to the TeV brane to obtain the proper couplings. The right-handed bottom quark will mix with Planck-brane localized fermions (or, alternatively, will have a large Planck-brane kinetic term) to split it from the top quark. The right-handed bottom then lives mostly on the

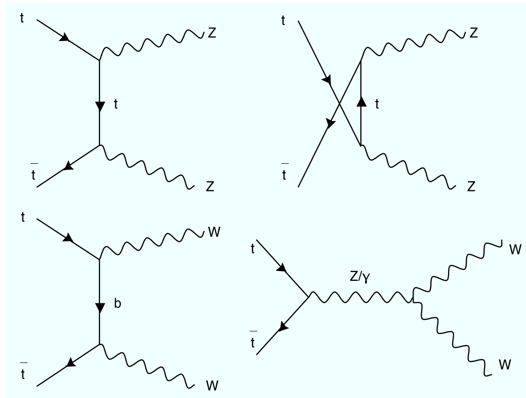

Figure 2.4: Scattering processes for $t\bar{t} \to V_L V_L$ of top anti-top pairs into longitudinal vector bosons. These processes determine the unitarity bound on the mass of the heavy top-Higgs boson in the higgsless—top-Higgs model.

Planck brane, and so will have the usual SM couplings. The problem arising in the original higgsless model was that a large mass $M_D R'$ on the IR brane caused much of the left-handed bottom quark to live in the $SU(2)_R$ multiplet. Note that a similar problem would arise in a model with a brane-localized Higgs and the same value of R'; the usual Randall-Sundrum models evade this problem with a large $1/R'$. In our new scenario, R'_t is significantly smaller than R'_w, so at tree level we are able to obtain the desired values of m_t and the SM couplings of the bottom. We show this explicitly in Figure 2.3. It corresponds to $R'^{-1}_w = 292$ GeV, $R'^{-1}_t = 3$ TeV, $R^{-1}_w = R^{-1}_t \approx 10^6$ GeV, $v_{\mathit{eff},t} = 50$ GeV, and a light reference fermion on the old side for which $c_L = .515$ (for these parameters we find $S \approx -.066$, $T \approx -.032$, and $U \approx .010$). Note that we can accommodate a change in the tree-level value in either direction, so loop corrections from the top-pion are not a grave danger. Furthermore, the masses of the lightest top and bottom KK modes are at approximately 6 TeV, so they should not cause dangerous contributions to the T parameter.

We would like now to investigate the phenomenology of this model in more detail. A novelty with respect to the usual higgsless models is the presence of light scalars. The top-Higgs on the AdS_t side will couple strongly to tops and give a small contribution to the Wmass, hence the name top-Higgs. Its tree level mass is determined by the quartic coupling on the IR brane $m_h^2 = \lambda_t v_{\mathit{eff},t}^2$, although large corrections could arise due to the strong coupling with the top. An important parameter controlling this model is $\gamma = \frac{v_{\mathit{eff},t}}{v}$, the ratio of the (warped down) top-Higgs VEV to the usual SM Higgs VEV. If $\gamma \ll 1$, we can have confidence that perturbative unitarity in WW scattering is restored by the KK modes in the higgsless bulk, provided they have masses in the 700 GeV range. In this case, the top-Higgs is not needed for perturbative unitarity in WW scattering, but there is still a unitarity bound on its mass. This bound arises from considering $t\bar{t} \to V_L V_L$ scattering, where V_L denotes a longitudinal W or Z boson. The relevant tree-level diagrams are shown in Figure 2.4. This bound sets the scale of new physics, Λ_{NP}, to be (as in [57])

$$\Lambda_{NP} \leq \frac{4\pi\sqrt{2}}{3G_F m_t} \approx 2.8 \text{ TeV}. \tag{2.68}$$

This is computed in an effective theory given by the Standard Model with the Higgs boson removed and the Yukawa coupling of the top replaced with a Dirac mass. The resonances that couple to the top in our model are predominantly those on the new side, which have a mass set by $R_t'^{-1}$, which is large. Thus these resonances make little contribution to the scattering in question, and we can view Λ_{NP} as a rough upper bound on the possible mass of the top-Higgs boson in our model. It is clear anyway that a heavy top-Higgs is allowed. The other set of scalars is an $SU(2)$ triplet of pseudo-Goldstone bosons that we called top-pions. We estimated their mass in Eq.2.64. It is set by the scale $1/R_t'$, so that we expect them to be quite heavy too, at least heavy enough to avoid direct bounds on charged scalars.

Table 2.1: Leading branching ratio estimates (subject to possibly order 1 corrections) for the heavy top-Higgs (assuming $M_{ht} \approx 1$ TeV and $\gamma \approx 0.2$). In the case $M_{ht} \gg M_{\pi t}$, we have assumed $M_{\pi t} = 400$ GeV for the purpose of calculation. These can receive large corrections, but the qualitative hierarchy (associated with $\gamma = \frac{v_{eff,t}}{v}$) should persist. Using the Pythia cross-section $\sigma \approx 88$ fb for a 1 TeV Higgs, rescaled by a factor of $\gamma^{-2} = 25$ to take into account enhanced production, we find an estimate of ≈ 1000 ZZ events in 100 fb^{-1}, but only about 1 $\gamma\gamma$ event. However, for $M_{ht} \approx 500$ GeV, we expect a larger cross section, $\approx \gamma^{-2} \times 1700$ fb, and there could be about 100 $\gamma\gamma$ events in 100 fb^{-1}. Note that the branching ratio estimates for the neutral top-pion will be essentially the same (with $M_{\pi t}$ and M_{ht} reversed in the above table).

Decay Mode	$M_{ht} \gg M_{\pi t}$	$M_{ht} \ll M_{\pi t}$	Remarks
$t\bar{t}$	34%	98%	Large background, but consider associated $t\bar{t}H$.
$W^{\pm}\pi_t^{\mp}$	43%	–	Similar to $t\bar{t}$.
$Z\pi_t^0$	22%	–	Interesting. $t\bar{t}Z$: four leptons, two b jets.
W^+W^-	.35%	1.0%	Rare and probably difficult.
ZZ	.17%	.50%	Usual "golden" mode, but very rare.
gg	.06%	.16%	
$b\bar{b}$	.01%	.03%	
$\gamma\gamma$	2.1×10^{-4} %	6.1×10^{-4} %	Very rare, but sometimes accessible.

In the following we will assume a wide range of possibilities for the scalar masses, although the agreement of the $Zb\bar{b}$ coupling would suggest that they are heavy, likely above the $t\bar{t}$ threshold.

The phenomenology of this model is still characterized by the presence of W and Z resonances that unitarize the WW scattering amplitude: this sector of the theory

Table 2.2: Leading branching ratio estimates (subject to possibly order 1 corrections) for the top-Higgs when M_{ht} is below the $t\bar{t}$ threshold and also below the top-pion threshold. These are calculating from rescaling the SM branching ratios using $M_{ht} \approx 300$ GeV. The number of events is estimated via the Pythia cross-section, $\sigma = 3.9$ pb for $M_{ht} = 300$ GeV, rescaled by a factor of $\gamma^{-2} = 25$ to take into account enhanced production. Alternatively, these can be viewed as approximate branching ratios of the neutral top-pion when its mass is below the $t\bar{t}$ threshold.

Decay Mode	BR	Events in 100 fb^{-1}	Remarks
W^+W^-	40%	4.0×10^6	Probably difficult.
$b\bar{b}$	22%	2.2×10^6	Large QCD background.
gg	20%	2.0×10^6	Large QCD background.
ZZ	18%	1.8×10^6	Usual "golden" mode, now rarer.
$\gamma\gamma$	.07%	7000	Light Higgs "golden" mode, still visible.

is under perturbative control so it is possible to make precise statements. For a detailed analysis of the collider signatures of higgsless models, see Reference [29]. Although the strong coupling regime does not allow us to make precise calculations, the presence of top-Higgses can provide interesting collider phenomenology for these models. The case of a top-Higgs has already been considered in the literature in more traditional scenarios [45, 58, 59]. The key feature of this model is the strong coupling with the top, determined by the large Yukawa coupling $m_t/v_{eff,t}$ that is enhanced by a factor $1/\gamma = v/v_{eff,t}$ with respect to the SM Higgs case. On the other hand, the couplings with massive gauge bosons are suppressed by a factor γ, as the contribution of the top-Higgs sector to electroweak symmetry breaking is small. There will be a coupling with the bottom, suppressed by m_b/m_t with respect to the top coupling. We also have couplings $h_t W^{\pm\mu}\pi_t^{\mp}$ and $h_t Z\pi_t^0$ of the top-higgs with SM gauge bosons and the top-pions. These arise from a term $2gh_t A_\mu^a \partial^\mu \pi_t^a$ in the Lagrangian.

Let us first discuss the decays of the neutral scalars h_t and π_t^0. If their mass is above the $t\bar{t}$ threshold, they will often decay into tops. Notably, if the mass is large, say 1 TeV, multiple top decays will not be suppressed due to the strong coupling. However, for $M_{ht} \gg M_{\pi t}$, the width of the cascade decay $h_t \to W^- \pi_t^+$ is $\frac{g^2}{16\pi}\frac{M_h^3}{M_W^2}$, becoming quite large for a very heavy top-Higgs, and even surpassing the enhanced decay to tops. There is a suppressed tree-level decay to the weak gauge bosons. Virtual tops will also induce loop decays into gauge bosons, $\gamma\gamma$, W^+W^-, ZZ, gg, and we generically expect them to be suppressed by a factor of about $\left(\frac{\alpha}{3\pi}\right)^2$ or $\left(\frac{\alpha_s}{3\pi}\right)^2$ with respect to the $t\bar{t}$ channel. A simple estimate shows that the $\gamma\gamma$ decay is suppressed, but it could still be present in a measurable number of events at the LHC in decays of the neutral top-pion or even of the top-Higgs if it is not too heavy. We summarize the various modes in Tables 2.1 and 2.2. The widths are calculated at leading order. That is, the tree-level decays are calculated by rescaling the tree-level SM widths by appropriate powers of γ, except for the decays through a top-pion, which are computed directly in our model. (The decay to $b\bar{b}$ is computed with the running b quark mass.) Loop-level decays are calculated by rescaling one-loop SM results by appropriate powers of γ. These estimates should provide the right qualitative picture, though the large couplings of the top could induce order one changes. Other interesting channels could open up if we consider flavor violating decays, like for example $t\bar{c}$. It might be relevant or even dominant in the case of a relatively light scalar (below the $t\bar{t}$ threshold) [59]. However, these channels are highly model dependent: in this scenario the flavor physics is generated on the Planck brane via mixings in non-diagonal localized kinetic terms. For instance, we can choose the parameters so that there is no $\pi_t t\bar{c}$ coupling at all. So, we will not consider this possibility further. Regarding the

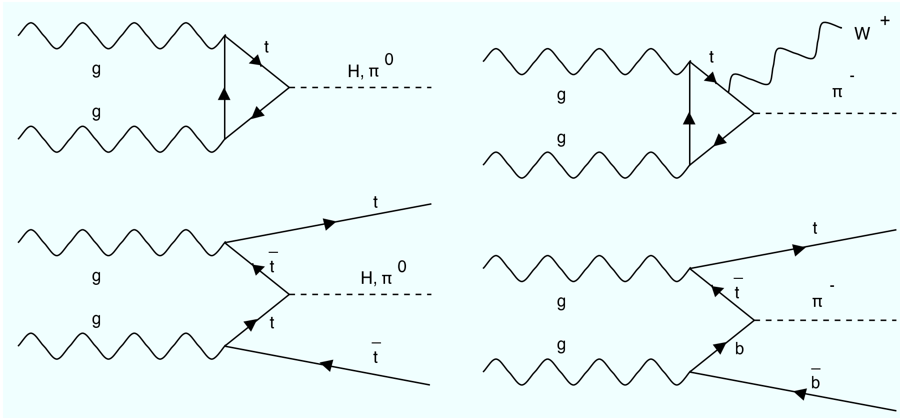

Figure 2.5: Gluon-gluon fusion processes producing top-higgs and top-pion bosons at the LHC.

charged top-pion, it will mostly decay into tb pairs, though at loop level there will be rare decays to $W^{\pm}\gamma$ and $W^{\pm}Z$, which could lead to interesting signatures.

At the Large Hadron Collider (LHC) we expect a lot of top-Higgses and top-pions to be produced (see Figure 2.5), via the usual gluon fusion or top fusion, now enhanced with respect to the SM one by the large Yukawa coupling. If the mass is larger than $2m_t$, the main decay channel is in $t\bar{t}$, or multiple tops. The QCD background is large, however it is probably realistic to search the spectrum of the $t\bar{t}$ events due to the enhanced production rate in gluon-fusion. A golden channel is represented by the decay into two photons or two Z bosons. We expect a substantial number of ZZ events throughout a wide range of masses. To observe $\gamma\gamma$ events, which are enhanced relative to the SM by the large Yukawa and by the enhanced production of neutral top-pion and of top-Higgs, we need relatively small masses. At $M_{ht} \approx 1$ TeV, the cross-section is expected to be too small to observe a substantial number of events. We have used Pythia [60] to estimate the SM cross-section for a Higgs produced by gluon-gluon fusion. This cross-section, suitably enhanced by the large Yukawa, was used to estimate numbers of LHC events per 100 fb^{-1} in Tables 2.1 and 2.2. Of course, strong coupling will modify our estimates of cross-sections and branching ratios, so the numbers we present should be taken as order-of-magnitude guides. We expect the neutral top-pion to have a mass somewhat below the TeV scale, so optimistically one should see the photon-photon channel from the neutral top-pion irrespective of the top-Higgs mass. We stress that, for a mass in the 500 GeV region, one can expect roughly one photon-photon event per fb^{-1}, while the number of ZZ events should be of order one thousand times larger. At the LHC it will be relatively easy to see peaks in the two photons or $\ell^+\ell^-\ell^+\ell^-$ channels, due to the reduced background. Thus, a heavy resonance in $\gamma\gamma$, associated with an anomalous production of multi-top events would be a striking signature of these models. If cascade decays of the top-Higgs into the top-pion are allowed, we could also observe interesting $Zt\bar{t}(t\bar{t})$ channels that could lead to striking 6 leptons 4 b events. If the masses are below the top threshold, the main decay channels will be into b's and gauge bosons. The golden channels are again $\gamma\gamma$ and ZZ. The high rate of $b\bar{b}$ events even above the WW threshold could help distinguish a light scalar in our model from a heavy SM Higgs. Also, if the rate of $\gamma\gamma$ is not too far below the rate of $\ell^+\ell^-\ell^+\ell^-$ events it would suggest a large top-loop induced coupling, since in the SM this ratio is fixed to be roughly $\left(\frac{\alpha}{3\pi}\right)^2$.

Finally, the charged top-pion would be harder to study: its production is suppressed as we do not have a gluon fusion channel producing solely a top pion. It will be produced in association with a tb pair or with a W boson. It will then most likely decay into tb, so that its signal will suffer from a large QCD pollution. An interesting effect could be an anomalous production of multi b-jet events. The loop-level decays to $W\gamma$ and WZ could produce interesting multi-gauge-boson events, but these have a suppression comparable to the $\gamma\gamma$ decays of the neutral top-pion.

2.6.3 Phenomenology of the higgsless—higgsless model

Finally, we summarize the tree-level numerical results for the higgsless—higgsless limit. Let us first discuss how to fix the values of the parameters corresponding to a potentially interesting theory. First of all, we would like one of the sides to be a higgsless model as in [37], with low enough KK masses for the gauge bosons to ensure perturbative unitarity of the WWscattering amplitudes. This can be achieved if the first resonance mass is around 600-700 GeV, thus fixing $R_w'^{-1} \sim 300$ GeV. The value of the W mass will fix the $\log R_w'/R_w \sim 10$, so that a natural value for R_w^{-1} is around 10^8 GeV. On the new side we need the IR scale to be large enough to accomodate the top mass, so that $R_t'^{-1} \sim 2 - 5\ TeV \gg R_w'^{-1}$. However, we want to do it without a low-scale violation of perturbative unitarity. Since the KK modes on the new side will be very heavy $>$ TeV, this is only possible if the new side does not contribute a lot to the W mass itself. From (2.67) we can see that this can be achieved by choosing a smaller curvature radius for the new side $R_t \ll R_w(R_t'/R_w')^2$. For simplicity we will also assume the 5D gauge couplings are the same in both bulks, and that $g_{5L} = g_{5R}$. Then, for any given value of R_t' and of the contribution of the new side to the W mass (that will determine the perturbative unitarity breakdown scale), we determine the scales $R_{w,t}$ and R_w', while $\tilde{g}_5$ is fixed by the Z mass.

We also choose a "reference" bulk fermion in the old bulk as in [37] to fix the wave-function normalizations. As in the one-bulk case, when this reference fermion has an approximately flat wave-function ($c_L \sim 0.5$) the tree-level precision electroweak parameters S, T, and U can all be made small.

To fix the actual numerical values we choose $R_t'^{-1} = 3$ TeV, and we allow the new bulk to contribute 5% of the W mass, $f_{\pi,t}^2 \approx 0.05(f_{\pi,w}^2 + f_{\pi,t}^2)$. Then for the values quoted above, we get $R_w'^{-1} \approx 276$ GeV, and the curvature scales are $R_w^{-1} \approx \times 10^8$ GeV and $R_t^{-1} \approx 2 \times 10^{11}$ GeV. Choosing our reference quark to be a massless left-handed quark with $c_L = 0.46$, we find at tree-level $S \approx -0.08$, $T \approx -0.04$, and $U \approx 0.01$.

The first task after fixing the parameters is to verify that the scale of perturbative unitarity violation is indeed pushed above the usual SM scale of 1 TeV. For this we can study the sum rules [10] that the KK modes masses and couplings have to satisfy in order for terms in the scattering amplitudes that grow with a powers of the energy to cancel. We solve numerically for the Kaluza-Klein resonances of the Z boson. The first one is at $M_Z^{(1)} \approx 676$ GeV $\approx 2.45 R_w'^{-1}$, as expected. Summing the KK modes up to 8 TeV, we find that the E^4 sum rule is satisfied to a precision of 2×10^{-6} and the E^2 sum rule to a precision of 5×10^{-3}. In order to find the unitarity violation scale we have shown in Figure 2.6 the s-wave partial-wave amplitude a_0 as a function of energy, which is obtained by numerically solving for all KK mode masses and couplings below 8 TeV, and then approximating the rest of the tower by an additional heavy mode so the graph does not misbehave at high energy (note that this approximation has no effect below 8 TeV). We can see that the unitarity bound from a_0 is around 5 TeV, well above the SM scale, and a scale likely inaccessible to the LHC. As explained in [31], we should only rely on the low-energy linear behavior of this function, which tells us that the effective theory is valid up to 5 TeV.

After fixing the parameters in the gauge sector, we are finally ready to consider

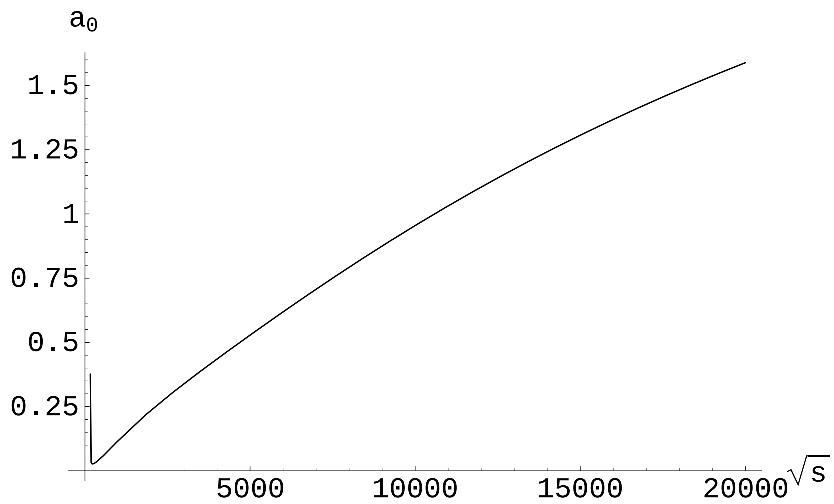

Figure 2.6: Examining perturbative unitarity: the leading partial-wave amplitude a_0, as a function of center-of-mass energy.

the physics of the third generation quarks. These particles are assumed to live on the new side, but the mass generation mechanism for them would be just like for the other fermions: a Dirac mass M_D on the new IR brane would give a common mass to top and bottom, and the bottom mass would then be suppressed by a large kinetic term on the Planck brane for b_R whose coefficient is ξ_b. We can then proceed in the following way: for a given choice of bulk masses c_L, c_R, we can solve for the requisite Dirac mass M_D to get the correct top mass m_t, and then for the mixing ξ_b needed on the Planck brane to get the correct bottom mass.

We can then numerically find the $Zb\bar{b}$ coupling as a function of c_L, c_R. We show a plot of the deviation of the $Zb_l\bar{b}_l$ coupling from the Standard Model value in Figure 2.3. Note that there is a band, where $c_L \approx 0.46$, where for a wide range of choices of c_R, $Zb_l\bar{b}_l$ is consistent with the SM value. This exactly corresponds to picking c_L equal to the reference value of the light fermions on the old side. On one side of the band the coupling is larger, and on the other side it is smaller. Thus a wide range of loop corrections to the $Zb_l\bar{b}_l$ coupling from the top-pion contribution can be accommodated in this model by changing the values of $c_{L,R}$, and tuning the sum of the tree-level plus loop corrections to equal the SM value. Thus we conclude that while in this model there is no *a priori* reason to expect this coupling to take on its SM value, parameters can likely be chosen such that the SM value could be accommodated.

Since we cannot calculate the loop corrections, for concreteness we will examine a case in which the tree-level value of the $Zb_l\bar{b}_l$ coupling agrees with the SM. We take $c_L = 0.46$, $c_R = -0.1$. We then find that we need to take $M_D \approx 610$ GeV and $\xi_b \approx 6000$ to obtain $m_t = 175$ GeV and $m_b = 4.5$ GeV. The tree-level $Zb_l\bar{b}_l$ coupling then deviates from the SM value by only .03%. We calculate now the various couplings of the pseudo-Goldstones to the top and bottom. We find that the couplings involving the right-handed bottom are small: $g_{\pi_t^0\bar{b}_l b_r} \approx g_{\pi_t^+\bar{t}_l b_r} \approx -0.106$. However, as expected, the couplings involving the right-handed top are large: $g_{\pi_t^0\bar{t}_l t_r} \approx g_{\pi_t^-\bar{b}_l t_r} \approx -4.16$. Thus the top-pion coupling is four times larger than the SM Higgs coupling.

2.7 Conclusions

We have considered extra dimensional descriptions of topcolor-type models. From the 4D point of view these would correspond to theories where two separate strongly interacting sectors would contribute to electroweak symmetry breaking. In the 5D picture these would be two separate AdS bulks with their own IR branes, and the two bulks intersecting on the common Planck brane. The motivation for considering such models is the need to separate the dynamics that gives most of electroweak symmetry breaking from that responsible for the top quark mass (which is the main problematic aspect of higgsless models of electroweak symmetry breaking).

We have described how to find the appropriate matching and boundary conditions for the fields that propagate in both sides, and gave a description of electroweak symmetry breaking if both IR branes have localized Higgs fields. We have considered both the cases when the Higgs VEVs are small or large (the higgsless limit). We discussed the CFT interpretation of all of these limits, and also showed that a light pseudo-Goldstone boson ("top-pion") has to emerge in these setups. Depending on the limit considered, the top-pion could be mostly contained in one of the brane Higgses or in A_5 in the higgsless limit.

Finally, we have used these models to try to resolve the issues surrounding the third generation quarks in the higgsless theories. In these models one of the bulks is like a generic higgsless model as in [37] with only the light fermions propagating there, while the new bulk will contain the top and bottom quark, but will not be the dominant source of electroweak symmetry breaking. The suppression of the contribution to the W mass from the new side is either obtained by a small top-Higgs VEV (higgsless—top-Higgs models) or via a small curvature radius in the new bulk. A generic issue in all of these cases will be that the top-pions (and eventually the top-Higgs) are strongly coupled to the top and bottom quarks. In the higgsless—higgsless case the small curvature radius will also imply that the KK modes dominantly living on the new side will be strongly coupled among themselves. In both limits the tree-level top mass and $Zb\bar{b}$ couplings can be made to agree with the experimental results, however, due to the coupling of the top-pion one also needs to worry about large shifts from loop corrections. We have discussed the basic phenomenological consequences of both limits. The top-pion and top-Higgs are expected to be largely produced at LHC. Their main signature would be an observable heavy resonance in the $\gamma\gamma$ channel in association with an anomalously large rate of multi-top events.

Chapter 3

A Braneless Approach to Holographic QCD

3.1 Introduction

QCD is a perennially problematic theory. Despite its decades of experimental support, the detailed low-energy physics remains beyond our calculational reach. The lattice provides a technique for answering nonperturbative questions, but to date there remain open questions that have not been answered. For instance, the low-energy scalar spectrum is a puzzle. There are a lot of experimentally observed states, however their composition (glueball vs. quarkonium) and their mixings are not well understood. The difficulty for any theory trying to make progress in this direction is to understand the interaction between the scalar states and the vacuum condensates of QCD. In this paper we attempt to incorporate the effects of the vacuum condensates into the holographic model of QCD as a first step toward understanding the scalar sector in the context of these models.

The SVZ sum rules [61] are a powerful theoretical tool for relating theoretically solid facts about perturbative QCD with experimental data in the low-energy region. The basic observation is that the correlator for a current J

$$\Pi(q) = i \int \mathrm{d}^4x \; e^{iqx} \left\langle 0 \left| T J(x) J(0) \right| 0 \right\rangle \tag{3.1}$$

may be expanded in a Wilson OPE that is valid up to some power (where instanton corrections begin to invalidate the local expansion), $(1/Q^2)(\mathrm{d}\Pi/\mathrm{d}Q^2) = \sum C_{2d} \left\langle \mathcal{O}_{2d} \right\rangle$, where the $\mathcal{O}_{2d}$ are gauge-invariant operators of dimension $2d$. In the deep Euclidean domain the coefficients C_{2d} are calculable. On the other hand, the correlator relates to observable quantities; for instance if J is the vector current then $\text{Im}\Pi(s)$ is directly proportional to the spectral density $\rho(s)$, measurable from the total cross section $\sigma(e^+e^- \to \text{hadrons})$. The SVZ sum rules, then, relate Wilson OPE expansions to measurable quantities. They can become useful for understanding detailed properties of the first resonances when one takes a Borel transform that suppresses the effect of higher resonances. As it turns out, keeping only low orders of perturbation theory in the coefficients C_{2d}, one still obtains reasonable agreement with data, so the assumption that the largest corrections arise from the condensates is a good one.

Recently another technique for understanding the properties of low-lying mesons has arisen in the form of AdS/QCD. The phenomenological model constructed on this basis [55] takes as its starting points the OPE (much as in the SVZ sum rules) and the AdS/CFT correspondence [7, 42]. The idea is straightforward: rather than attempting to deform the usual Type IIB on $AdS_5 \times S^5$ to obtain a theory more like QCD, one starts with QCD and attempts to build a holographic dual. Of course in detail such a program is bound to eventually run up against difficulties from α' corrections, g_s corrections, the geometry of the five compact dimensions (or the proper definition of a noncritical string theory) and other issues. However, one can set aside these problems, begin with a relatively small set of fields needed to model the low-lying states in QCD, and see how well the approximation works. In this phenomenological approach with bulk fields placed in the Randall-Sundrum background [2], and the AdS space cut off with a brane at a fixed $z = z_c$ in the infrared a surprisingly good agreement with the physics of the pions, ρ, and a_1 mesons has been found [55] (for more recent work on AdS/QCD see [62, 63]). However, there are several obvious limitations to this model. For example, it does not take into account the power corrections to the OPE in the UV (the effects of the vacuum condensates), or the corrections coming from logarithmic running. Also, the theory has a single mass scale (set by the location of IR cutoff brane) which determines the mass scales in all the different sectors in QCD.

Here we show how the effect of the vacuum condensates and of asymptotic freedom can be simply incorporated into the model. We will limit ourselves to pure Yang-Mills theory without quarks, though we expect that most of the aspects of incorporating quarks should be relatively straightforward. For the vacuum condensates one has to introduce a dynamical scalar with appropriate mass term and potential coupled to gravity. There will be a separate field for every gauge invariant operator of QCD, and a non-zero condensate will lead to a non-trivial profile of the scalar in the bulk. While the effects of these condensates on the background close to the UV boundary are small (though not always negligible), they will become the dominant source in the IR and effectively shut off the space in a singularity (and without having to cut the space off by hand). This resolves several ambiguities in choosing the boundary conditions at the IR brane, and the various condensates will also be able to set different mass scales in the different sectors of QCD. Asymptotic freedom can be achieved by properly choosing the potential for the scalar corresponding to the Yang-Mills gauge coupling. Incorporating asymptotic freedom will have an important effect on the glueball sector since a massless Goldstone field will pick up a mass from the anomalous breaking of scale invariance.

It is natural to ask why we should expect QCD to have any useful holographic dual. The most well understood examples of holography are in the limit of large N and $g_{YM}^2 N \gg 1$, far from the regime of real-world QCD, which apparently would be a completely intractable string theory with a large value of α'. On the other hand, the large N approximation has frequently been applied in QCD phenomenology. A major concern in applying holography to QCD is whether the dual should be local, or whether it can have higher-derivative α' corrections. The α' corrections are associated with the massive stringy excitations in the bulk, which once integrated out yield complicated Lagrangians for the remaining light fields. However, we can think of this physics in a different way. Holography is closely related to the renormalization group [64]; the coordinate z can be identified with μ^{-1}, where μ is the renormalization group scale. AdS/CFT identifies massive bulk fields with higher-dimension operators in the field theory. From this point of view, α' corrections to the physics of light bulk fields are associated with the effects of higher-dimension operators coupled to the low-dimension operators in the RG flow. The OPE tells us that, at large Q^2, the effects of these higher-dimension operators in the field theory are controllably small. From this point of view, despite the apparently large α' corrections, it is reasonable to begin with a local bulk action in terms of fields corresponding to the low dimension operators of QCD. The example of SVZ gives us hope that this can correctly capture physics of light mesons. For highly excited mesons, which in QCD look like extended flux tubes, and thus feel long distances, it is more probable that the strong IR physics will mix different operators and that our neglect of α'-like corrections will become more troublesome. It is a general problem, in fact, that physics of highly excited hadrons is troublesome in these models [65]. Recent proposals for backgrounds with correct Regge physics [66] offer some hope of addressing this problem. We will offer some further comments on how a closed-string tachyon might give a dynamical explanation of such a background, but there is clearly much more to be done along these lines.

The paper is organized as follows. In section 2 we will remind the reader of the basic formulation of AdS/QCD on Randall-Sundrum backgrounds as in Reference[55], and discuss some of its shortcomings. In section 3 we will discuss a class of models that do not incorporate the running of the QCD coupling, but do incorporate the effects of the lowest vacuum condensate. We will point out that these backgrounds too have some shortcomings, but improve on the RS backgrounds, and may provide a useful setting for exploring some questions. We calculate the gluon condensate and point out that there is a zero mode in the glueball spectrum due to the spontaneous breaking of scale invariance. In section 4 we discuss the construction of 5D theories with asymptotic freedom. We calculate the gluon condensate and provide a first estimate of masses for the 0^{++} glueballs from these backgrounds. We point out a possible interpretation of the running coupling where α_s remains finite for all energies. In section 5 we show how to systematically incorporate the effects of the higher condensates. We give a background including the effect of $\langle \mathrm{TrF}^3 \rangle$ and show how it affects the gluon condensate and the glueball spectrum. In section 6 we discuss the difficulties of reproducing the correct Regge physics, and speculate that closed string tachyon dynamics could perhaps be responsible for reproducing the necessary IR background. Finally we provide the conclusions and some outlook in section 7.

3.2 AdS/QCD on Randall-Sundrum backgrounds

We briefly review the AdS/QCD model on Randall-Sundrum backgrounds [55]. One assumes that the metric is exactly AdS in a finite region $z = 0$ to $z = z_c$, i.e.

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right), \ 0 \le z \le z_c, \tag{3.2}$$

where $z_c \sim \Lambda_{QCD}^{-1}$ determines the scale of the mass spectrum. It is assumed that there is a brane (the "infrared brane") at $z = z_c$; in practice one assumes a UV boundary at $z = \epsilon$ and sends $\epsilon \to 0$ at the end of calculations. One puts a field $\phi_{\mathcal{O}}$ in the bulk for every gauge invariant operator $\mathcal{O}$ in the gauge theory. If $\mathcal{O}$ is a p-form of dimension Δ, ϕ has a 5D mass $m_5^2 = (\Delta + p)(\Delta + p - 4)$. This operator has UV boundary conditions dictated by the usual AdS/CFT correspondence. On the other hand, IR boundary conditions are less clear, and one in principle can add localized terms on the IR brane.

For instance, in the original papers the treatment involves the rho and a_1 mesons and the pions. There are bulk gauge fields, A_M^L and A_M^R (where M is a 5D Lorentz index) coupling to the operators $\bar{q}_L \gamma^\mu t^a q_L$ and $\bar{q}_R \gamma^\mu t^a q_R$. There is also a bulk scalar $X^{\alpha\beta}$ coupling to the operator $\bar{q}_R^\alpha q_L^\beta$, where α and β are flavor indices. The scalar $X^{\alpha\beta}$ is *assumed* to have a profile proportional to $\delta^{\alpha\beta} \left(m_\alpha z + \langle q_\alpha \bar{q}_\alpha \rangle z^3 \right)$, based on the Klebanov-Witten result [43] that a nonnormalizable term in a scalar profile corresponds to a perturbation of the Lagrangian by a relevant operator (in this case, $m\bar{q}q$), while a normalizable term corresponds to a spontaneously generated VEV for the corresponding field theory operator. The profile for $X(z)$ couples differently to the vector and axial vector mesons and achieves the $\rho - a_1$ mass splitting. Furthermore, the pions arise as pseudo-Nambu-Goldstone modes of broken chiral symmetry, and the Gell-Mann–Oakes–Renner relationship is satisfied. Several other constants in the chiral Lagrangian are determined to around 10% accuracy.

There are several limitations to this model. One is that it does not take into account the power corrections to the OPE in the UV, or the corrections coming from logarithmic running. The SVZ sum rules work fairly well with just the leading power correction taken into account [61], and these power corrections can be incorporated into the form of the metric with a simple ansatz[63]. However, back-reaction has not been taken into account in such studies, so the Einstein equations will not be exactly satisfied. Also, in the SVZ sum rule approach, leading corrections to the OPE essentially determine the mass scale of the lightest resonance in the corresponding channel. In the Randall-Sundrum approach, the leading correction to the OPE and the IR wall at z_c *both* influence the corresponding mass scale. This has effects on the spectrum.

For instance, in QCD the mass scales associated with mesons made from quarks and mesons made from gluons are very different [67]. One can see this in the OPE. For instance, in the tensor 2^{++} channel, the leading corrections to the OPE come from the same operator in the $q\bar{q}$ case as in the G^2 case, but the coefficients are very different. In the Randall-Sundrum approach, without turning on a background field VEV the mass scale for both of these channels is set by z_c^{-1}. One can turn on a background VEV and give the fields different couplings to it to reproduce the difference in the OPEs, but z_c^{-1} will continue to play a role in setting their masses.

Another limitation is that these backgrounds lack asymptotic freedom. If one defines the theory with a cutoff at which α_s has some finite value, this might not appear to be of central importance. On the other hand, we understand that in real QCD the values of the condensates are determined by the QCD scale at which the perturbative running coupling blows up, so achieving asymptotic freedom can allow such a relationship and make the model less *ad hoc*. Furthermore, we will see that the lack of proper conformal symmetry breaking can lead to a massless scalar glueball state (i.e., the model has a "radion problem"), which is most satisfactorily resolved by incorporating asymptotic freedom.

In summary, the AdS/QCD models on hard-wall backgrounds work surprisingly well for some quantities, but have obvious drawbacks. There are ambiguities in IR boundary conditions, and the existence of a single IR wall influencing all fields obscures the relationship between masses of light resonances and power corrections discovered by Shifman, Vainshtein, and Zakharov. Luckily, there is a simple remedy to these difficulties: we simply remove the IR brane, and allow the growth of the condensates to dynamically cut the space off in the infrared.

3.3 Vacuum condensates as IR cutoff

To model the pure gauge theory we begin with the action for five-dimensional gravity coupled to a dilaton (in the Einstein frame):

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \sqrt{g}(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi). \tag{3.3}$$

Here κ^2 is the 5 dimensional Newton constant and R is the AdS curvature (related to the the (negative) bulk cosmological constant as $R^{-2} = -\frac{\kappa^2}{6}\Lambda$. Note, that ϕ is dimensionless here. The dilaton will couple to the gluon operator $G_{\mu\nu}G^{\mu\nu}$. The fact that there is a non-vanishing gluon condensate in QCD is expressed by the fact that the dilaton will have a non-trivial background. We can find the most general such background by solving the coupled system of the dilaton equation of motion and the Einstein's equation, under the ansatz that we preserve four-dimensional Lorentz invariance while the fifth dimension has a warp factor:

$$\begin{aligned} ds^2 &= e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \\ \phi &= \phi(y). \end{aligned} \tag{3.4}$$

The coupled equations for $A(y), \phi(y)$ will then be

$$
\begin{aligned}
&4A'^2 - A'' = 4R^2 \\
&A'^2 = \frac{\phi'^2}{24} + \frac{1}{R^2} \\
&\phi'' = 4A'\phi'.
\end{aligned}
\tag{3.5}
$$

A simple way of solving these equations is to use the superpotential method [68, 69], that is define the function $W(\phi)$ such that

$$
\begin{aligned}
&A'(y) = W(\phi(y)) \\
&\phi'(y) = 6\frac{\partial W}{\partial \phi}.
\end{aligned}
\tag{3.6}
$$

This is always possible, and the equation determining the superpotential is given by

$$
V = 18\left(\frac{\partial W}{\partial \phi}\right)^2 - 12W^2 = -\frac{12}{R^2}. \tag{3.7}
$$

To solve for the most general superpotential consistent with our chosen (constant) potential, it is useful to parameterize it in terms of a "prepotential" w as [69]

$$
\begin{aligned}
&W = \frac{1}{R}\left(w + \frac{1}{w}\right) \\
&W' = \sqrt{\frac{2}{3}}\frac{1}{R}\left(w - \frac{1}{w}\right),
\end{aligned}
\tag{3.8}
$$

which is chosen such that (3.7) is automatically satisfied. The consistency condition of the two equations in (3.8) implies a simple equation for the prepotential:

$$
w' = \sqrt{\frac{2}{3}}w, \tag{3.9}
$$

and thus for the superpotential we find

$$
W(\phi) = \frac{1}{R}(Ce^{\sqrt{\frac{2}{3}}\phi} + C^{-1}e^{-\sqrt{\frac{2}{3}}\phi}). \tag{3.10}
$$

With the superpotential uniquely determined (up to a constant C) we can then go ahead and integrate the equations in (3.6). The result is

$$\phi(y) = \sqrt{\frac{3}{2}} \log\left[C \tanh 2(y_0 - y)/R\right],$$
$$A(y) = -\frac{1}{4} \log\left[\cosh 2(y_0 - y)/R \sinh 2(y_0 - y)/R\right] + A_0, \qquad (3.11)$$

where y_0 and A_0 are integration constants. These are the solutions also found in [69, 70]. In order to have $A(y)$ asymptotically equal to y (for large negative y), we will fix $A_0 = \frac{y_0}{R} - \frac{1}{2}\log 2$. Note, that there is another branch of the solution where in which tanh is replaced by coth in the solution for ϕ. In order to have a form of the solution that is more familiar and useful, we make a coordinate transformation $e^{\frac{y-y_0}{R}} = z/z_c$. This will recast the solution in a form that matches the usual conformal coordinates x^μ, z near $z = 0$:

$$\mathrm{d}s^2 = \left(\frac{R}{z}\right)^2 \left(\sqrt{1 - \left(\frac{z}{z_c}\right)^8}\, \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right) \qquad (3.12)$$

$$\phi(z) = \sqrt{\frac{3}{2}} \log\left(\frac{1 + \left(\frac{z}{z_c}\right)^4}{1 - \left(\frac{z}{z_c}\right)^4}\right) + \phi_0. \qquad (3.13)$$

Here z_c is a new parameter determining the IR scale, and we expect $z_c \sim \Lambda_{QCD}^{-1}$. The point $z = z_c$ is a naked singularity, which we must imagine is resolved in the full string theory. Also note that the first correction to the AdS_5 metric goes as z^8.

3.3.1 Gluon condensate

We have seen that the metric incorporates power corrections to the pure AdS solution, which we want to identify with the effects of the gluon condensate. Below we would like to make this statement more precise. According to the general rules of the AdS/CFT correspondence given some field $\varphi_{\mathcal{O}}$ on this background representing an operator $\mathcal{O}$ in QCD, the deep-Euclidean correlator of $\mathcal{O}$ will have a Q^{-8} correction if $\varphi_{\mathcal{O}}$ has no dilaton coupling, and a Q^{-4} correction if $\varphi_{\mathcal{O}}$ couples to ϕ. Note that near $z = 0$, the dilaton behaves $\phi_0 + \sqrt{6}\frac{z^4}{z_c^4}$. This is in agreement with the expectation that a field coupling to an operator with dimension Δ has two solutions, $z^{\Delta-d}$ and z^{Δ}. In our case $\Delta = d = 4$, and we expect the constant piece to correspond to the source for the operator $\mathrm{Tr}G^2$ and coefficient of the z^4 to give the gluon condensate. The precise statement [43] from AdS/CFT is that if solution to the classical equations of motion Φ has the form near the boundary

$$\Phi(x, z) \to z^{d-\Delta}[\Phi_0(x) + \mathcal{O}(z^2)] + z^{\Delta}[A(x) + \mathcal{O}(z^2)] \tag{3.14}$$

then the condensate (one-point function) of the operator $\mathcal{O}$ is given by

$$\langle \mathcal{O}(x) \rangle = (2\Delta - d)A(x). \tag{3.15}$$

However, to apply this to the solution in (3.13) we need to make sure that the appropriate normalization of the fields is used. The expression (3.15) is derived assuming a scalar field action of the form $1/2 \int d^4x dz/z^3 (\partial_M \phi)^2$. Comparing this with the action used here (3.3) we find

$$\langle \mathrm{Tr}G^2 \rangle = 4\sqrt{3}\sqrt{\frac{R^3}{\kappa^2}}\frac{1}{z_c^4}. \tag{3.16}$$

In order to be able to relate this expression for the condensate we need to find an expression for R^3/κ^2. This can be done by requiring that the leading term in the OPE of the gluon operator G^2 is correctly reproduced in the holographic theory. One can simply calculate the leading term in the OPE in QCD [71]

$$\int d^4x \langle G^2(x) G^2(0) \rangle e^{iqx} = -\frac{(N^2-1)}{4\pi^2} q^4 \log\frac{q^2}{\mu^2} + \dots \tag{3.17}$$

The same quantity can be calculated in the gravity theory by evaluating the action for the scalar field with a give source $\phi_0(q)$ in the UV. The general expression for the action is obtained (after integrating by parts and using the bulk equation of motion)

$$S_{5D} = \frac{1}{2\kappa^2}\int d^4x \frac{R^3}{z^3}\frac{1}{2}\phi\partial_z\phi|_{z\to 0}. \tag{3.18}$$

In order to find the action one can use the bulk equation of motion close to the UV for the scalar field given by

$$\phi'' + q^2\phi - \frac{3}{z}\phi' = 0. \tag{3.19}$$

Requiring that the wave function approaches 1 around $z = 0$ will fix the leading terms in the wave function:

$$\phi(z) = 1 - \frac{1}{32}q^4 z^4 \log q^2 z^2 + \frac{1}{4}q^2 z^2 + \dots. \tag{3.20}$$

Taking the second derivative of the action we find

$$\int d^4x \langle G^2(x) G^2(0)\rangle e^{iqx} = -\frac{R^3}{16\kappa^2} q^4 \log q^2/\mu^2 + \dots, \tag{3.21}$$

from which we get the identification

$$\frac{R^3}{\kappa^2} = \frac{4(N^2-1)}{\pi^2}. \tag{3.22}$$

Using this result we get a prediction for the gluon condensate

$$\langle \mathrm{Tr} G^2 \rangle = \frac{8}{\pi z_c^4}\sqrt{3(N^2-1)}. \tag{3.23}$$

3.3.2 The Glueball Spectrum

Now we wish to solve for the scalar glueball spectrum, which is associated with scalar fluctuations about the dilaton–metric background. The analogous calculation in the supergravity model has been performed in [72]. We should solve the coupled radion–dilaton equations, as often there is a light mode from the radion. In other words, we should solve for eigenmodes of the coupled Einstein-scalar system. This has been worked out in detail for a generic scalar background in [73]: the linearized metric and scalar ansatz is given by:

$$\begin{aligned} ds^2 &= e^{-2A(y)}(1-2F(x,y))dx^\mu dx^\nu \eta_{\mu\nu} - (1+4F(x,y))dy^2 \\ \phi(x,y) &= \phi_0(y) + \frac{3}{\kappa^2 \phi_0}\left(F'(x,y) - 2A'(y)F(x,y)\right). \end{aligned} \tag{3.24}$$

This will satisfy the coupled Einstein-scalar equations if $F = F(y)e^{iq\cdot x}$ with $q^2 = m^2$ and $F(y)$ satisfies the differential equation

$$F'' - 2A'F' - 4A''F - 2\frac{\phi_0''}{\phi_0'}F' + 4A'\frac{\phi_0''}{\phi_0'}F = -e^{-2A}m^2 F. \tag{3.25}$$

Using the solutions for $A(y)$ and $\phi_0(y)$ from Eq. 3.11, and an ansatz $F(x,y) = F(y)e^{iq\cdot x}$, with $q^2 = -m^2$, this becomes:

$$F''(y) + \frac{10}{R}\coth\frac{4(y-y_0)}{R}F'(y) + \left(\frac{16}{R^2} + m^2\frac{e^{2y_0/R}}{\sqrt{2\sinh\frac{4(y_0-y)}{R}}}\right)F(y) = 0. \tag{3.26}$$

Demanding a normalizable solution, we need that (in the z coordinates) $\int dz\sqrt{g}|\varphi(z)|^2$ and $\int dz\sqrt{g}g^{55}|\partial_z\varphi(z)|^2$ be finite. Thus we need $\varphi(z) \sim z^4$ at small z. We need to be somewhat more careful about solving for F: in the z coordinates, we have that

$$z \to 0 : z\frac{dF}{dz} - 2z\frac{dA}{dz}F \to z\frac{dF}{dz} - 2F \sim \varphi(z), \tag{3.27}$$

so that $F(z) \sim z^2$ near $z = 0$ is compatible with our assumptions on the behavior of $\varphi(z)$. As usual, this equation can be solved using the shooting method: the differential equation is solved numerically starting from the UV boundary with arbitrary normalization the BC following from (3.27) (the numerics is very insensitive to the choice of the actual UV BC for the higher modes) for varying values of m^2. For discrete values of m^2 the wave function will be normalizable (which numerically is equivalent to requiring a Neumann BC at the location of the singularity). This way we find (in units of z_c^{-1}) glueballs with masses 6.61, 9.84, 12.94, and 15.98 (and so forth, with regular spacing in mass).

There is also a serious problem: we find a massless mode for the radion (with the $F(z) \sim z^2$ UV boundary conditions). This can be understood as follows: in real QCD, the classical conformal symmetry is broken by the scale anomaly. However, in our model, it is broken by the z^4 profile of the dilaton. AdS/CFT tells us we should understand turning on such a normalizable background in the UV as a spontaneous symmetry breaking. In Randall-Sundrum models, one similarly has a radion problem and needs to invoke (for instance) a Goldberger-Wise stabilization [9] to avoid a massless mode. The radion has not been part of previous investigations of glueballs on Randall-Sundrum backgrounds [74], so such studies have essentially assumed that the stabilization mechanism removes a light mode from the spectrum. However, an added Goldberger-Wise field does not seem to correspond to an operator of QCD, so it goes against the spirit of the AdS/CFT correspondence. It is apparent that a palatable solution of the radion problem in AdS/QCD demands a 5D treatment of scale dependence that mirrors that in 4D. This motivates us to search for backgrounds incorporating asymptotic freedom, which also allows us to begin approaching a more detailed matching to perturbative QCD.

3.4 Incorporating Asymptotic freedom

We have shown in the previous section a background that incorporates the lowest QCD condensate $\mathrm{Tr}F^2$ and which automatically provides an IR cutoff via the backreaction of the metric. However, this setup is certainly too simplistic even to just produce the main features of QCD: for example asymptotic freedom is not reproduced in that setup. It is the dilaton field that also sets the QCD coupling constant, and by approaching a constant value the model in the previous section actually describes a theory that approaches a conformal fixed point in the UV, rather than QCD. One may think that this is not an important difference for the IR physics, but this is not quite right. For example as we have seen it introduces a “radion problem.”

Thus we set out to find a potential for the dilaton that will reproduce the logarithmic running of the coupling. We assume that similarly to string theory the gauge coupling is actually given by $e^{b\phi(z)}$ (where b is a numerical constant). We will find a result consistent with expectations from string theory.[1]

We assume that our action is:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \sqrt{g} (-\mathcal{R} - V(\phi) + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi), \tag{3.28}$$

where now we will try to determine $V(\phi)$ such that we reproduce asymptotic freedom. If we require that the coupling runs logarithmically, and as usual identify the energy scale with the inverse of the AdS coordinate z we need to have a solution of the form

$$e^{b\phi(z)} = \frac{1}{\log \frac{z_0}{z}}, \tag{3.29}$$

where $z_0 = \Lambda_{QCD}^{-1}$ and we do not fix b *a priori*. Then going to the y coordinates as usual via the definition $e^{y/R} = z/R$, we will find

$$e^{b\phi(y)} = \frac{R}{y_0 - y}. \tag{3.30}$$

[1]Other discussions of backgrounds with logarithmically running coupling can be found in [75, 76].

If we now assume that this solution follows from a superpotential W then $\phi'(y) = 6\frac{\partial W}{\partial \phi} = \frac{1}{b(y_0-y)} = \frac{1}{bR}e^{b\phi}$. This implies that $W(\phi) = \frac{1}{6Rb^2}e^{b\phi} + W_0$. Now that we have found the form of the superpotential, we can easily solve for the warp factor: $A'(y) = W(\phi(y)) = \frac{1}{6b^2(y-y_0)} + W_0$, and hence $A(y) = A_0 + W_0 y + \frac{1}{6b^2}\log\frac{R}{y_0-y}$. In z coordinates, this becomes:

$$A(z) = A_0 + W_0 R \log z/R - \frac{1}{6b^2}\log\log z_0/z, \tag{3.31}$$

and hence $e^{-2A(z)} = e^{-2A_0}(R/z)^{2W_0R}(\log z_0/z)^{1/(3b^2)}$. From this we conclude that we should take $A_0 = 0$ and $W_0 = \frac{1}{R}$ to get a solution that looks AdS-like up to some powers of $\log z_0/z$.

The potential corresponding to this superpotential is then given by

$$V(\phi) = 18\left(\frac{\partial W}{\partial \phi}\right)^2 - 12W^2 = -\frac{1}{3b^2R^2}\left(\left(\frac{1}{b^2} - \frac{3}{2}\right)e^{2b\phi} + 12e^{b\phi} + 36b^2\right). \tag{3.32}$$

This is *particularly* simple in the case that $b = \pm\sqrt{\frac{2}{3}}$. In that case we have simply

$$V(\phi) = -\frac{6}{R^2}e^{\pm\sqrt{\frac{2}{3}}\phi} - \frac{12}{R^2} \tag{3.33}$$

$$W(\phi) = \frac{1}{R}\left(\frac{1}{4}e^{\pm\sqrt{\frac{2}{3}}\phi} + 1\right) \tag{3.34}$$

$$\phi = \mp\sqrt{\frac{3}{2}}\log\frac{y_0-y}{R} = \mp\sqrt{\frac{3}{2}}\log\log\frac{z_0}{z} \tag{3.35}$$

$$A = \frac{y}{R} + \frac{1}{4}\log\frac{R}{y_0-y} = \log\frac{z}{R} - \frac{1}{4}\log\log\frac{z_0}{z}, \tag{3.36}$$

and thus the metric will be

$$ds^2 = \left(\frac{R}{z}\right)^2\left(\sqrt{\log\frac{z_0}{z}}dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right) = \frac{e^{-2\frac{y}{R}}}{\sqrt{y_0-y}}dx^\mu dx^\nu \eta_{\mu\nu} - dy^2. \tag{3.37}$$

Our dilaton in (3.28) is normalized in an unusual way, nevertheless (3.33) is recognizable as a potential that commonly occurs in nonsupersymmetric string theory backgrounds: a cosmological constant plus a term exponential in the dilaton.

This is quite reasonable from the string theory perspective, where such a term can arise from dilaton tadpoles in critical backgrounds or from the central charge in noncritical backgrounds. In fact, the factor $\sqrt{2/3}$ arises from string theory considerations in a simple way, which may be an amusing coincidence or may have more significance. Suppose that there is a noncritical string theory in 5 dimensions. Its action in string frame has the form [77]

$$S = \frac{1}{2\kappa_0^2}\int d^5x(-G)^{1/2}e^{-2\Phi}\left(C + R + 4\partial_\mu\Phi\partial^\mu\Phi + \cdots\right), \tag{3.38}$$

where C is proportional to the central charge and is nonvanishing since we are dealing with a noncritical string. Now we go to Einstein frame:

$$S = \frac{1}{2\kappa^2}\int d^5x(-\tilde{G})^{1/2}\left(Ce^{4\tilde{\Phi}/3} + \tilde{R} - \frac{4}{3}\partial_\mu\tilde{\Phi}\tilde{\partial}^\mu\tilde{\Phi} + \cdots\right), \tag{3.39}$$

and finally we note that comparing to our normalization above, $\tilde{\Phi} = \sqrt{3/8}\phi$, so that $e^{4\tilde{\Phi}/3} = e^{\sqrt{2/3}\phi}$.

3.4.1 The Glueball Spectrum

To calculate the glueball spectrum we can apply Eq. (3.25) for the background in (3.35-3.36). This equation (transformed to z coordinates) reduces to (in units of R)

$$z^2F''(z) - z\left(1 + \frac{5}{2\log\frac{z_0}{z}}\right)F'(z) + \left(\frac{4}{\log\frac{z_0}{z}} + \frac{m^2z^2}{\sqrt{\log\frac{z_0}{z}}}\right)F(z) = 0. \tag{3.40}$$

Using the shooting method again we find (in units of z_0^{-1}) glueballs at 2.52, 5.45, 8.16, and 10.81. In particular this background seems to have a light mode from the radion, but not any zero mode.

Lattice estimates put the first 0^{++} glueball in pure SU(3) gauge theory at approximately 1730 MeV, and the second at about 2670 MeV, with uncertainties of order 100 MeV [78]. Thus they put the ratio of the first and second scalar glueball masses about about 1.54, whereas we find a significantly larger value of 2.16. While the lattice errors are still fairly large, this probably indicates that we are not so successful at precisely determining properties of the second scalar glueball resonance. Since we undoubtedly fail to properly describe highly excited resonances, this is not so surprising. If we set z_0^{-1} to match the lattice estimate for the first glueball mass, we find

$$z_0^{-1} \approx 680 \text{ MeV}. \tag{3.41}$$

One can also calculate the spectrum of spin 2^{++} glueball masses by solving the fluctuations of the Einstein equation around the background. The resulting differential equation we find is

$$z \log \frac{z_0}{z} f''(z) - (1 + 3 \log \frac{z_0}{z}) f'(z) + m^2 z \sqrt{\log \frac{z_0}{z}} f(z) = 0. \tag{3.42}$$

Using the shooting method (and imposing Dirichlet BC on the UV boundary) we find the lightest modes at 4.03, 6.56, ... in units of $1/z_0$. It is important to point out that the lowest spin 2^{++} glueball is naturally heavier in this setup than the spin 0^{++} glueball due to the mixing of the radion with the dilaton. In the usual supergravity solutions the spin 0^{++} and 2^{++} glueballs usually end up degenenerate (in contradiction to lattice simulations). This is for example the case in the AdS black hole solution of Witten analyzed in [72] (the additional light scalar modes identified in [79] do not correspond to QCD modes).

3.4.2 Power Corrections and gluon condensate

We would now like to evaluate the gluon condensate in this theory assuming that the the IR scale z_0 is fixed by the value of the lightest glueball mass. As usual one needs to calculate the 5D action corresponding to a fixed source term turned on for the QCD coupling and to get the condensate (one -point function) we need to differentiate the 5D action with respect to the source. Ordinarily, the computation of the 5D action on a given solution in AdS/QCD reduces to simply a boundary term. However, in our case it is not so simple: our potential is not just a mass term, so that the bulk piece $V - \frac{\phi}{2}\frac{\partial V}{\partial \phi}$ is not set to zero by the equations of motion. As a result, the action also has a "bulk" piece.

We evaluate the 5D action as a function of z_0, imposing a UV cutoff at ϵ. The action is given by:

$$\frac{1}{2\kappa^2}\int_{z=\epsilon}^{z_0}\left(\frac{R}{z}\right)^5 \log\frac{z_0}{z}\left(-\mathcal{R} - \frac{1}{2}z^2\phi'(z)^2 + \frac{12}{R^2} + \frac{6}{R^2}e^{\sqrt{\frac{2}{3}}\phi(z)}\right)dz. \tag{3.43}$$

This integral can be performed explicitly:

$$\frac{1}{2\kappa^2}\left[\frac{1}{2z^4} + \frac{2\log\frac{z_0}{z}}{z^4}\right]_{z=\epsilon}^{z_0} \tag{3.44}$$

We drop the UV divergent terms (the $1/\epsilon^4$ pieces) assuming that there will be counter terms absorbing these. Then the explicit expression for the action will be

$$S(z_0) = \frac{1}{4\kappa^2 z_0^4}. \tag{3.45}$$

Note that this is more easily calculated as

$$S(z_0) = \frac{1}{2\kappa^2}\int d^4x\ 2\sqrt{g_{4D}}\ W(\phi), \tag{3.46}$$

evaluated at the boundary $z = z_0$, where g_{4D} is the induced 4D metric at the boundary. (This observation has been made before in Reference [80].) To see this, we use the following relations:

$$-\mathcal{R}(y) = -20A'(y)^2 + 8A''(y) \;=\; -20W(\phi)^2 + 48\left(\frac{\partial W}{\partial \phi}\right)^2 \qquad (3.47)$$

$$-\frac{1}{2}\phi'(y)^2 \;=\; -18\left(\frac{\partial W}{\partial \phi}\right)^2 \qquad (3.48)$$

$$-V(\phi) \;=\; -18\left(\frac{\partial W}{\partial \phi}\right)^2 + 12W^2 \qquad (3.49)$$

to see that $\sqrt{g}S_{5D}$, evaluated on the solution, is

$$e^{-4A(y)}\left(-8W^2 + 12\left(\frac{\partial W}{\partial \phi}\right)^2\right) = 2\frac{d}{dy}\left(e^{-4A(y)}W(\phi(y))\right). \qquad (3.50)$$

This makes it clear that we can use the superpotential as a counterterm on the UV boundary to cancel the terms diverging as $\epsilon \to 0$ (which we dropped above.)

It turns out that $\frac{1}{\kappa^2}$ is almost precisely as in the background without asymptotic freedom (because, for the fluctuating modes, corrections to the wavefunction near $z = 0$ are small α_s corrections), so one can still use (3.22) to find the value of R^3/κ^2. However, there is a slight subtlety: we found a value for $\frac{1}{\kappa^2}$ assuming a source coupled to $\mathrm{Tr}G^2$. In fact in our case we have fixed the numerical factor in the correspondence of $e^{\sqrt{2/3}\phi}$ based on its asymptotic behavior. Using the expression for the coupling in a pure YM theory

$$\alpha_{YM}(Q) = \frac{2\pi}{\frac{11}{3}N_c \log\frac{Q}{\Lambda_{QCD}}} \qquad (3.51)$$

and identifying $\Lambda_{QCD} = \frac{1}{z_0}$ and $Q = \frac{1}{z}$, we have at the cutoff $z = \epsilon = \frac{1}{\Lambda}$:

$$e^{\sqrt{\frac{2}{3}}\phi(\epsilon)} = \frac{11N_c}{6\pi}\alpha_{YM}(\Lambda) = \frac{11N_c}{24\pi^2}g^2_{YM}(\Lambda). \qquad (3.52)$$

Now, for a fluctuation $\varphi(z)$, we have

$$e^{\sqrt{2/3}(\phi(z)+\varphi(z)e^{iq\cdot x})} \approx e^{\sqrt{2/3}\phi(z)}(1 + \sqrt{2/3}\varphi(z)e^{iq\cdot x}). \qquad (3.53)$$

Now, $\varphi(z)$ near $z = 0$ behaves like any massless scalar fluctuation on an AdS background, and thus we have that it shifts the action by an amount

$$S_{5D} = \frac{1}{2\kappa^2}\varphi(\epsilon)^2 \frac{-1}{32} q^4 \log q^2/\mu^2 + \cdots . \tag{3.54}$$

The key now is to understand precisely which field theory correlator corresponds to taking the second derivative of this expression with respect to $\varphi(\epsilon)$. The field theory action is $-\frac{1}{4g_{YM}^2}F^2$; effectively, we are adding a source by taking the coefficient to be instead $-\frac{1}{4g_{YM}^2}(1+\delta e^{iq\cdot x})$. Comparing this to Eq. 3.53, we see that $\delta = -\sqrt{\frac{2}{3}}\varphi(\epsilon)$. Now, we have the two-point correlator for $G^2 = g_{YM}^{-2}F^2$:

$$\int d^4x \langle \frac{1}{4}G^2(x)\frac{1}{4}G^2(0)\rangle e^{iqx} = -\frac{(N_c^2-1)}{64\pi^2} q^4 \log\frac{q^2}{\mu^2} + \ldots , \tag{3.55}$$

which should correspond to taking a second derivative with respect to δ of our above result, and so we find:

$$\frac{R^3}{\kappa^2} = \frac{64}{3}\frac{N_c^2-1}{64\pi^2} = \frac{(N_c^2-1)}{3\pi^2}. \tag{3.56}$$

In particular, for $N_c = 3$, this means that

$$S_{5D} = \frac{2}{3\pi^2 z_0^4}. \tag{3.57}$$

In order to find the actual condensate, we have to differentiate the action with respect to the value of the source on the boundary. In our case the source is just the QCD coupling itself g_{YM}^{-2}. Using the expression for the coupling in a pure YM theory

$$\alpha_{YM}(Q) = \frac{2\pi}{\frac{11}{3}N_c \log\frac{Q}{\Lambda_{QCD}}} \tag{3.58}$$

and identifying $\Lambda_{QCD} = \frac{1}{z_0}$ we find that the derivative with respect to g_{YM}^{-2} (viewing g_{YM} as a function of z_0) is the same as $\frac{24\pi^2}{11N_c} z_0 \frac{d}{dz_0}$.

Putting all this together, we find that:

$$\left\langle \frac{1}{4}\mathrm{Tr}F^2 \right\rangle = \frac{(N_c^2-1)}{12\pi^2}\frac{24\pi^2}{11N_c} 4 z_0^{-4} \approx (1.19 z_0^{-1})^4. \tag{3.59}$$

Using our estimate of z_0 from the glueball mass in Eq. 3.41, we obtain:

$$\left\langle \frac{1}{4\pi^2}\mathrm{Tr}F^2 \right\rangle = \frac{1}{\pi^2}(1.19 \times 680 \text{ MeV})^4 \approx 0.043 \text{ GeV}^4. \tag{3.60}$$

For comparison, an SU(3) lattice calculation found $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \approx 0.10$ GeV^4 [81]. Our result is of the same order but slightly smaller. Most phenomenological estimates are smaller, beginning with the SVZ result of $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \approx 0.012$ GeV^4, but for pure Yang-Mills the value is expected to increase [61].

3.4.3 Relation to Analytic Perturbation Theory

The QCD perturbation series is an asymptotic expansion of some unknown function, and the divergence at Λ_{QCD} signals only a breakdown of perturbation theory, not a meaningful infinity. In particular, it has been proposed that the pole of the logarithm be cancelled by additional terms to produce an "analytic perturbation theory." See, for instance, the work of Shirkov and Solovtsov [82] and related literature (of which there is too much to give an exhaustive account here). It is interesting that our holographic equations produce a result along these lines, when interpreted in a particular way.

To see this, note that our identification of the coordinate z with the inverse of a renormalization group scale μ is only clearly defined in the far UV (near $z = 0$). In fact, when we take as a metric ansatz $\mathrm{d}s^2 = \exp(-2A(y))\mathrm{d}x^2 + \mathrm{d}y^2$, it is more reasonable to interpret $A(y)$ as $-\log \mu R$, so that the 4D part of the metric goes like $\mu^2 \mathrm{d}x^2$. Our expression for the QCD coupling in terms of the dilaton $\exp \sqrt{2/3}\phi(y)$ is given by $\frac{R}{y_0 - y}$, which blows up at a finite coordinate in exactly the way that the one-loop QCD beta function tells us α_s should blow up at $\mu = \Lambda_{QCD}$. On the other hand, using the modified identification of the energy scale explained above,

$$A(y) \leftrightarrow -\log \mu R,$$

that is by identifying *the warp factor* (instead of y) as the logarithm of the energy scale, we find that $y \to y_0$ corresponds to $\mu \to 0$. Thus we can view $\exp \sqrt{2/3}\phi(y)$ as providing a formula for $\alpha_s(\mu)$ that is smoothly defined at all μ, which blows up as a power law in the deep infrared $\mu \to 0$ (instead of $\mu \to \Lambda_{QCD}$) and reduces to the perturbative result at large μ.

In particular, one can solve for $\alpha_s(\mu)$ according to this prescription. The relevant expressions

$$\frac{6\pi}{11N_c}\alpha_s^{-1}(\mu) = (y_0 - y(\mu))/R \tag{3.61}$$

$$-\log \mu R = \frac{y(\mu)}{R} + \frac{1}{4}\log \frac{R}{y_0 - y(\mu)} \tag{3.62}$$

$$\Lambda_{QCD} = z_0^{-1} = \frac{1}{R}e^{-y_0/R} \tag{3.63}$$

can be inverted to find

$$\frac{1}{\alpha_s(\mu)} = \frac{11N_c}{24\pi}W(4\mu^4/\Lambda_{QCD}^4), \tag{3.64}$$

where $W(y)$ is the Lambert W-function [83], that is, the principal value of the solution to $y = x\exp(x)$. In fact, the Lambert W-function appears similarly in the analytic perturbation theory approach [84], although the form of $\alpha_s(\mu)$ is slightly different there. Nonetheless, our results are suggestive of a role for the backreaction on the metric as enforcing good analytic properties, which deserves further attention.

3.5 Effects of the $\mathrm{Tr}(F^3)$ condensate

In pure Yang-Mills, there is an operator of dimension 6, $\mathcal{O}_6 = f^{abc}F^a_{\mu\nu}F^b_{\nu\rho}F^c_{\rho\mu}$. This operator will get a condensate which modifies the OPE at higher orders. We wish to investigate the size of the corrections on the glueball masses in our approach, to understand how stable the numerics are. Thus we add a new field χ, and we wish to modify the superpotential. We should still have in the potential terms Λ and $C\exp(b\phi)$, as before. However, now we should also have a mass term for χ, with $m_\chi^2 = 6(6-4) = 12$ by the AdS/CFT correspondence. On the other hand, our theory is not conformal, and $\mathcal{O}_6$ has an anomalous dimension proportional to α_s, suggesting we should also have terms in the potential that couple ϕ and χ.

For now, we will make no attempt to constrain all the higher-order terms in the 5D action coupling ϕ and χ. Instead, we seek a superpotential with the properties discussed above, as a first approximation. Luckily, there is a superpotential which allows the profiles of the scalars and the warp factor to be found analytically. Our motivation for this particular choice is that it allows an analytic solution and has the desired properties. On the other hand, it resembles certain superpotentials that arise in gauged supergravity [85]:

$$W(\phi,\chi) = \frac{1}{4}\exp\left(\sqrt{\frac{2}{3}}\phi\right) + \cosh(B\chi). \tag{3.65}$$

(We will see shortly that $B = 1$.) The corresponding potential is then:

$$\begin{aligned} V(\phi,\chi) &= 18\left[\left(\frac{\partial W}{\partial\phi}\right)^2 + \left(\frac{\partial W}{\partial\chi}\right)^2\right] - 12W^2 \\ &= -12 - 6e^{\sqrt{\frac{2}{3}}\phi} + (18B^4 - 12B^2)\chi^2 - 3B^2 e^{\sqrt{\frac{2}{3}}\phi}\chi^2 + \mathcal{O}(\chi^4). \end{aligned} \tag{3.66}$$

We find that $\phi(y)$ is as before, whereas $\chi(y)$ is given by $\chi'(y) = 6\frac{\partial W}{\partial\chi} = 6B\sinh(B\chi)$. But this is nearly identical to the equation we solved to find $\phi(y)$ in the case *without* log running. In particular, this means that

$$\chi(y) = \log\left(\tanh\frac{3(y_1-y)}{R}\right), \tag{3.67}$$

where we have chosen $B=1$ to ensure that at small z, $\chi(z)\sim z^6$. In fact we can check this, as in the potential above we expect $(18B^4-12B^2)\chi^2 = 6\chi^2$, confirming that we want $B=1$.

Finally, we evaluate the warp factor, using $A'(y) = W(\phi(y),\chi(y))$:

$$A(y) = -\frac{1}{6}\log\left(\cosh\frac{3(y_1-y)}{R}\sinh\frac{3(y_1-y)}{R}\right) + \frac{1}{4}\log\left(\frac{R}{y_0-y}\right) + \frac{y_1}{R} - \frac{\log 2}{3}, \tag{3.68}$$

where the first term replaces the y of our solution without the inclusion of $\mathcal{O}_6$, but deviates from it in the infrared. (The constant terms correct for a constant difference between y and the first term, in the far UV.) The solution in the z coordinates is given by

$$ds^2 = \left(\frac{R}{z}\right)^2\left[\left(\log\frac{z_0}{z}\right)^{\frac{1}{2}}\left(1-\frac{z^{12}}{z_1^{12}}\right)^{\frac{1}{3}} dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right] \tag{3.69}$$

$$\phi(z) = -\sqrt{\frac{3}{2}}\log\log\frac{z_0}{z} \tag{3.70}$$

$$\chi(z) = \log\frac{1-\frac{z^6}{z_1^6}}{1+\frac{z^6}{z_1^6}}. \tag{3.71}$$

At this point the reader should be notice certain issues in our calculation. First, while it is true that the potential $V(\phi,\chi)$ couples ϕ and χ and includes a term suggestive of an anomalous dimension, the solutions for $\phi(y)$ and $\chi(y)$ themselves are completely decoupled! This, however, is not really a concern: the backreaction on the metric feels all of the terms in the potential. In other words, it is *only through A(y)* that the anomalous dimension is manifest. If, as suggested earlier, we interpret $A(y)$ as $-\log\mu$, then $\phi(\mu)$ and $\chi(\mu)$ will feel the effect of the anomalous dimension.

Another issue is that we have sacrificed precise agreement with perturbation theory for the sake of having a simple, solvable example. We have arranged to get the logarithmic running of α_s, the proper scaling dimension of $\mathcal{O}_6$, and an anomalous dimension term for $\mathcal{O}_6$ which is proportional to α_s. On the other hand, we have not been careful to match the coefficient in this anomalous dimension. Details of the OPE and anomalous dimension for this operator can be found in Reference [86]; eventually one would want a model that matches them. Of course, in our discussion of $\alpha_s(\mu)$ earlier, we also had a second β function coefficient that did not match. This suggests that our analytically solvable superpotentials, while useful for a preliminary study, probably need to be replaced by a more detailed numerical study based on a more careful matching of the holographic renormalization group. Nonetheless, the disagreement appears only at higher orders of perturbation theory, and we can already use our preliminary superpotential $W(\phi, \chi)$ to get some sense of the stability of spectra calculated in holographic models.

3.5.1 Gubser's Criterion: Constraining z_1/z_0

In our solution, $\phi(z)$ blows up at $z = z_0$ while $\chi(z)$ blows up at $z = z_1$. The space will shut off at whichever of these is encountered first. Intuitively it is clear that if the dimension 6 condensate is to make a relatively small correction to the results we have already obtained using only the dimension 4 condensate, we should have $z_1 > z_0$, so that $\chi(z)$ remains finite over the interval where the solution is defined.

In fact there is a conjecture that will enforce this condition. Namely, Gubser in Reference [87] has proposed that curvature singularities of the type arising in the geometries we are considering are allowed only if the scalar potential is bounded above when evaluated on the solution. By "allowed", one should understand that in these cases one expects the singularity to be resolved in the full string theory; geometries violating the criterion are somehow pathological. The conjecture is based on some nontrivial consistency checks involving considerations of finite temperature and examples from the Coulomb branch in AdS/CFT, but it is not proven. In any case we will assume for now that it holds for any geometry that can be properly thought of as dual to a field theory. It is clear that our original solution involving only ϕ satisfies the criterion: in that case we had $V(\phi) = -12 - 6e^{\sqrt{2/3}\phi} < 0$.

The case with χ is more subtle. We have

$$\begin{aligned} V(\phi(z), \chi(z)) &= -15 - 6e^{\sqrt{\frac{2}{3}}\phi(z)} \cosh\chi(z) + \cosh(2\chi(z)) \\ &= -6 \frac{\left(1 - \left(\frac{z}{z_1}\right)^{24}\right)}{\left(1 - \left(\frac{z}{z_1}\right)^{12}\right)^2 \log\frac{z_0}{z}} - 12 \frac{1 - 4\left(\frac{z}{z_1}\right)^{12} + \left(\frac{z}{z_1}\right)^{24}}{\left(1 - \left(\frac{z}{z_1}\right)^{12}\right)^2} \end{aligned} \quad (3.72)$$

Clearly as $z_1 \to \infty$ we recover the previous solution and Gubser's criterion is satisfied. On the other hand, as soon as $z_1 < z_0$, V begins to attain large positive values as $z \to z_1$. The reason is simple: the function $1 - 4x^{12} + x^{24}$ has a zero at $x \approx 0.9$, so the second term above can attain positive values when $z \approx z_1$, and the denominator will attain arbitrarily small values provided the singularity at z_1 is reached (i.e. $z_1 < z_0$). In fact large positive values of V are attained if and only if $z_1 < z_0$.

In short, Gubser's criterion limits us to precisely those solutions which can be viewed in some sense as a perturbation of our existing solution.

3.5.2 Condensates

To calculate the condensate we need to again evaluate the classical action for the solution, which in our case is given by

$$\begin{aligned} S = \quad & \frac{1}{2\kappa^2}\int_\epsilon^{z_0}\left(\frac{R}{z}\right)^6 \log\frac{z_0}{z}\left(1-\frac{z^{12}}{z_1^{12}}\right)^{\frac{2}{3}}\left[-\frac{1}{2}(\phi'^2+\chi'^2)\left(\frac{z}{R}\right)^2\right. \\ & \left. +12+6e^{\sqrt{\frac{2}{3}}\phi}\cosh\chi+6\sinh^2\chi-\mathcal{R}\right]. \end{aligned} \tag{3.73}$$

Again dropping the UV divergent terms we find either using (3.46) or by direct integration

$$S = \frac{1}{2\kappa^2}\frac{(z_1^{12}-z_0^{12})^{\frac{2}{3}}}{2z_0^4 z_1^8} \tag{3.74}$$

Following the steps for calculating the condensate for the single field case we find that the modified condensate is given by

$$\left\langle \frac{1}{4}\mathrm{Tr}F^2\right\rangle = \frac{(N_c^2-1)}{3\pi^2}\frac{24\pi^2}{11N_c}z_0\frac{d}{dz_0}\left[\frac{(z_1^{12}-z_0^{12})^{\frac{2}{3}}}{4z_0^4 z_1^8}\right] \approx (1.19z_0^{-1})^4\frac{1+\left(\frac{z_0}{z_1}\right)^{12}}{\left[1-\left(\frac{z_0}{z_1}\right)^{12}\right]^{\frac{1}{3}}}. \tag{3.75}$$

One can see that for $z_1 > z_0$ (as expected from the criterion of Sec. 3.5.1) this condensate is very insensitive to the actual value of z_1. In order to actually fix z_1 one would have to calculate the second condensate $\langle \mathrm{Tr}F^3\rangle$ and compare it to the lattice results. However, there are no reliable lattice estimates for this condensate available.

It is also plausible that if we account more properly for the anomalous dimension of $\mathrm{Tr}F^3$ and for matching of perturbative corrections to the OPE, we will be able to select a solution without this ambiguity. However, constructing such a solution appears to require a numerical study, which we will leave for future work.

3.5.3 Glueball spectra

Now that we have the background deformed by condensates of Tr F^2 and Tr F^3, we can again compute the glueball spectrum. What we would like to check is how sensitive the results are to the value of z_1. To simplify the numerical problem we are assuming that the low-lying glueball modes are still predominantly contained in the ϕ and A fields, and that the leading effect of turning on the χ field is to modify the gravitational background $A(y)$. Using this approximation we find the following equation satisfied by the glueball wave functions (again in z coordinates and units of R):

$$z^2 F''(z) - z \left(1 + \frac{5}{2 \log \frac{z_0}{z}} - \frac{4}{1 - \frac{z_1^{12}}{z^1 2}} \right) F'(z) +$$

$$\left(\frac{4}{\log \frac{z_0}{z}} \frac{1 + \frac{z^{12}}{z_1^{12}}}{1 - \frac{z^{12}}{z_1^{12}}} + \frac{m^2 z^2}{\left(1 - \frac{z^{12}}{z_1^{12}}\right)^{\frac{1}{3}} \sqrt{\log \frac{z_0}{z}}} - \frac{96 z^{12}}{z_1^{12} \left(1 - \frac{z^{12}}{z_1^{12}}\right)^2} \right) F(z) = 0. \quad (3.76)$$

One can see that the equation reduces to (3.40) in the limit when $z_1 \gg z_0 \geq z$. By again numerically solving this equation for various values of $z_1/z_0 > 1$ we find that the glueball eigenvalues are very insensitive to the actual value of z_1 as long as z_1 is not extremely close to z_0. For example, the lightest eigenvalue at $2.52/z_0$ increases by less than a percent while lowering z_1/z_0 from ∞ to 1.5. For $z_1/z_0 = 1.1$ the lightest mass grows by 3 percent, while for the extreme value of $z_1/z_0 = 1.01$ the growth is still just 9 percent. The glueball mass ratios are even less sensitive to z_1: the ratio of the first excited state to the lightest modes decreases by about 3 percent when changing z_1/z_0 from ∞ to 1.01. Thus we conclude that the predictions for glueball spectra presented in the previous section are quite robust against corrections from higher condensates.

3.6 Linearly confining backgrounds?

One of the most problematic aspects of holographic QCD is the deep IR physics: one expects Regge behavior from states of high angular momentum, and a linear confining potential. The solutions presented in this paper show many qualitative and quantitative similarities with ordinary QCD. However, they do not properly describe the highly excited glueball states. Since there is a singularity at a finite distance, the characteristic mass relation for highly excited glueballs will be that of ordinary KK theories (in this respect the theory is similar to the models with an IR brane put in by hand) $m_n^2 \sim n^2/z_0^2$, instead of the expected Regge-type behavior $m_n^2 \sim n/z_0^2$ [65]. Experimental and lattice data suggest that linear confinement effects persist further into the UV than one might expect, and already the light resonances observed in QCD seem to fall on Regge trajectories. Regge physics arises naturally from strings; in our approach, the more massive excitations of the 5D string correspond to high-dimension operators on the field theory side. To describe linear confinement and Regge physics accurately, then, it is conceivable that one must take into account the effects of a large number of operators. Thus we are led to seek alternative, but still well-motivated, approaches to the deep IR physics. One approach is simply to demand that the 5D fields have IR profiles that provide the desired behavior, as in Reference[66]. However, one would like to have a dynamical model of this effect. Here we first check that the backgrounds used in sections 4-5 do not reproduce such IR profiles. However in [66] it was suggested suggested that tachyon condensation might provide the appropriate dynamics. We provide a simple modification of our model possibly substantiating this idea, and speculate on its relation to known gauge theory effects: namely, UV renormalons and other $1/Q^2$ corrections as discussed by Zakharov and others [90].

3.6.1 No linear confinement in the dilaton-graviton system

We first try to find other solutions that would not have a singularity at a finite distance, even for the action (3.28) since until now we have found only a particular solution for (3.28) using a superpotential. However in general there should be an infinite family of superpotentials giving the same potential. For the general case we cannot find the other solutions analytically for all values of the coordinates. We can, however, attempt to solve the equations of motion analytically close to the UV boundary. A similar approach has been taken in ref.[88] for a type 0 string theory containing a tachyon, considered to be dual to non-supersymmetric SU(N) Yang-Mills with six adjoint scalars. In our case we will not consider a tachyon for now, as we consider 5D fields to be in one-to-one correspondence with gauge invariant operators in the 4D dual. There are some orientifold theories in type 0 that have no bulk tachyon, so our approach is not *a priori* meaningless.

In order to avoid the change of variables needed to achieve an asymptotically AdS metric (plus power corrections), we solve the equations beginning from the explicitly conformally flat parametrization of the metric,

$$\mathrm{d}s^2 = e^{-2A(z)}\left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right). \tag{3.77}$$

Then for the Einstein equations and the equation of motion for ϕ we find (using $'$ to denote derivatives with respect to z):

$$\phi''(z) - 3A'(z)\phi'(z) - e^{-2A(z)}\partial V(\phi)/\partial\phi = 0, \tag{3.78}$$

$$6A'(z)^2 - (1/2)\phi'(z)^2 + e^{-2A(z)}V(\phi) = 0, \tag{3.79}$$

$$3A'(z)^2 - 3A''(z) + (1/2)\phi'(z)^2 + e^{-2A(z)}V(\phi) = 0. \tag{3.80}$$

We again use the same potential $V(\phi) = -6e^{\sqrt{2/3}\phi(z)} - 6$ (but no longer assume the simple expression for the superpotential), and solve the equations in the UV to obtain

$$\phi(z) = \sqrt{\frac{3}{2}}\left(-\log\log\frac{1}{z} + \frac{3}{4}\frac{\log\log\frac{1}{z}}{\log\frac{1}{z}} + \frac{\gamma}{\log\frac{1}{z}} + \cdots\right), \tag{3.81}$$

$$A(z) = \log\frac{1}{z} + \frac{1}{2\log\frac{1}{z}} - \frac{3}{8}\frac{\log\log\frac{1}{z}}{\left(\log\frac{1}{z}\right)^2} + \frac{\frac{\gamma}{2}+\frac{33}{48}}{\left(\log\frac{1}{z}\right)^2} + \cdots. \tag{3.82}$$

The omitted terms are those which are smaller as $z \to 0$, i.e. power corrections in z or higher powers in $(-\log z)^{-1}$. Here γ is a parameter that is undetermined by the UV equations of motion, just as in the solution of ref.[88]. Further, we find that if we add small perturbations to $\exp(\sqrt{2/3}\phi)$ and to $A(z)$, an $\mathcal{O}(z^4)$ power correction to the former is allowed while only constant corrections to the latter are allowed. It is reassuring that we find a power correction suitable for the gluon condensate. Ref.[88] found also a correction of order z^2 and interpreted it as a UV renormalon[2] (whereas the power correction we have considered may be thought of as an IR renormalon). The tachyon was crucial to the appearance of the UV renormalon term, which might have interesting implications.

In order to find a linearly confining solution, we would need to connect these UV solutions to solutions in the $z \to \infty$ IR region, which would also give linear confinement. Suppose we then search for a solution that is valid at large z. In the case that there is no cosmological constant, there is a linear dilaton solution; this is clear since our equations are those for a noncritical string. The linear dilaton persists in the presence of a cosmological constant as a solution at large z; the equations are satisfied up to terms exponentially small at large z by a linear dilaton on flat 5D space. Such a linear dilaton background does *not* give a confining solution. Furthermore, one can check that no other power-law growth in z is allowed. In particular, the z^2 behavior as in Ref. [66] is not a solution to our equations of motion. Thus we conclude that none of the solutions of the action

[2]For a review of the physics of the UV renormalon, see Ref. [89].

(3.28) will result in a linearly confining background.

3.6.2 Linear confinement from the tachyon-dilaton-graviton system?

Although we do not understand how to *systematically* approach the study of UV renormalon corrections, it is at least superficially plausible that they are associated with a closed string tachyon. This is a natural scenario to consider, since the UV renormalon is associated with Q^{-2} corrections. Ref. [88] found such an association in a concrete string theory model. The study of such corrections has been discussed in great detail in the QCD literature, which links the idea of the UV renormalon to the quadratic corrections associated with the QCD string tension and with a hypothetical nonperturbative tachyonic gluon mass, as well as with monopoles and vortices; we can only refer to a small sample of that literature [90]. Holographically, effects associated with dimension 2 corrections would appear to be associated with tachyonic physics. Interestingly, it has been proposed that the UV renormalon is associated with a nonvanishing value of $\langle A^2 \rangle_{min}$ where the minimization is over gauge choices [91]. Such a dimension 2 condensate could plausibly be associated holographically to a closed string tachyon (one that saturates the Breitenlohner-Freedman bound [92]), though it does contradict the picture of holographic fields as being associated to *gauge-invariant, local* operators in the field theory. The minimum value of the A^2 condensate can be expressed in terms of *nonlocal*, gauge-invariant operators, beginning with $F_{\mu\nu}(D^2)^{-1}F^{\mu\nu}$ [93]. The nonlocality of this operator is perhaps suggestive of the long-distance, stringy effects of the flux tube needed to describe excited hadrons.

All of these hints of the importance of Q^{-2} corrections suggest that we take the idea of a closed string tachyon seriously, despite the lack of a gauge-invariant local operator for it to match to. Perhaps this indicates that certain 5D corrections can somehow be "resummed" into the effects of a tachyonic field. The linear confining potential of QCD has been suggested to relate to an $\exp cz^2$ background in 5D [66]. We would like to have a dynamical solution incorporating both asymptotic freedom in the UV and linear confinement in the IR. We have observed that the cosmological constant does not destroy the existence of a linear dilaton solution at large z (up to small corrections). This suggests that if we have a theory with a quadratic tachyon profile at zero cosmological constant, such a solution may persist in the presence of a cosmological constant.

The action of the bosonic noncritical string theory including the leading α' corrections, using a sigma model approach, is (after transforming to Einstein frame), according to [94]:

$$\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left[e^{\frac{4}{3}\Phi}\frac{4}{\alpha'}(\lambda-1)-2\mathcal{R}+\frac{8}{3}(\partial\Phi)^2+e^{-t}\left(\frac{4}{\alpha'}(1+t-\frac{1}{2}\lambda)e^{\frac{4}{3}\Phi}+\mathcal{R}\right.\right.$$
$$\left.\left.-\frac{4}{3}(\partial\Phi)^2-\frac{4}{3}\partial\Phi\partial t+(\partial t)^2\right)\right]. \tag{3.83}$$

To this action we are adding a cosmological constant (which we assume could come from a 5-form flux) and adjust the parameters such that with the $t\to 0$ substitution we recover the action considered in the previous section. The resulting action is:

$$\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left[e^{\frac{4}{3}\Phi}\frac{12}{R^2}\frac{\lambda-1}{\lambda}+\frac{12}{R^2}-2\mathcal{R}+\frac{8}{3}(\partial\Phi)^2+e^{-t}\left(\frac{12}{R^2\lambda}(1+t-\frac{1}{2}\lambda)e^{\frac{4}{3}\Phi}+\mathcal{R}\right.\right.$$
$$\left.\left.-\frac{4}{3}(\partial\Phi)^2-\frac{4}{3}\partial\Phi\partial t+(\partial t)^2\right)\right]. \tag{3.84}$$

The resulting equations of motion for the metric ansatz $ds^2 = e^{2A(z)}(dx^2-dz^2)$

are:

$$\frac{4}{3}e^{2A}e^{\frac{4}{3}\Phi}\frac{12}{R^2\lambda}(\lambda-1)+\frac{16}{3}(3A'\Phi'+\Phi'')+e^{-t}\Bigg(e^{\frac{4}{3}\Phi+2A}\frac{12}{R^2}\frac{1}{\lambda}(1+t-\frac{1}{2}\lambda)\frac{4}{3}+\frac{8}{3}t'\Phi'$$
$$-8A'\Phi'-\frac{8}{3}\Phi''+\frac{4}{3}t'^2-4A't'\Bigg)=0 \qquad (3.85)$$
$$\frac{12}{R^2\lambda}e^{\frac{4}{3}\Phi+2A}(\frac{\lambda}{2}-t)-12A'^2-8A''-\frac{4}{3}\Phi'^2-t'^2-4A'\Phi'-\frac{4}{3}\Phi''+2t''+6A't'=0 \qquad (3.86)$$

$$-6A'^2(2-e^{-t})+\frac{6}{R^2}e^{2A}(1+\frac{1}{\lambda}e^{\frac{4}{3}\Phi}(\lambda-1+e^{-t}(1-\frac{\lambda}{2}+t))+\Phi'^2(\frac{2}{3}-\frac{4}{3}e^{-t})+$$
$$\frac{1}{2}e^{-t}(-\frac{4}{3}\Phi't'+t'^2)=0. \qquad (3.87)$$

Here $\lambda=(26-D)/3=21/3$. One can show that for large z the leading order solution to these equations is given by

$$\Phi\rightarrow\Phi_0,\quad A\rightarrow A_0,\quad t\rightarrow-\frac{3}{\lambda}e^{2A_0+\frac{4}{3}\Phi_0}z^2+\frac{1}{2}(\lambda-2). \qquad (3.88)$$

In order to serve our purposes there must be a solution interpolating between logarithmic running on AdS in the UV and the above confining background (with flat space and constant dilaton) in the IR. It would be interesting to numerically study these equations, as well as similar systems in the type 0 string. The IR solution above has the property that the e^{-t} terms are *growing* at large z, whereas the proposed background in Ref. [66] has an exponential *shutoff* of the metric, which is not a solution of the above equations. However, the sign does not appear to affect the existence of Regge physics. Regardless, we intend these remarks not as a definitive statement, but as a provocative hint that further studies of tachyon dynamics could be useful for understanding QCD.

3.7 Conclusions and Outlook

In this paper we have clarified a number of aspects of AdS/QCD, and sketched a concrete program for computing in the holographic model and estimating associated errors. We have also explained that deep IR physics, associated with high radial excitations or large angular momentum, is troublesome in this framework. The underlying reason is that the OPE does not reproduce the quadratic corrections associated with this physics. We have suggested that a closed string tachyon can reproduce much of the underlying dynamics, but at this point that is a toy model and it is not clear how to systematically apply the idea to computations.

Our results suggest a number of directions for new work. One obvious task is to extend these results to theories with flavor. This should be straightforward, although there are potential numerical difficulties. It would be particularly interesting to see if one can obtain results for mixing of glueballs with $q\bar{q}$ mesons without large associated uncertainties.

Another direction is to make more explicit the connection between the renormalization group and the holographic dual. The 5D action has an interpretation as the 4D generating functional $W[J]$, whereas some numerical attempts involving truncations of exact RGEs have focused on computing the Legendre transform of this quantity, $\Gamma[\phi]$. There are subtle issues of nonperturbative gauge-invariant regulators that must be considered in such studies, but it is conceivable that such existing work could be reinterpreted as a computation of background profiles for various fields, about which we could then compute the spectrum of excitations and couplings with the usual 5D techniques. We hope to clarify this relationship in a future paper. It is also interesting in this context to think about how the holographic renormalization group might relate to the "analytic perturbation theory" framework.

Finally, the intricate story relating $1/q^2$ corrections, UV renormalons, vortices, linear confinement, and the closed string tachyon is very interesting and still rather poorly understood. A better understanding of these quantities and their relationships could elucidate the confining dynamics of QCD, and also shed light on closed string tachyon condensation in general (with possible applications to cosmology).

Chapter 4

The S-parameter in Holographic Technicolor Models

4.1 Introduction

One of the outstanding problems in particle physics is to understand the mechanism of electroweak symmetry breaking. Broadly speaking, models of natural electroweak symmetry breaking rely either on supersymmetry or on new strong dynamics at some scale near the electroweak scale. However, it has long been appreciated that if the new strong dynamics is QCD-like, it is in conflict with precision tests of electroweak observables [6]. Of particular concern is the S parameter. It does not violate custodial symmetry; rather, it is directly sensitive to the breaking of SU(2). As such, it is difficult to construct models that have S consistent with data, without fine-tuning.

The search for a technicolor model consistent with data, then, must turn to non-QCD-like dynamics. An example is "walking" [95], that is, approximately conformal dynamics, which can arise in theories with extra flavors. It has been argued that such nearly-conformal dynamics can give rise to a suppressed or even negative contribution to the S parameter [96]. However, lacking nonperturbative calculational tools, it is difficult to estimate S in a given technicolor theory.

In recent years, a different avenue of studying dynamical EWSB models has opened up via the realization that extra dimensional models [2] may provide a weakly coupled dual description to technicolor type theories [8]. The most studied of these higgsless models [10, 11] is based on an AdS_5 background in which the Higgs is localized on the TeV brane and has a very large VEV, effectively decoupling from the physics. Unitarization is accomplished by gauge KK modes, but this leads to a tension: these KK modes cannot be too heavy or perturbative unitarity is lost, but if they are too light then there are difficulties with electroweak precision: in particular, S is large and positive [18]. In this argument the fermions are assumed to be elementary in the $4D$ picture (dual to them being localized on the Planck brane). A possible way out is to assume that the direct contribution of the EWSB dynamics to the S-parameter are compensated by contributions to the fermion-gauge boson vertices [97, 98]. In particular, there exists a scenario where the fermions are partially composite in which $S \approx 0$ [37, 38, 39], corresponding to almost flat wave functions for the fermions along the extra dimension. The price of this cancellation is a percent level tuning in the Lagrangian parameter determining the shape of the fermion wave functions. Aside from the tuning itself, this is also undesirable because it gives the model-builder very little freedom in addressing flavor problems: the fermion profiles are almost completely fixed by consistency with electroweak precision.

While Higgsless models are the closest extra-dimensional models to traditional technicolor models, models with a light Higgs in the spectrum do not require light gauge KK modes for unitarization and can be thought of as composite Higgs models. Particularly appealing are those where the Higgs is a pseudo-Nambu-Goldstone boson [53, 41, 99, 100]. In these models, the electroweak constraints are less strong, simply because most of the new particles are heavy. They still have a positive S, but it can be small enough to be consistent with data. Unlike the Higgsless models where one is forced to delocalize the fermions, in these models with a higher scale the fermions can be peaked near the UV brane so that flavor issues can be addressed.

Recently, an interesting alternative direction to eliminating the S-parameter constraint has been proposed in [101]. There it was argued, that by considering holographic models of EWSB in more general backgrounds with non-trivial profiles of a bulk Higgs field one could achieve $S < 0$. The aim of this paper is to investigate the feasibility of this proposal. We will focus on the direct contribution of the strong dynamics to S. In particular, we imagine that the SM fermions can be almost completely elementary in the $4D$ dual picture, corresponding to them being localized near the UV brane. In this case, a negative S would offer appealing new prospects for model-building since such values of S are less constrained by data than a positive value [102]. Unfortunately we find that the $S > 0$ quite generally, and that backgrounds giving negative S appear to be pathological.

The outline of the paper is as follows. We first present a general plausibility argument based purely on 4D considerations that one is unlikely to find models where $S < 0$. This argument is independent from the rest of the paper, and the readers interested in the holographic considerations may skip directly to section 4.3. Here we first review the formalism to calculate the S parameter in quite general models of EWSB using an extra dimension. We also extend the proof of $S > 0$ for BC breaking [18] in arbitrary metric to the case of arbitrary *kinetic functions* or localized kinetic mixing terms. These proofs quite clearly show that no form of boundary condition breaking will result in $S < 0$. However, one may hope that (as argued in [101]) one can significantly modify this result by using a bulk Higgs with a profile peaked towards the IR brane to break the electroweak symmetry. Thus, in the crucial section 4.4, we show that $S > 0$ for models with bulk breaking from a scalar VEV as well. Since the gauge boson mass is the lowest dimensional operator sensitive to EWSB one would expect that this is already sufficient to cover all interesting possibilities. However, since the Higgs VEV can be very strongly peaked, one may wonder if other (higher dimensional) operators could become important as well. In particular, the kinetic mixing operator of L, R after Higgs VEV insertion would be a direct contribution to S. To study the effect of this operator in section 4.5, it is shown that the bulk mass term for axial field can be converted to kinetic functions as well, making a unified treatment of the effects of bulk mass terms and the effects of the kinetic mixing from the higher-dimensional operator possible. Although we do not have a general proof that $S > 0$ including the effects of the bulk kinetic mixing for a general metric and Higgs profile, in section 4.5.2 we present a detailed scan for AdS metric and for power-law Higgs vev profile using the technique of the previous section for arbitrary kinetic mixings. We find $S > 0$ once we require that the higher-dimensional operator is of NDA size, and that the theory is ghost-free. We summarize and conclude in section 4.6.

4.2 A plausibility argument for $S > 0$

In this section we define S and sketch a brief argument for its positivity in a general technicolor model. The reader mainly interested in the extra-dimensional constructions can skip this section since it is independent from the rest of the paper. However, we think it is worthwhile to try to understand why one might expect $S > 0$ on simple physical grounds. The only assumptions we will make are that we have some strongly coupled theory that spontaneously breaks $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ down to $\mathrm{SU}(2)_V$, and that at high energies the symmetry is restored. With these assumptions, $S > 0$ is plausible. $S < 0$ would require more complicated dynamics, and might well be impossible, though we cannot prove it.[1]

Consider a strongly-interacting theory with SU(2) vector current V_μ^a and SU(2) axial vector current A_μ^a. We define (where J represents V or A):

$$i \int d^4x \; e^{-iq\cdot x} \left\langle J_\mu^a(x) J_\mu^b(0) \right\rangle = \delta^{ab} \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi_J(q^2). \tag{4.1}$$

We further define the left-right correlator, denoted simply $\Pi(q^2)$, as $\Pi_V(q^2) - \Pi_A(q^2)$. In the usual way, Π_V and Π_A are related to positive spectral functions $\rho_V(s)$ and $\rho_A(s)$. Namely, the Π functions are analytic functions of q^2 everywhere in the complex plane except for Minkowskian momenta, where poles and branch points can appear corresponding to physical particles and multi-particle thresholds. The discontinuity across the singularities on the $q^2 > 0$ axis is given by a spectral function. In particular, there is a dispersion relation

$$\Pi_V(q^2) = \frac{1}{\pi} \int_0^\infty ds \; \frac{\rho_V(s)}{s - q^2 + i\epsilon}, \tag{4.2}$$

with $\rho_V(s) > 0$, and similarly for Π_A.

Chiral symmetry breaking establishes that $\rho_A(s)$ contains a term $\pi f_\pi^2 \delta(s)$. This is the massless particle pole corresponding to the Goldstone of the spontaneously broken SU(2) axial flavor symmetry. (The corresponding pions, of course, are eaten once we couple the theory to the Standard Model, becoming the longitudinal components of the $W^\pm$ and Z bosons. However, for now we consider the technicolor sector decoupled from the Standard Model.) We define a subtracted correlator by $\bar{\Pi}(q^2) = \Pi(q^2) + \frac{f_\pi^2}{q^2}$ and a subtracted spectral function by $\bar{\rho}_A(s) = \rho_A(s) - \pi f_\pi^2 \delta(s)$. Now, the S parameter is given by

$$S = 4\pi \bar{\Pi}(0) = 4 \int_0^\infty ds \; \frac{1}{s} \left(\rho_V(s) - \bar{\rho}_A(s)\right). \tag{4.3}$$

[1]For a related discussion of the calculation of S in strongly coupled theories, see [103].

Interestingly, there are multiple well-established nonperturbative facts about $\Pi_V - \Pi_A$, but none are sufficient to prove that $S > 0$. There are the famous Weinberg sum rules [105]

$$\frac{1}{\pi}\int_0^\infty ds\ (\rho_V(s) - \bar{\rho}_A(s)) \ = \ f_\pi^2, \tag{4.4}$$

$$\frac{1}{\pi}\int_0^\infty ds\ s\,(\rho_V(s) - \bar{\rho}_A(s)) \ = \ 0. \tag{4.5}$$

Further, Witten proved that $\Sigma(Q^2) = -Q^2(\Pi_V(Q^2) - \Pi_A(Q^2)) > 0$ for all Euclidean momenta $Q^2 = -q^2 > 0$ [106]. However, the positivity of S seems to be more difficult to prove.

Our plausibility argument is based on the function $\Sigma(Q^2)$. In terms of this function, $S = -4\pi\Sigma'(0)$. (Note that in $\Sigma(Q^2)$ the $1/Q^2$ pole from Π_A is multiplied by Q^2, yielding a constant that does not contribute when we take the derivative. Thus when considering Σ we do not need to subtract the pion pole as we did in $\bar{\Pi}$.) We also know that $\Sigma(0) = f_\pi^2 > 0$. On the other hand, we know something else that is very general about theories that spontaneously break chiral symmetry: at very large Euclidean Q^2, we should see symmetry restoration. More specifically, we expect behavior like

$$\Sigma(Q^2) \to \mathcal{O}\left(\frac{1}{Q^{2k}}\right), \tag{4.6}$$

where k is associated with the dimension of some operator that serves as an order parameter for the symmetry breaking. (In some 5D models the decrease of $\Pi_A - \Pi_V$ will actually be *faster*, e.g. in Higgsless models one has exponential decrease.) While we are most familiar with this from the OPE of QCD, it should be very general. If a theory did not have this property and Π_V and Π_A differed significantly in the UV, we would not view it as a spontaneously broken symmetry, but as an explicitly broken one. Now, in this context, positivity of S is just the statement that, because $\Sigma(Q^2)$ begins at a positive value and eventually becomes very small, the smoothest behavior one can imagine is that it simply decreases monotonically, and in particular, that $\Sigma'(0) < 0$ so that $S > 0$.[2] The alternative would be that the chiral symmetry breaking effects push $\Sigma(Q^2)$ in different directions over different ranges of Q^2. We have not proved that this is impossible in arbitrary theories, but it seems plausible that the simpler case is true, namely that chiral symmetry restoration always acts to decrease $\Sigma(Q^2)$ as we move to larger Q^2. Indeed, we will show below that in a wide variety of perturbative holographic theories S is positive.

4.3 Boundary-effective-action approach to oblique corrections. Simple cases with boundary breaking

In this section we review the existing results and calculational methods for the electroweak precision observables (and in particular the S-parameter) in holographic models of electroweak symmetry breaking. There are two equivalent formalisms for calculating these parameters. One is using the on-shell wave function of the W/Z bosons [51, 22], and the electroweak observables are calculated from integrals over the extra dimension involving these wave functions. The advantage of this method is that since it uses the physical wave functions it is easier to find connections to the Z and the KK mass scales. The alternative formalism proposed by Barbieri, Pomarol and Rattazzi [18] (and later extended in [24] to include observables off the Z-pole) uses the method of the boundary effective action [42], and involves off-shell wave functions of the boundary fields extended into the bulk. This latter

[2]For a related discussion of the behaviour of $\Sigma\left(Q^2\right)$ in the case of large-N_c QCD, see [104].

method leads more directly to a general expression of the electroweak parameters, so we will be applying this method throughout this paper. Below we will review the basic expressions from [18].

A theory of electroweak symmetry breaking with custodial symmetry has an $SU(2)_L\times SU(2)_R$ global symmetry, of which the $SU(2)_L\times U(1)_Y$ subgroup is gauged (since the S-parameter is unaffected by the extra $B-L$ factor we will ignore it in our discussion). At low energies, the global symmetry is broken to $SU(2)_D$. In the holographic picture of [18] the elementary $SU(2)\times U(1)$ gauge fields are extended into the bulk of the extra dimension. The bulk wave functions are determined by solving the bulk EOM's as a function of the boundary fields, and the effective action is just the bulk action in terms of the boundary fields.

In order to first keep the discussion as general as possible, we use an arbitrary background metric over an extra dimension parametrized by $0 < y < 1$, where $y = 0$ corresponds to the UV boundary, and $y = 1$ to the IR boundary. In order to simplify the bulk equations of motion it is preferential to use the coordinates in which the metric takes the form [3] [18]

$$ds^2 = e^{2\sigma}dx^2 + e^{4\sigma}dy^2 \ . \tag{4.7}$$

The bulk action for the gauge fields is given by

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left((F^L_{MN})^2 + (F^R_{MN})^2\right). \tag{4.8}$$

The bulk equations of motion are given by

$$\partial_y^2 A_\mu^{L,R} - p^2 e^{2\sigma} A_\mu^{L,R} = 0, \tag{4.9}$$

or equivalently the same equations for the combinations $V_\mu, A_\mu = (A_{\mu L} \pm A_{\mu R})/\sqrt{2}$.

[3] In this paper, we use a $(- + \ldots +)$ signature. 5D bulk indices are denoted by capital Latin indices while we use Greek letters for 4D spacetime indices. 5D indices will be raised and lowered using the 5D metric while the 4D Minkowski metric is used for 4D indices.

We assume that the (light) SM fermions are effectively localized on the Planck brane and that they carry their usual quantum numbers under $SU(2)_L \times U(1)_Y$ that remains unbroken on the UV brane. The values of these fields on the UV brane have therefore a standard couplings to fermion and they are the 4D interpolating fields we want to compute an effective action for. This dictates the boundary conditions we want to impose on the UV brane

$$A_\mu^{L\,a}(p^2,0) = \bar{A}_\mu^{L\,a}(p^2),\ A_\mu^{R\,3}(p^2,0) = \bar{A}_\mu^{R\,3}(p^2),\ A_\mu^{R\,1,2}(p^2,0) = 0. \tag{4.10}$$

$A_R^{1,2}$ are vanishing because they correspond to ungauged symmetry generators. The solutions of the bulk equations of motion satisfying these UV BC's take the form

$$V_\mu(p^2,y) = v(y,p^2)\bar{V}_\mu(p^2),\ A_\mu(p^2,y) = a(y,p^2)\bar{A}_\mu(p^2). \tag{4.11}$$

where the interpolating functions v and a satisfy the bulk equations

$$\partial_y^2 f(y,p^2) - p^2 e^{2\sigma} f(y,p^2) = 0 \tag{4.12}$$

and the UV BC's

$$v(0,p^2) = 1,\ a(0,p^2) = 1. \tag{4.13}$$

The effective action for the boundary fields reduces to a pure boundary term since by integrating by parts the bulk action vanishes by the EOM's:

$$\mathcal{S}_{\mathit{eff}} = \frac{1}{2g_5^2}\int d^4x (V_\mu \partial_y V^\mu + A_\mu \partial_y A^\mu)|_{y=0} = \frac{1}{2g_5^2}\int d^4p (\bar{V}_\mu^2 \partial_y v + \bar{A}_\mu^2 \partial_y a)|_{y=0} \tag{4.14}$$

And we obtain the non-trivial vacuum polarizations for the boundary vector fields

$$\Sigma_V(p^2) = -\frac{1}{g_5^2}\partial_y v(0,p^2),\ \Sigma_A(p^2) = -\frac{1}{g_5^2}\partial_y a(0,p^2). \tag{4.15}$$

The various oblique electroweak parameters are then obtained from the momentum expansion of the vacuum polarizations in the effective action,

$$\Sigma(p^2) = \Sigma(0) + p^2\Sigma'(0) + \frac{p^4}{2}\Sigma''(0) + \ldots \tag{4.16}$$

For example the S-parameter is given by

$$S = 16\pi\Sigma'_{3B}(0) = 8\pi(\Sigma'_V(0) - \Sigma'_A(0)). \tag{4.17}$$

A similar momentum expansion can be performed on the interpolating functions v and a: $v(y,p^2) = v^{(0)}(y) + p^2 v^{(1)}(y) + \ldots$, and similarly for a. The S-parameter is then simply expressed as

$$S = -\frac{8\pi}{g_5^2}(\partial_y v^{(1)} - \partial_y a^{(1)})|_{y=0}. \tag{4.18}$$

The first general theorem was proved in [18]: for the case of boundary condition breaking in a general metric, $S \geq 0$. The proof uses the explicit calculation of the functions $v^{(n)}, a^{(n)}, n = 0, 1$. First, the bulk equations (4.9) write

$$\partial_y^2 v^{(0)} = \partial_y^2 a^{(0)} = 0,\ \partial_y^2 v^{(1)} = e^{2\sigma} v^{(0)},\ \partial_y^2 a^{(1)} = e^{2\sigma} a^{(0)}. \tag{4.19}$$

And the p^2-expanded UV BC's are

$$v^{(0)} = a^{(0)} = 1, v^{(1)} = a^{(1)} = 0 \text{ at } y = 0 \tag{4.20}$$

Finally, we need to specify the BC's on the IR brane that correspond to the breaking $SU(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_D$

$$\partial_y V_\mu = 0,\ A_\mu = 0, \tag{4.21}$$

which translates into simple BC's for the interpolating functions

$$\partial_y v^{(n)} = a^{(n)} = 0,\ n = 0, 1. \tag{4.22}$$

The solution of these equations are $v^{(0)} = 1, a^{(0)} = 1 - y, v^{(1)} = \int_0^y dy' \int_0^{y'} dy'' e^{2\sigma(y'')} - y \int_0^1 dy' e^{2\sigma(y')}$, $a^{(1)} = \int_0^y dy' \int_0^{y'} dy'' e^{2\sigma(y'')}(1 - y'') - y \int_0^1 dy' \int_0^{y'} dy'' e^{2\sigma(y'')}(1 - y'')$. Consequently

$$S = \frac{8\pi}{g_5^2}\left(\int_0^1 dy e^{2\sigma(y)} dy - \int_0^1 dy \int_0^y dy'(1 - y') e^{2\sigma(y')}\right) \tag{4.23}$$

which is manifestly positive.

4.3.1 $S > 0$ for BC breaking with boundary kinetic mixing

The first simple generalization of the BC breaking model is to consider the same model but with an additional localized kinetic mixing operator added on the TeV brane (the effect of this operator has been studied in flat space in [18] and in AdS space in [51, 22]). The localized Lagrangian is

$$-\frac{\tau}{4g_5^2}\int d^4x\sqrt{-g}V_{\mu\nu}^2. \tag{4.24}$$

This contains only the kinetic term for the vector field since the axial gauge field is set to zero by the BC breaking. In this case the BC at $y = 1$ for the vector field is modified to $\partial_y V_\mu + \tau p^2 V_\mu = 0$. In terms of the wave functions expanded in small momenta we get $\partial_y v^{(1)} + \tau v^{(0)} = 0$. The only change in the solutions will be that now $v^{(1)'} = -\tau - \int_y^1 e^{2\sigma(y')}dy'$, resulting in

$$S = \frac{8\pi}{g_5^2}\left(\int_0^1 e^{2\sigma(y)}dy - \int_0^1 dy\int_0^y (1-y')e^{2\sigma(y')}dy' + \tau\right) \tag{4.25}$$

Thus as long as the localized kinetic term has the proper sign, the shift in the S-parameter will be positive. If the sign is negative, there will be an instability in the theory since fields localized very close to the TeV brane will feel a wrong sign kinetic term. Thus we conclude that for the physically relevant case S remains positive.

4.3.2 $S > 0$ for BC breaking with arbitrary kinetic functions

The next simple extension of the BPR result is to consider the case when there is an arbitrary y-dependent function in front of the bulk gauge kinetic terms. These could be interpreted as effects of gluon condensates modifying the kinetic terms in the IR. In this case the action is

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left(\phi_L^2(y)(F^L_{MN})^2 + \phi_R^2(y)(F^R_{MN})^2\right). \tag{4.26}$$

$\phi_{L,R}(y)$ are arbitrary profiles for the gauge kinetic terms, which are assumed to be the consequence of some bulk scalar field coupling to the gauge fields. Note that this case also covers the situation when the gauge couplings are constant but $g_{5L} \neq g_{5R}$. The only assumption we are making is that the gauge kinetic functions for L, R are strictly positive. Otherwise one could create a wave packet localized around the region where the kinetic term is negative which would have ghost-like behavior.

Due to the y-dependent kinetic terms it is not very useful to go into the V, A basis. Instead we will directly solve the bulk equations in the original basis. The bulk equations of motion for L, R are given by

$$\partial_y(\phi^2_{L,R}\partial_y A^{L,R}_\mu) - p^2 e^{2\sigma}\phi^2_{L,R}A^{L,R}_\mu = 0 \tag{4.27}$$

To find the boundary effective action needed to evaluate the S-parameter we perform the following decomposition:

$$\begin{aligned} A^L_\mu(p^2,y) &= \bar{L}_\mu(p^2)L_L(y,p^2) + \bar{R}_\mu(p^2)L_R(y,p^2), \\ A^R_\mu(p^2,y) &= \bar{L}_\mu(p^2)R_L(y,p^2) + \bar{R}_\mu(p^2)R_R(y,p^2). \end{aligned} \tag{4.28}$$

Here $\bar{L}, \bar{R}$ are the boundary fields, and the fact that we have four wave functions expresses the fact that these fields will be mixing due to the BC's on the IR brane. The UV BC's (4.10) and the IR BC's (4.21) can be written in terms of the interpolating functions as

$$\text{(UV)} \qquad L_L(0,p^2)=1,\ L_R(0,p^2)=0,\ R_L(0,p^2)=0,\ R_R(0,p^2)=1. \tag{4.29}$$

$$\text{(IR)} \qquad \begin{array}{l} L_L(1,p^2)=R_L(1,p^2),\ L_R(1,p^2)=R_R(1,p^2), \\ \partial_y(L_L(1,p^2)+R_L(1,p^2))=0,\ \partial_y(L_R(1,p^2)+R_R(1,p^2))=0. \end{array} \tag{4.30}$$

The solution of these equations with the proper boundary conditions and for small values of p^2 is rather lengthy, but straightforward. The end result is that

$$S=-\frac{8\pi}{g_5^2}\left(\phi_L^2\partial_y L_R^{(1)}+\phi_R^2\partial_y R_L^{(1)}\right)|_{y=0}=-\frac{8\pi}{g_5^2}(a_{L_R}+a_{R_L}), \tag{4.31}$$

where the constants a_{R_L} are negative as their explicit expressions show. Therefore S is positive.

4.4 $S>0$ in models with bulk Higgs

Having shown than $S>0$ for arbitrary metric and EWSB through BC's, in this section, we switch to considering breaking of electroweak symmetry by a bulk scalar (Higgs) vev. We begin by neglecting the effects of kinetic mixing between $SU(2)_L$ and $SU(2)_R$ fields coming from higher-dimensional operator in the $5D$ theory, expecting that their effect, being suppressed by the $5D$ cut-off, is subleading. We will return to a consideration of such kinetic mixing effects in the following sections.

We will again use the metric (4.7) and the bulk action (4.8). Instead of BC breaking we assume that EWSB is caused by a bulk Higgs which results in a y-dependent profile for the axial mass term

$$-\int d^5x\sqrt{-g}\,\frac{M^2(y)}{2g_5^2}A_M^2. \tag{4.32}$$

Here M^2 is a positive function of y corresponding to the background Higgs VEV. The bulk equations of motion are:

$$(\partial_y^2 - p^2 e^{2\sigma})V_\mu = 0, \quad (\partial_y^2 - p^2 e^{2\sigma} - M^2 e^{4\sigma})A_\mu = 0. \tag{4.33}$$

On the IR brane, we want to impose regular Neumann BC's that preserve the full SU(2)$_L\times$ SU(2)$_R$ gauge symmetry

$$(IR) \;\; \partial_y V_\mu = 0, \quad \partial_y A_\mu = 0. \tag{4.34}$$

As in the previous section, the BC's on the UV brane just define the 4D interpolating fields

$$(UV) \;\; V_\mu(p^2, 0) = \bar{V}_\mu(p^2), \quad A_\mu(p^2, 0) = \bar{A}_\mu(p^2). \tag{4.35}$$

The solutions of the bulk equations of motion satisfying these BC's take the form

$$V_\mu(p^2, y) = v(y, p^2)\bar{V}_\mu(p^2), \quad A_\mu(p^2, y) = a(y, p^2)\bar{A}_\mu(p^2), \tag{4.36}$$

where the interpolating functions v and a satisfy the bulk equations

$$\partial_y^2 v - p^2 e^{2\sigma} v = 0, \quad \partial_y^2 a - p^2 e^{2\sigma} a - M^2 e^{4\sigma} a = 0. \tag{4.37}$$

As before, these interpolating functions are expanded in powers of the momentum: $v(y, p^2) = v^{(0)}(y) + p^2 v^{(1)}(y) + \ldots$, and similarly for a. The S-parameter is again given by the same expression

$$S = -\frac{8\pi}{g_5^2}(\partial_y v^{(1)} - \partial_y a^{(1)})|_{y=0}. \tag{4.38}$$

We will not be able to find general solutions for $a^{(1)}$ and $v^{(1)}$ but we are going to prove that $\partial_y a^{(1)} > \partial_y v^{(1)}$ on the UV brane, which is exactly what is needed to conclude that $S > 0$.

First at the zeroth order in p^2, the solution for $v^{(0)}$ is simply constant, $v^{(0)} = 1$, as before. And $a^{(0)}$ is the solution of

$$\partial_y^2 a^{(0)} = M^2 e^{4\sigma} a^{(0)}, \quad a^{(0)}|_{y=0} = 1, \quad \partial_y a^{(0)}|_{y=1} = 0. \tag{4.39}$$

In particular, since $a^{(0)}$ is positive at $y = 0$, this implies that $a^{(0)}$ remains positive: if $a^{(0)}$ crosses through zero it must be decreasing, but then this equation shows that the derivative will continue to decrease and can not become zero to satisfy the other boundary condition. Now, since $a^{(0)}$ is positive, the equation of motion shows that it is always concave up, and then the condition that its derivative is zero at $y = 1$ shows that it is a decreasing function of y. In particular, we have for all y

$$a^{(0)}(y) \leq v^{(0)}(y), \tag{4.40}$$

with equality only at $y = 0$.

Next consider the order p^2 terms. What we wish to show is that $\partial_y a^{(1)} > \partial_y v^{(1)}$ at the UV brane. First, let's examine the behavior of $v^{(1)}$: the boundary conditions are $v^{(1)}|_{y=0} = 0$ and $\partial_y v^{(1)}\big|_{y=1} = 0$. The equation of motion is:

$$\partial_y^2 v^{(1)} = e^{2\sigma} v^{(0)} = e^{2\sigma} > 0, \tag{4.41}$$

so the derivative of $v^{(1)}$ must increase to reach zero at $y = 1$. Thus it is negative everywhere except $y = 1$, and $v^{(1)}$ is a monotonically decreasing function of y. Since $v^{(1)}|_{y=0} = 0$, $v^{(1)}$ is strictly negative on $(0, 1]$.

For the moment suppose that $a^{(1)}$ is also strictly negative; we will provide an argument for this shortly. The equation of motion for $a^{(1)}$ is:

$$\partial_y^2 a^{(1)} = e^{2\sigma} a^{(0)} + M^2 e^{4\sigma} a^{(1)}. \tag{4.42}$$

Now, we know that $a^{(0)} < v^{(0)}$, so under our assumption that $a^{(1)} < 0$, this means that

$$\partial_y^2 a^{(1)} \leq \partial_y^2 v^{(1)}, \tag{4.43}$$

with equality only at $y = 0$. But we also know that $\partial_y v^{(1)} \partial_y a^{(1)}$ at $y = 1$, since they both satisfy Neumann boundary conditions there. Since the derivative of $\partial_y a^{(1)}$ is strictly smaller over $(0,1]$, it must start out at a higher value in order to reach the same boundary condition. Thus we have that

$$\partial_y a^{(1)}\Big|_{y=0} > \partial_y v^{(1)}\Big|_{y=0} . \tag{4.44}$$

The assumption that we made is that $a^{(1)}$ is strictly negative over the interval $(0,1]$. The reason is the following: suppose that $a^{(1)}$ becomes positive at some value of y. Then as it passes through zero it is increasing. But then we also have that $\partial_y^2 a^{(1)} = e^{2\sigma} a^{(0)} + M^2 e^{4\sigma} a^{(1)}$, and we have argued above that $a^{(0)} > 0$. Thus if $a^{(1)}$ is positive, $\partial_y a^{(1)}$ remains positive, because $\partial_y^2 a^{(1)}$ cannot become negative. In particular, it becomes impossible to reach the boundary condition $\partial_y a^{(1)} = 0$ at $y = 1$. This fills the missing step in our argument and shows that the S parameter must be positive.

In the rest of this section we show that the above proof for the positivity of S remains essentially unchanged in the case when the bulk gauge couplings for the $SU(2)_L$ and $SU(2)_R$ gauge groups are not equal. In this case (in order to get diagonal bulk equations of motion) one needs to also introduce the canonically normalized gauge fields. We start with the generic action (metric factors are understood when contracting indices)

$$\int d^5x \sqrt{-g} \left(-\frac{1}{4g_{5L}^2} (F_{MN}^L)^2 - \frac{1}{4g_{5R}^2} (F_{MN}^R)^2 - \frac{h^2(z)}{2} (L_M - R_M)^2 \right) \tag{4.45}$$

To get to a canonically normalized diagonal basis we redefine the fields as

$$\tilde{A} = \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} (L - R) \ , \ \tilde{V} = \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} \left(\frac{g_{5R}}{g_{5L}} L + \frac{g_{5L}}{g_{5R}} R \right) . \tag{4.46}$$

To get the boundary effective action, we write the fields $\tilde{V}, \tilde{A}$ as

$$\tilde{A}(p^2, z) = \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} \left(\bar{L}(p^2) - \bar{R}(p^2) \right) \tilde{a}(p^2, z) \ , \tag{4.47}$$

$$\tilde{V}(p^2, z) = \frac{1}{\sqrt{g_{5L}^2 + g_{5R}^2}} \left(\frac{g_{5R}}{g_{5L}} \bar{L}(p^2) + \frac{g_{5L}}{g_{5R}} \bar{R}(p^2) \right) \tilde{v}(p^2, z) \ . \tag{4.48}$$

Here $\bar{L}, \bar{R}$ are the boundary effective fields (with non-canonical normalization exactly as in [18]), while the profiles $\tilde{a}, \tilde{v}$ satisfy the same bulk equations and boundary conditions as a, v in (4.33)–(4.35) with an appropriate replacement for $M^2 = (g_{5L}^2 + g_{5R}^2)h^2$. In terms of the canonically normalized fields, the boundary effective action takes its usual form

$$\mathcal{S}_{\text{eff}} = \frac{1}{2}\int d^4x \left(\tilde{V}\partial_y\tilde{V} + \tilde{A}\partial_y\tilde{A}\right)_{y=0}. \tag{4.49}$$

And we deduce the vacuum polarization

$$\Sigma_{L3B}(p^2) = -\frac{1}{g_{5L}^2 + g_{5R}^2}(\partial_y\tilde{v}(0,p^2) - \partial_y\tilde{a}(0,p^2)) \tag{4.50}$$

And finally the S-parameter is equal to

$$S = -\frac{16\pi}{g_{5L}^2 + g_{5R}^2}(\partial_y\tilde{v}^{(1)} - \partial_y\tilde{a}^{(1)}) \tag{4.51}$$

Since $\tilde{a}^{(n)}, \tilde{v}^{(n)}, n = 0, 1$ satisfy the same equations (4.33)–(4.35) as before, the proof goes through unchanged and we conclude that $S > 0$.

4.5 Bulk Higgs and bulk kinetic mixing

Next, we wish to consider the effects of kinetic mixing from higher-dimensional operator in the bulk involving the Higgs VEV – as mentioned earlier, this kinetic mixing is suppressed by the $5D$ cut-off and hence expected to be a sub-leading effect. The reader might wonder why we neglected it before, but consider it now? The point is that, although the leading effect on S parameter is positive as shown above, it can be accidentally suppressed so that the *formally* sub-leading effects from the bulk kinetic mixing can be important, in particular, such effects could change the sign of S. Also, the Higgs VEV can be large, especially when the Higgs profile is "narrow" such that it approximates BC breaking, and thus the large VEV can (at least partially) compensate the suppression from the $5D$ cut-off. Of course, in this limit of BC breaking (δ-function VEV), we know that kinetic mixing gives $S < 0$ only if tachyons are present in the spectrum, but we would like to cover the cases intermediate between BC breaking limit and a broad Higgs profile as well. In this section, we develop a formalism, valid for arbitrary metric and Higgs profile, to treat the bulk mass term and kinetic mixing on the same footing and then we apply this technique to models in AdS space and with power-law profiles for Higgs VEV in the next section.

We first present a discussion of how a profile for the y-dependent kinetic term is equivalent to a bulk mass term. This is equivalent to the result [101] that a bulk mass term can be equivalent to an effective metric. However, we find the particular formulation that we present here to be more useful when we deal with the case of a kinetic mixing. Assume we have a Lagrangian for a gauge field that has a kinetic term

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\phi^2(y)F_{MN}^2 \tag{4.52}$$

We work in the axial gauge $A_5 = 0$ and again the metric takes the form (4.7). We redefine the field to absorb the function ϕ: $\tilde{A}(y) = \phi(y)A(y)$. The action in terms of the new field is then written as

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\left(e^{2\sigma}\tilde{F}_{\mu\nu}^2 + 2(\partial_y\tilde{A}_\mu)^2 + 2\frac{\phi'^2}{\phi^2}\tilde{A}_\mu^2 - 4(\partial_y\tilde{A}_\mu)\tilde{A}^\mu\frac{\phi'}{\phi}\right) \tag{4.53}$$

To see that the kinetic profile ϕ is equivalent to a mass term, we integrate by parts in the second term

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left(\tilde{F}_{MN}^2 + 2e^{-4\sigma}\frac{\phi''}{\phi}\tilde{A}_\mu^2\right) + \frac{1}{2g_5^2}\int d^4x\,\frac{\phi'}{\phi}\tilde{A}_\mu^2\Big|_0^1 \tag{4.54}$$

Thus we find that a bulk kinetic profile is equivalent to a bulk mass plus a boundary mass. The bulk equations of motion for the new variables will then be

$$\partial_y^2 \tilde{A}_\mu - e^{2\sigma} p^2 \tilde{A}_\mu - \frac{\phi''}{\phi} \tilde{A}_\mu = 0, \tag{4.55}$$

and the boundary conditions become

$$\partial_y \tilde{A}_\mu = \frac{\phi'}{\phi} \tilde{A}_\mu. \tag{4.56}$$

Note, that despite the bulk mass term, there is still a massless mode whose wave-function is simply $\phi(z)$. Now we can reverse the argument and say that a bulk mass must be equivalent to a profile for the bulk kinetic term plus a boundary mass term.

4.5.1 The general case

We have seen above how to go between a bulk mass terms and a kinetic function. We will now use this method to discuss the general case, when there is electroweak symmetry breaking due to a bulk higgs with a sharply peaked profile toward the IR brane, and the same Higgs introduces kinetic mixing between L and R fields corresponding to a higher dimensional operator from the bulk. For now we assume that the Higgs fields that breaks the electroweak symmetry is in a (2,2) of $SU(2)_L \times SU(2)_R$, with a VEV $\langle H \rangle = \mathrm{diag}(h(z), h(z))/\sqrt{2}$.[4] This Higgs profile h has dimension 3/2. The 5D action is given by

$$\int d^5x \sqrt{-g} \left[-\frac{1}{4g_5^2} \left[(F_{MN}^L)^2 + (F_{MN}^R)^2 \right] - (D_M H)^\dagger (D^M H) + \frac{\alpha}{\Lambda^2} \mathrm{Tr}(F_{MN}^L H^\dagger H F^{MN\ R}) \right]. \tag{4.57}$$

Here α is a coefficient of $\mathcal{O}(1)$ and Λ is the 5D cutoff scale, given approximately by $\Lambda \sim 24\pi^3/g_5^2$. The kinetic mixing term just generates a shift in the kinetic terms

[4] An alternative possibility would be to consider a Higgs in the (3,3) representation of $SU(2)_L \times SU(2)_R$.

of the vector and axial vector field, and we will write the bulk mass term also as a shift in the kinetic term for the axial vector field. The exact form of the translation between the two forms is given by answering the question of how to redefine the field with an action (note that m^2 has a mass dimension 3)

$$-\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left(wF_{MN}^2 + m^2 2g_5^2 A_\mu A^\mu\right) \tag{4.58}$$

to a theory with only a modified kinetic term. The appropriate field redefinition $A = \rho\tilde{A}$ will be canceling the mass term if ρ satisfies

$$\partial_y(w\partial_y\rho) = m^2 g_5^2 e^{4\sigma}\rho, \tag{4.59}$$

together with the boundary conditions $\rho'|_{y=1} = 0, \rho|_{y=0} = 1$. The relation between the new and the old expression for w will be $\tilde{w} = \rho^2 w$. The action in this case is given by

$$-\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\tilde{w}\tilde{F}_{MN}^2 + \int d^4x\frac{\tilde{w}(0)}{2g_5^2}(\partial_y\rho)\tilde{A}^2|_{y=0} \tag{4.60}$$

This last boundary term is actually irrelevant for the S-parameter: since it does not contain a derivative on the field it can not get an explicit p-dependence so it will not contribute to S, so for practical purposes this boundary term can be neglected.

With this expression we now can calculate S. For this we need the modified version of the formula from [101], where the breaking is not by boundary conditions but by a bulk Higgs. The expression is

$$S = \frac{8\pi}{g_5^2}\int_0^1 dy e^{2\sigma}(w_V - \tilde{w}_A). \tag{4.61}$$

In our case $w_V = 1 - \frac{\alpha h^2(y)2g_5^2}{\Lambda^2}$ while $\tilde{w}_A = w_A\rho^2 = (1 + \frac{\alpha h^2(y)2g_5^2}{\Lambda^2})\rho^2$.

This formula also gives another way to see that $S > 0$ in the absence of kinetic mixing, without analyzing the functions $v^{(1)}$ and $a^{(1)}$ from Section 4.4 in detail.

Without kinetic mixing, $w_V = 1$ and $\tilde{w}_A = \rho^2$, and the equation of motion for ρ is simply $\partial_y^2 \rho = m^2 g_5^2 e^{4\sigma} \rho$. In that case ρ is just the function we called $a^{(0)}$ in Section 4.4. Since we showed there that $a^{(0)} \leq 1$, we see that our expression 4.61 gives an alternative argument that $S > 0$ without kinetic mixing, because it is simply an integral of $e^{2\sigma}(1 - \rho^2) \geq 0$.

4.5.2 Scan of the parameter space for AdS backgrounds

Having developed the formalism for a unified treatment of bulk mass terms and bulk kinetic mixing, we then apply it to the AdS case with a power-law profile for the Higgs vev. Requiring (i) calculability of the 5D theory, i.e., NDA size of the higher-dimensional operator, (ii) that excited W/Z's are heavier than a few 100 GeV, and (iii) a ghost-free theory, i.e., positive kinetic terms for both V and A fields, we find that S is always positive in our scan for this model. We do not have a general proof that $S > 0$ for an arbitrary background with arbitrary Higgs profiles, if we include the effects of the bulk kinetic mixing, but we feel that such a possibility is quite unlikely based on our exhaustive scan. For this scan we will take the parametrization of the Higgs profile from [107]. Here the metric is taken as AdS space

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) , \tag{4.62}$$

where as usual $R < z < R'$. The bulk Higgs VEV is assumed to be a pure monomial in z (rather than a combination of an increasing and a decreasing function). The reason for this is that we are only interested in the effect of the strong dynamics on the electroweak precision parameters. A term in the Higgs VEV growing toward the UV brane would mean that the value of bulk Higgs field evaluated on the UV brane gets a VEV, implying that there is EWSB also by a elementary Higgs (in addition to the strong dynamics) in the $4D$ dual. We do not want to consider such a case. The form of the Higgs VEV is then assumed to be

$$v(z) = \sqrt{\frac{2(1+\beta)\log R'/R}{(1-(R/R')^{2+2\beta})}} \frac{gV}{g_5} \frac{R'}{R} \left(\frac{z}{R'}\right)^{2+\beta} , \tag{4.63}$$

where the parameter β characterizes how peaked the Higgs profile is toward the TeV brane ($\beta \to -1$ corresponds to a flat profile, $\beta \to \infty$ to an infinitely peaked one). The other parameter V corresponds to an "effective Higgs VEV", and is normalized such that for $V \to 246$ GeV we recover the SM and the KK modes decouple ($R' \to \infty$ irrespective of β). For more details about the definitions of these parameters see [107].[5]

We first numerically fix the R' parameter for every given V, β and kinetic mixing parameter α by requiring that the W-mass is reproduced. We do this approximately, since we assume the simple matching relation $1/g^2 = R\log(R'/R)/g_5^2$ to numerically fix the value of g_5, which is only true to leading order, but due to wave function distortions and the extra kinetic term will get corrected. Then, ρ can be numerically calculated by solving (4.59), and from this S can be obtained via (4.61).

We see that S decreases as we increase α. On the the hand, the kinetic function for vector field (w_V) also decreases in this limit. So, in order to find the minimal value of S consistent with the absence of ghosts in the theory, we find numerically the maximal value of α for every value of V, β for which the kinetic function of the vectorlike gauge field is still strictly positive. We then show contour plots for the minimal value of S taking this optimal value of α as a function of V, β in Figure 4.5.2. In the first figure we fix $R' = 10^{-8}$ GeV^{-1}, which is the usual choice

[5]References [108] also considered similar models.

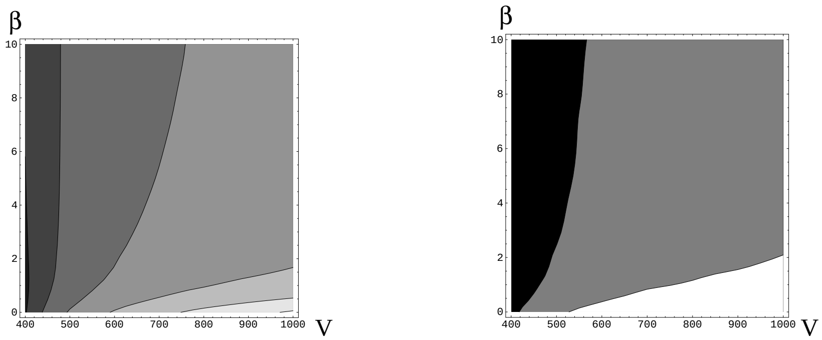

Figure 4.1: The contours of models with fixed values of the S-parameter due to the electroweak breaking sector. In the left panel we fix $1/R = 10^8$ GeV, while in the right $1/R = 10^{18}$ GeV. The gauge kinetic mixing parameter α is fixed to be the maximal value corresponding to the given V, β (and R' chosen such that the W mass is approximately reproduced). In the left panel the contours are $S = 1, 2, 3, 4, 5, 6,$ while in the right $S = 1$, 1.5, 2.

for higgsless models with light KK W' and Z' states capable of rendering the model perturbative. In the second plot we choose the more conventional value $R = 10^{-18}$ GeV^{-1}. We can see the S is positive in both cases over all of the physical parameter space.

We can estimate the corrections to the above matching relation from the wavefunction distortion and kinetic mixing as follows. The effect from wavefunction distortion is expected to be $\sim g^2 S/(16\pi)$ which is $\lesssim 10\%$ if we restrict to regions of parameter space with $S \lesssim 10$. Similarly, we estimate the effect due to kinetic mixing by simply integrating the operator over the extra dimension to find a deviation $\sim g^6 (VR')^2 \log^2 (R'/R) \,/\, (24\pi^3)^2$. So, if restrict to $V \lesssim 1$ TeV and $1/R' \gtrsim 100$ GeV, then this deviation is also small enough. We see that both effects are small due to the deviation being non-zero only near IR brane – even though it is O(1) in that region, whereas the zero-mode profile used in the matching relation is spread throughout the extra dimension.

In order to be able to make a more general statement (and to check that the neglected additional contributions to the gauge coupling matching from the wave function distortions and the kinetic mixing indeed do not significantly our results) we have performed an additional scan over AdS space models where we do not require the correct physical value of M_W to be reproduced. In this scan we then treat R' as an independent free parameter. In this case the correct matching between g and g_5 is no longer important for the sign of S, since at every place where g_5 appears it is multiplied by a parameter we are scanning over anyway (V or α).

We performed the scan again for two values of the AdS curvature, $1/R = 10^8$ and 10^{18} GeV. For the first case we find that if we restrict $\alpha < 10, 1/R' < 1$ TeV there is no case with $S < 0$. However, there are some cases with $S < 0$ for $\alpha > 10$, although in these cases the theory is likely not predictive. For $1/R = 10^{18}$ GeV we find that $S < 0$ only for $V \sim 250$ GeV and $\beta \sim 0, 1/R' \sim 1$ TeV. In this case α is of order one (for example $\alpha \sim 5$). This case corresponds to the composite Higgs model of [53, 41, 99] and it is quite plausible that at tree-level $S < 0$ if a large kinetic mixing is added in the bulk. However in this case EWSB is mostly due to a Higgs, albeit a composite particle of the strong dynamics, rather than *directly* by the strong dynamics, so it does not contradict the expectation that when EWSB is triggered directly via strong dynamics, then S is always large and positive. However, it shows that any general proof for $S > 0$ purely based on analyzing the properties of Eqs. (4.59)-(4.61) is doomed to failure, since these equations contain physical situations where EWSB is not due to the strong dynamics but due to a light Higgs in the spectrum. Thus any general proof likely needs to include more physical requirements on the decoupling of the physical Higgs.

4.6 Conclusions

In this paper, we have studied the S parameter in holographic technicolor models, focusing especially on its sign. The motivation for our study was as follows. An alternative (to SUSY) solution to the Planck-weak hierarchy involves a strongly interacting $4D$ sector spontaneously breaking the EW symmetry. One possibility for such a strong sector is a scaled-up version of QCD as in the traditional technicolor models. In such models, we can use the QCD data to "calculate" S finding $S \sim +O(1)$ which is ruled out by the electroweak precision data. Faced by this constraint, the idea of a "walking" dynamics was proposed and it can be then argued that $S < 0$ is possible which is much less constrained by the data, but the S parameter cannot be calculated in such models. In short, there is a dearth of calculable models of (non-supersymmetric) strong dynamics in $4D$.

Based on the AdS/CFT duality, the conjecture is that certain kinds of theories of strong dynamics in $4D$ are dual to physics of extra dimensions. The idea then is to construct models of EWSB in an extra dimension. Such constructions allow more possibilities for model-building, at the same time maintaining calculability if the $5D$ strong coupling scale is larger than the compactification scale, corresponding to large number of technicolors in the $4D$ dual.

It was already shown that $S > 0$ for boundary condition breaking for arbitrary metric (a proof for $S > 0$ for the case of breaking by a *localized* Higgs vev was recently studied in reference [109]). In this paper, we have extended the proof for boundary condition breaking to the case of arbitrary bulk kinetic functions for gauge fields or gauge kinetic mixing.

Throughout this paper, we have assumed that the (light) SM fermions are effectively localized near the UV brane so that flavor violation due to higher-dimensional operators in the $5D$ theory can be suppressed, at the same time allowing for a solution to the flavor hierarchy. Such a localization of the light SM fermions in the extra dimension is dual to SM fermions being "elementary", i.e., not mixing with composites from the $4D$ strong sector. It is known that the S parameter can be suppressed (or even switch sign) for a flat profile for SM fermions (or near the TeV brane) – corresponding to mixing of elementary fermions with composites in the $4D$ dual, but in such a scenario flavor issues could be a problem.

We also considered the case of bulk breaking of the EW symmetry motivated by recent arguments that $S < 0$ is possible with different effective metrics for vector and axial fields. For arbitrary metric and Higgs profile, we showed that $S > 0$ at leading order, i.e., neglecting effects from all higher-dimensional operators in the $5D$ theory (especially bulk kinetic mixing), which are expected to be sub-leading effects being suppressed by the cut-off of the $5D$ theory. We also note that boundary mass terms can generally be mimicked to arbitrary precision by localized contributions to the bulk scalar profile, so we do not expect a more general analysis of boundary plus bulk breaking to find new features. Obtaining $S < 0$ must then require either an unphysical Higgs profile or higher-dimensional operators to contribute effects larger than NDA size, in which case we lose calculability of the $5D$ theory.

To make our case for $S > 0$ stronger, we then explored effects of the bulk kinetic mixing between $SU(2)_{L,R}$ gauge fields due to Higgs vev coming from a higher-dimensional operator in the $5D$ theory. Even though, as mentioned above, this effect is expected to be sub-leading, it can nevertheless be important (especially for the sign of S) if the leading contribution to S is accidentally suppressed. Also, the large Higgs VEV, allowed for narrow profiles in the extra dimension (approaching the BC breaking limit), can compensate the suppression due to the cut-off in this operator. For this analysis, we found it convenient to convert bulk $(\text{mass})^2$ for gauge fields also to kinetic functions. Although a general proof for $S > 0$ is lacking in such a scenario, using the above method of treating the bulk mass for axial fields, we found that $S \sim +O(1)$ for AdS_5 model with power-law Higgs profile in the viable (ghost-free) and calculable regions of the parameter space.

In summary, our results combined with the previous literature strongly suggests that S is positive for *calculable* models of technicolor in $4D$ and $5D$. We also presented a plausibility argument for $S > 0$ which is valid in general, i.e., even for non-calculable models.

Chapter 5

Conclusions

Detailed conclusions have been presented in each chapter, so here we will confine ourselves to a brief summary and some general remarks about the prospects for the Randall-Sundrum framework to apply to the physics of electroweak symmetry breaking. As chapter 2 shows, it is possible to accommodate precision measurements (Peskin-Takeuchi parameters, the top mass, the $Zb\bar{b}$ coupling) in the Randall-Sundrum framework in a way that predicts new light resonances (KK W and Z bosons unitarizing WW scattering) and other interesting physics (top-pions). The model is not entirely satisfactory: it isn't completely free of strong couplings that render perturbative calculation uncertain, and it requires tuning of fermion profiles to cancel the large S parameter. The analysis of chapter 4 shows that such tuning should be generic in any five-dimensional approach to electroweak symmetry breaking. The strong dynamics seems to be always associated with a significant positive S. (It is possible that theories exist in which this is not the case, but such theories should involve large contributions from higher-dimension operators, suggesting a breakdown of calculability.) Allowing the fermions to mix strongly with composite fermions as in Reference [37] seems to be necessary to cancel such effects.

Chapter 3 examined the use of Randall-Sundrum-like frameworks as a model of QCD. By extension, any asymptotically weakly coupled technicolor-like theory would have a similar description. Much as in other studies of AdS/QCD (References [55, 62, 65]) the results are mixed. Key qualitative aspects of the physics come out correct, often to a surprisingly quantitatively accurate degree: confinement and the generation of a gluon condensate are the chief examples in our study. We showed that building a five-dimensional theory that explicitly incorporates asymptotic freedom lifts the light radion mode, as expected from general considerations. However, the mass spectrum remains very different from QCD, and adding a first example of a higher-dimension operator doesn't help. Although we have speculated on models incorporating a tachyon that might generate a more realistic spectrum, it seems clear that the real problem is that one does not expect a supergravity-like approximation with few fields to be a good model of the dual of QCD. AdS/CFT works well when the 't Hooft coupling is large and most operators get large anomalous dimensions, so that they are dual to very heavy string modes that can be integrated out. In QCD, at least over a very wide range of energies, most operators have *small* anomalous dimension, so the would-be "stringy" modes are relatively light. Only when we have full control of stringy physics in Ramond-Ramond backgrounds should we hope to be able to calculate a *true* dual of QCD.

Based on these considerations, we find ourselves in a curious position: we have interesting five-dimensional effective field theories that we can view as models of electroweak symmetry breaking. They're quite good at getting qualitative aspects of strong-coupling physics right, and we have two ways of thinking about them. On the one hand, we can view them as toy models of old-fashioned theories of technicolor based on known field theories. AdS/QCD tells us that they are good models in some ways, but not in others, and in particular not for the spectrum; still, the fact that we can do phenomenology with these models suggests that maybe there are viable technicolor field theories. The alternative point of view is to view these 5d models as theories in their own right, which are truly dual to some qualitatively new type of large 't Hooft coupling technicolor theory. The difficulty here is that we have only effective theories. Large 't Hooft coupling theories that break chiral symmetries are known in string theory (e.g. the Klebanov-Strassler theory [112]), but control of these solutions relies on supersymmetry and it is fair to say that the details of Randall-Sundrum model building (e.g. the ability to tune fermion bulk masses) are not guaranteed to be present in string theory. We also have no direct ability in field theory to construct large 't Hooft coupling, confining gauge theories. Thus, the theoretical existence of Randall-Sundrum-like technicolor theories remains unclear beyond low-energy effective field theory.

The exciting prospect, however, is that experiments could turn out to *require* such theories. Apart from the details of the mass spectrum, or the top-pion signals discussed in chapter 2, we can suggest a few general signs that might point to this. First is the roughly linear spacing of masses, as discussed in chapter 3. Along with this comes a lack of high-spin fields, since the stringy modes are parametrically heavier at large 't Hooft coupling (unlike in QCD). Yet another is that the continuum production of quarks and gluons (or other, as-yet-unknown partons) in these theories should look very different from QCD: large 't Hooft coupling suggests that we wouldn't see the characteristic jetty events arising from a parton shower, but instead more spherical events. (Recently detailed calculations of this sort of physics have been presented by Hofman and Maldacena [113].) More work remains to be done to try to sharpen the difference between Randall-Sundrum-like and old-technicolor-like theories (and to determine whether theories in some intermediate regime exist). The Large Hadron Collider turns on soon. If it discovers new strong interactions, these and other questions will become of central importance.

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